The ion velocity distribution in front of a neutralizing target

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Motivation

The understanding of the plasma-wall transition is essential for the modeling of the streaming of plasmas with direct wall contact and for the interpretation of probe data (Langmuir probes, Mach probes, etc.). Furthermore, the knowledge of the energy distribution of the ions bombarding the surface is of great importance for the choice of materials relevant to nuclear fusion.

Experiment

Laser-induced fluorescence (LIF) enables an experimental access to the problem. By scanning the wavelength of a tunable, narrow bandwidth laser, exciting the $3d^4F_9/2$ to $4p^4D_7/2$ transition at 664.3698 nm in the argon ion, and simultaneously detecting the fluorescence light from the decay to $4s^4P_5/2$ at 435 nm, the ion velocity distribution function (ivdf) can be measured non-invasively.

Fig. 1 shows the linear plasma generator PSI-2, where all measurements reported below were performed. Between the heated cylindrical cathode and the anode a stationary high current arc discharge is sustained. Here the primary plasma is produced. Confined by an axial magnetic field ($I_D \approx 100$ A) and driven by a pressure gradient it streams towards the neutralizer plate at the end of the device. A great portion of the primary plasma, however, is lost within the hollow anode region, but the fraction escaping from there is amplified due to ionization in a thin cylindrical zone extending over whole remaining part of the discharge. Due the shape of cathode and anode the plasma is maintained mainly within a hollow column of 10 cm in diameter. Typical radial profiles of density $n$ and electron temperature $T_e$ in the target chamber are shown in Fig. 2. Since the rate coefficient for ionization $R_i$ depends very sensitively on $T_e$ (according to Pots [1] $R_i = \exp(-E_i/T_e) \sqrt{T_e/E_i} \cdot 1.2 \cdot 10^{-13}$ m$^3$/s, where $E_i = 15.76$ eV) and as both, $T_e$ and $n_e$, 

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{LIF diagnostic installed at PSI-2}
\end{figure}
are peaked at a radial position of $r = 25$ mm, significant ionization takes place only in a region between $r_1 = 16$ mm and $r_2 = 33$ mm. A finite plasma density outside this source region can only be maintained by means of cross field diffusion.

The laser beam is launched axially into the target chamber where it hits the target, a circular disk (∅ = 100 mm) made of boron nitride, which is immersed in the plasma at a distance of about 80 cm to the neutralizer plate. The radial position of the laser beam is about $r = 25$ mm, i.e. in the source region, where the temperature and the (local) density were measured to be $T_e = 3.4$ eV and $n = 1.1 \cdot 10^{18}$ m$^{-3}$.

In contrast to measurements carried out three years ago [2], where the spatial resolution due to the type of laser applied was poor ($\Delta z = 50$ mm), the spatial resolution could be increased by more than a factor of 100 ($\Delta z = 0.4$ mm).

**Modeling of plasma streaming**

The stationary continuity equation in cylindrical coordinates, assuming cross field diffusion $\vec{\Gamma}_\perp = -D_\perp \nabla n$, reads

$$
\frac{1}{r \cdot n} \frac{\partial}{\partial r} D_\perp r \frac{\partial n}{\partial r} + v_i = \frac{1}{n \cdot u}(n \cdot u) =: \mu,
$$

where the ionization frequency $v_i = n_n \cdot R_i$ is assumed to be constant within the source region $r_1 \ldots r_2$ and 0 elsewhere. We assume quasi-neutrality $n = n_e = n_i$ and ambipolar fluxes, i.e. equal streaming velocities $u = u_i = u_e$. The equation is solved by separation of variables choosing $n(r,z) = R(r) \cdot Z(z)$, where $R(r)$ is composed of ordinary and modified Bessel functions

$$
R(r) = R(0) \begin{cases} 
I_0(k_\perp r) & \text{for } r \leq r_1 \\
c_1 J_0(k_S r) + c_2 Y_0(k_S r) & \text{for } r_1 < r \leq r_2 \\
c_3 K_0(k_\perp r) & \text{for } r > r_2
\end{cases}
$$

with $k_\perp = \sqrt{\mu/D_\perp}$ and $k_S = \sqrt{(v_i - \mu)/D_\perp}$. Continuity of $R$ and $R'$ at $r_1$ and $r_2$ yields 4 equations that define the constants $c_1$, $c_2$, $c_3$ and $k_S$. From the fit (thick curve in Fig.2) we determine in particular $k_\perp = 60$ m$^{-1}$ and $k_S = 67$ m$^{-1}$. We finally notice that because of $D_\perp = v_i/(k_\perp^2 + k_S^2)$ the perpendicular diffusion coefficient can be determined from the shape of the
density profile, provided the neutral density entering via \( n_i = n_0 R_i (T_e) \) is known. The remaining stationary one-dimensional continuity and momentum equations in the axial direction read

\[
\frac{\partial}{\partial z} (n \cdot u) = \mu n
\]

\[
0 = -\frac{\partial p_e}{\partial z} - enE_z
\]

\[
m_i n u \frac{\partial u}{\partial z} = -\frac{\partial p_i}{\partial z} + enE_z - m_i n u (v_i + v_{cx} + v_{el}) ,
\]

where \( E_z \) is the electric field and \( p_{e,i} \) are the pressures. \( v_{cx} = \sigma_{cx} \cdot \sqrt{2T_i/m_i} \) and \( v_{el} = \sigma_{el} \cdot \sqrt{2T_i/m_i} \) are the frequencies for charge exchange and elastic collisions. Rather than discussing the energy equation, electron and ion pressures are assumed to be isothermal, so that

\[
\frac{\partial p_e}{\partial z} = \frac{\partial n_i}{\partial z} = \frac{\partial p_i}{\partial z}.
\]

