

## Operational space for bremsstrahlung radiation dominated energy of runaway electrons in tokamak plasmas

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**Abstract** The role that bremsstrahlung radiation generated when electrons are deflected by ions can play in limiting the maximum runaway energy in disruption-mitigation experiments with massive injection of high  $Z$  impurities has been recently addressed in Ref.[1]. In this contribution, an extended analysis of the dynamics in momentum-space of relativistic test runaway electrons including the acceleration in the electric field, collisions with the plasma particles as well as deceleration due to synchrotron and bremsstrahlung radiation losses has been revisited to clarify the effect of bremsstrahlung on the runaway dynamics, and to determine the space of most optimum parameters for bremsstrahlung dominated radiation. The question about the role that bremsstrahlung could play in limiting the runaway energy during disruptions will be also addressed.

**1. Introduction** During tokamak disruptions large amounts of high energy runaway electrons can be generated, which could seriously affect the lifetime of the first wall materials. Large efforts are being devoted to reduce the number and energy of runaway electrons during disruptions. The conditions of high effective ion charge and plasma density during disruption mitigation by injection of high  $Z$  impurities into the plasma point out to bremsstrahlung radiation as a loss mechanism which could significantly contribute to reduce the maximum runaway energy.

Following this line of thought, Bakhtiari *et al.* [1] recently attempted the analysis of this process by means of extending a test particle model for the runaway dynamics [2] to include the bremsstrahlung radiation losses:

$$\frac{dq_{\parallel}}{dt} = D - \gamma(\alpha + \gamma)\frac{q_{\parallel}}{q^3} - \left(F_{gc} + F_{gy}\frac{q_{\perp}^2}{q^4}\right)\gamma^4\left(\frac{v}{c}\right)^3\frac{q_{\parallel}}{q} - F_{br}\alpha\gamma\left(\ln 2\gamma - \frac{1}{3}\right)\frac{q_{\parallel}}{q}, \quad (1)$$

$$\frac{dq}{dt} = D\frac{q_{\parallel}}{q} - \frac{\gamma^2}{q^2} - \left(F_{gc} + F_{gy}\frac{q_{\perp}^2}{q^4}\right)\gamma^4\left(\frac{v}{c}\right)^3 - F_{br}\alpha\gamma\left(\ln 2\gamma - \frac{1}{3}\right) \quad (2)$$

These two equations give the rate of change of the parallel and total electron momentum (normalized to  $m_e c$ ),  $q_{\parallel}$  and  $q$ , respectively, in the normalized parallel electric field  $D = E_{\parallel}/E_R$  ( $E_R = n_e e^3 \ln \Lambda / 4\pi \epsilon_0^2 m_e c^2$  is the critical electric field for runaway generation). The equations include three different loss mechanisms: 1) collisions (second term on each equation); 2) synchrotron radiation losses (third term) and 3) bremsstrahlung radiation losses (fourth term). The remaining parameters are:  $\gamma = (1+q^2)^{1/2}$ ;  $v$  is the electron velocity, related to  $\gamma$  through the relation  $v/c = (\gamma^2 - 1)^{1/2}/\gamma$ ;  $\tau = \nu_r t$  with  $\nu_r = n_e e^4 \ln \Lambda / 4\pi \epsilon_0^2 m_e^2 c^3$ ;  $\alpha = 1 + Z$ , where  $Z$  is the effective charge,  $Z = \sum_k n_k \ln \Lambda_{ek} Z_k^2 / (n_e \ln \Lambda)$ , with  $n_k$  being the density of the  $k$ th ion with atomic number  $Z_k$ ; and  $F_{gc}$ ,  $F_{gy}$  are parameters describing the two contributions to the synchrotron radiation losses coming from the guiding

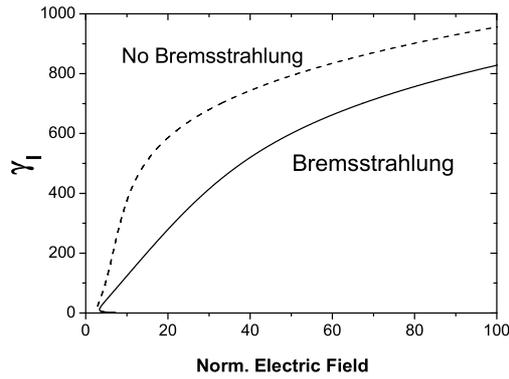


Figure 1: Comparison between the runaway energy limit  $\gamma_l$  calculated with and without including bremsstrahlung radiation losses. Plasma parameters:  $B_0 = 3$  T,  $R_0 = 3$  m,  $n_e = 10^{21} \text{ m}^{-3}$ ,  $Z = 54$  (Xe injection).

center motion and the electron gyromotion, respectively [2]:  $F_{gc} = F_{gy}(m_e c / e B_0 R_0)^2$ ,  $F_{gy} = 2\epsilon_0 B_0^2 / 3n_e \ln \Lambda m_e$ . Finally,  $F_{br} = (137\pi \ln \Lambda)^{-1}$  is the bremsstrahlung radiation parameter [1].

The essential features of the phase-space structure of the test relaxation equations [2] are retained when the bremsstrahlung loss term is included. Two singular points exist in momentum space: a saddle point, providing an estimate of the critical energy for runaway generation, and a stable focus, which gives the limiting energy that these runaways can achieve. While the saddle point (critical energy for runaway generation) is essentially unaffected by bremsstrahlung, the runaway electron energy (stable focus), for high enough values of  $n_e$  and  $Z$ , can be significantly reduced, as illustrated in Fig. 1.

**2. Conditions for bremsstrahlung dominated radiation losses** Fig. 2 shows, for the same conditions than Fig. 1, the limiting electron energy  $\gamma_l$  (calculated including both the electron synchrotron radiation as well as the bremsstrahlung radiation; full line) together with the limiting energies calculated taking into account each of the three radiative mechanisms separately (synchrotron radiation associated with the electron gyromotion,  $\gamma_{gy}$ , with the guiding center motion on toroidal paths,  $\gamma_{gc}$ , and bremsstrahlung radiation,  $\gamma_{br}$ ). It is observed that, at low electric fields ( $D < D_{gy}$ ;  $D_{gy}$  given by the intersection of the  $\gamma_l$  and  $\gamma_{gy}$  curves), the electron energy is determined by the electron gyromotion as, in this case, the collisions with the plasma particles are effective to increase enough the pitch angle to cause significant radiation ( $P_{gy} \propto \sin^2 \theta$  [2];  $P_{gy} \equiv$  power loss associated with the electron gyromotion). For intermediate electric field values, the radiation losses become dominated by bremsstrahlung, which is essentially proportional to  $\gamma_l$  while, for large enough electric fields ( $D > D_{gc}$ ;  $D_{gc}$  approximately given by the intersection of  $\gamma_l$  and  $\gamma_{gc}$ ), the electron energy is determined by the synchrotron radiation associated with the guiding center motion which increases as  $\gamma_l^4$ .

Hence, for given plasma parameters, there is a range of electric field values ( $D_{gy}, D_{gc}$ ) for which bremsstrahlung constitutes the main radiation loss mechanism and, therefore,

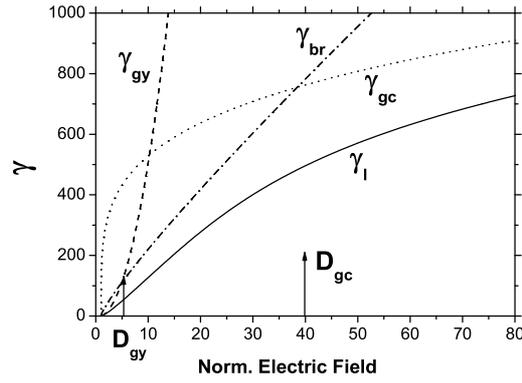


Figure 2: Runaway energy limit  $\gamma_l$  vs normalized electric field compared with the energy limits calculated assuming the radiation losses to be dominated by the electron gyromotion ( $\gamma_{gy}$ ), the guiding-center motion ( $\gamma_{gc}$ ) and the bremsstrahlung radiation ( $\gamma_{br}$ ). Plasma parameters are the same as in Fig. 1 ( $B_0 = 3$  T;  $R_0 = 3$  m;  $n_e = 10^{21}$  m $^{-3}$ ;  $Z = 54$ ).

the regimes in which bremsstrahlung dominates the electron radiation losses must satisfy  $D_{gy} \leq D_{gc}$ . The implications of this condition are illustrated in Fig. 3 in which, for given plasma parameters ( $B_0 = 3$  T,  $R_0 = 3$  m,  $Z = 18$ ),  $D_{gy}$  and  $D_{gc}$  are plotted as function of the electron density. Clearly, a minimum critical value of the density,  $n_{ec}$ , must be overcome (at which  $D_{gy} = D_{gc}$ ;  $n_{ec} \sim 3 \times 10^{20}$  m $^{-3}$  in the figure), if bremsstrahlung radiation is to become the dominant mechanism ( $D_{gy} \leq D_{gc}$ ). More precisely, at any given density  $n_e \geq n_{ec}$ , bremsstrahlung will dominate for the range of electric fields ( $D_{gy}(n_e), D_{gc}(n_e)$ ), of amplitude increasing with density (see Fig. 3). From the condition  $D_{gy}(n_{ec}) = D_{gc}(n_{ec})$ , a parametric dependence  $n_{ec} \sim B_0^{2/3} R_0^{-1/2} Z^{-1}$  is found.