We then add Eq.(4) and Eq.(5), replace \( \frac{\partial n}{\partial z} \) by means of Eq.(3), multiply by \( u/(nm_i c_s^3) \), where \( c_s = \sqrt{(T_e + \gamma_i T_i)/m_i} \) (here \( \gamma_i = 1 \)) is the speed of sound, and to get the relation

\[
\frac{\partial M}{\partial z} = \mu \frac{1 + \alpha M^2}{c_s \left( 1 - M^2 \right)} , \quad \text{where} \quad \alpha = \frac{v_i + v_{cx} + v_{el}}{\mu} .
\]

Integrating this equation leads to an expression for \( z(M) \)

\[
z(M) = \frac{-c_s}{v_i + v_{cx} + v_{el}} \left( 1 + \frac{\alpha}{\sqrt{\alpha}} \arctan \left( \sqrt{\alpha} M \right) - M \right) + z_0 ,
\]

where \( z_0 \) is the position where \( M = 0 \) and \( \alpha \) is chosen such that the Bohm criterion \( z(M = 1) = 0 \) is satisfied. The density \( n(M) \) as a function of the Mach Number can be obtained from the equation of continuity (3), determining \( \partial n/\partial z \) and then dividing this expression by \( n \cdot \partial M/\partial z \).

Both sides of the equation obtained can be expressed by logarithmic derivatives, so that we find

\[
n(M) = n_0 (1 + \alpha M^2)^{-\frac{\alpha + 1}{2\alpha}} ,
\]

where \( n_0 \) is the density in the bulk plasma. Note that the electric field can also easily be obtained from Eq. (4), which, however, is not addressed here for the purpose of brevity.

**Measurements, evaluation and discussion**

Fig. 3 (left) shows the measured ivdfs at different distances \( z \) to the target plate. Shifted Maxwellian velocity distributions were fitted to the curves and the evaluated Mach numbers and densities are shown in the right hand graph. The points for \( M \) and \( n \) at \( z = 800 \) mm correspond to the measurement when the disk is removed, since the neutralizer plate then constitutes the target. For the sake of comparability this ivdf is shown reduced by a factor of 6.

It is observed that the plasma is accelerated to the speed of sound when approaching the target, as predicted by the Bohm criterion. The change from moderate Mach numbers of the order
Figure 3: (left): measured ivdfs in front of a target plate, (right): evaluated Mach numbers and densities

\[ M = 0.4 \text{ to } M = 1 \] takes place on a scale of about \( \lambda_M = 50 \text{ mm} \).

The data in Fig. 3 (right) can be fitted by formulae (7) and (8) very well when the parameters are chosen as \( \alpha = 170 \) and \( v_i + v_{cx} + v_{el} = 60 \text{ kHz} \). While the rate coefficient for ionization is known accurately (see above), the values for \( \sigma_{cx} \) and \( \sigma_{el} \) are less reliable. Whereas the cross sections according to Pots [1] are \( \sigma_{cx} = (6.83 - 0.78 \ln(E/\text{eV})^2) \cdot 10^{-20} \text{ m}^2 \) and \( \sigma_{el} = 0.36 \cdot \sigma_{cx} \) those for charge exchange given by Hornbeck et al. [3] \( \sigma_{cx} = 1.34 \cdot 10^{-18} \text{ m}^2 \) are a factor of about 3.5 larger. Inserting \( D_\perp = \frac{v_i}{k_i^2 + k_\perp^2} \) the quantity \( \alpha \) is linked to the ratio of collision frequencies via

\[ \alpha = \left(1 + \frac{k_i^2}{k_\perp^2}\right) \cdot \left(1 + \frac{v_{cx} + v_{el}}{v_i}\right). \]

In order to explain \( \alpha = 170 \) we have to postulate \( v_{cx} + v_{el} = 75 \), which is much higher than the predicted value of 2.87. The ionization frequency is then as low as \( v_i = \frac{v_{cx} + v_{el}}{75 + 1} = 790 \text{ Hz} \) and thus a neutral density of \( n_n = 1.04 \cdot 10^{18} \text{ m}^{-3} \), which would imply a rather high gas temperature of 3500 K. With \( v_i = 790 \text{ Hz} \), \( D_\perp = 0.1 \text{ m}^2/\text{s} \) results, which is a factor 16 smaller than previously assumed [4]. However, the large inconsistency with the published rate coefficients needs further attention. Furthermore, the assumption of a constant neutral gas density in the whole region may not be justified because of the large recycling at the target, so that \( \alpha \) may be overestimated. Apart from these open questions the experimental verification of the Bohm criterion is most the essential result of this paper.

References