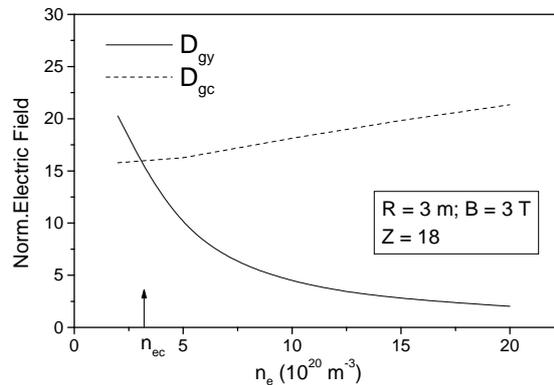


Figure 3: Normalized electric fields  $D_{gy}$  and  $D_{gc}$  vs density for  $B_0 = 3$  T,  $R_0 = 3$  m, and  $Z = 18$ . The regions for bremsstrahlung dominated radiation correspond to  $D_{gy} \leq D_{gc}$ . The critical density is also indicated ( $n_{ec} \sim 3 \times 10^{20}$  m $^{-3}$ ).

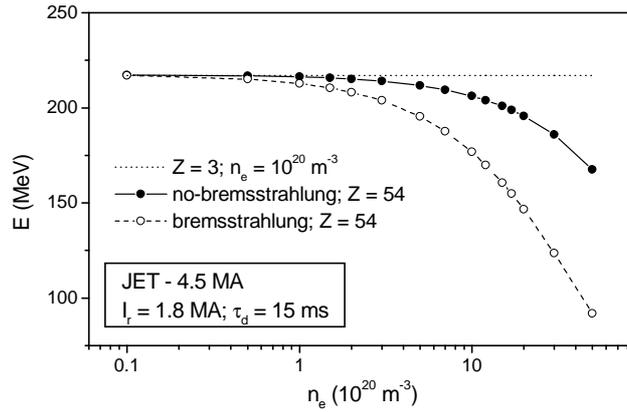


Figure 4: Runaway electron energy vs density for a 4.5 MA disruption in JET. The calculations have been performed assuming  $Z = 54$  (Xe injection) (black circles). For comparison, calculations without the assumption of impurity injection ( $Z = 3$ ;  $n_e = 10^{20} \text{ m}^{-3}$ ; horizontal line) and for  $Z = 54$  but neglecting the effect of bremsstrahlung (open circles) are also included.

In summary, it is concluded that the conditions  $n_e \geq n_{ec}$  and  $D \in (D_{gy}, D_{gc})$  must be met for bremsstrahlung radiation power to exceed the electron synchrotron losses and, therefore, to play a main role in determining the final runaway energy.

**3. Disruption generated runaways** Finally, it will be discussed if, under typical conditions of high  $Z$  and plasma density as those found during disruption mitigation by injection of high  $Z$  impurities, bremsstrahlung can significantly contribute to limit the maximum runaway energy. For simplicity, a constant average electric field during the disruption current quench will be assumed, estimated by  $E_{\parallel} \approx (L/2\pi R_0) \cdot (I_o - I_r)/\tau_d$  ( $I_o$ : pre-disruption plasma current;  $I_r$ : runaway current plateau;  $\tau_d$ : current decay time;  $L$ : plasma inductance). Eqs. (1) and (2) are then used to calculate the time evolution of the electron energy and the maximum attained energy during the current decay. An example is illustrated in Fig. 4, which shows the runaway energy vs density for a 4.5 MA JET disruption ( $R_0 = 3 \text{ MA}$ ,  $a = 1 \text{ m}$ ,  $B_0 = 3 \text{ T}$ ,  $T_e = 5 \text{ eV}$ ,  $L \simeq 4 \mu\text{H}$ ,  $\tau_d = 15 \text{ ms}$ ), with 2 MA runaway current. The results obtained assuming  $Z = 54$  (Xe injection; black circles) are shown for comparison with the calculations for "standard" disruption conditions ( $Z = 3$ ,  $n_e = 10^{20} \text{ m}^{-3}$ ; horizontal line), without the assumption of impurity injection, together with those performed for  $Z = 54$  but neglecting the bremsstrahlung contribution (open circles). The effect of bremsstrahlung is predicted to be larger for the ITER device where, due to its larger size, the electron synchrotron radiation should be lower.

## References

- [1] M. Bakhtiari et al., Phys. Plasmas **12**, 102503 (2005).
- [2] J.R. Martín-Solís et al., Phys. Plasmas **5**, 2370 (1998).