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On possibility of turbulence wave number spectra reconstruction using radial correlation reflectometry data.

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The radial correlation reflectometry (RCR), using simultaneously different frequencies for probing is often used nowadays for the tokamak turbulence characterization. The coherence decay of two scattering signals with growing difference of probing frequencies is studied in this diagnostic and applied for estimation of the turbulence radial correlation length.

However already in 1D numerical Born approximation analysis [1] a role of small angle scattering in reflectometry was shown, leading to slow decay of coherence in RCR. This effect was described in RCR linear analytic theory in 1D and slab 2D model [2-4]. It was also resolved in 1D full wave numerical modeling [5] at small level of density perturbations.

In the present paper the RCR CCF is analyzed in linear, Born approximation but, unlike [1-5], in cylinder geometry, which allows to describe the magnetic surface curvature effects. The procedure for the turbulence CCF reconstruction from the RCR CCF is proposed.

The theoretical model.

We analyze the O mode RCR using the 2D cylinder geometry model in which the microwave electric field is parallel to the magnetic field and is described by the following equation:

$$\frac{\partial^2 E}{\partial r^2}+\frac{1}{r}\frac{\partial E}{\partial r}+\frac{1}{r^2}\frac{\partial^2 E}{\partial \vartheta^2}+\frac{1}{c^2}\left\{\omega_i^2-\frac{4\pi e^2\left[n(r)+\delta n(r,\vartheta)\right]}{m_e}\right\}E=0 \qquad (1)$$

where ω_i stands for the variable probing frequency in the RCR signal channel (whereas ω_0 stands for the frequency in the reference channel); $n(r)$ is the density profile and $\delta n(r,\vartheta)$ is a random density perturbation which is given by a superposition of harmonics

$$\delta n^i(r,\theta)=\frac{n(r)}{n_0}\sum_{k,l}\delta n_{kl}\cos(\kappa_k r+q_l r\theta+\varphi_{kl}^i) \qquad (2)$$

possessing wave numbers $\kappa_k=\frac{k}{L};q_l=\frac{l}{r}$ and random phase φ_{kl}^i. The length *L*, determining the maximal fluctuation radial wave length ($2\pi L$) was taken L=135 cm. The density perturbation level is supposed to be proportional to the local density, whereas the fluctuation spectrum δn_{kl}^2 is assumed Gaussian given by the following expression:

$\delta n_{kl}^2 = \exp(-\left(\frac{kl_c}{L}\right)^2)\exp(-\left(\frac{ll_c}{r}\right)^2)$. The RCR performance modeling is performed for three of the density profile (“linear”, “convex”, “concave”) shown in figure 1 and for small (40 cm minor radius) plasma. The probing frequency in the reference channel was taken equal to 63.1 GHz. We utilize the most convenient expression for the reflectometry signal picked up by the ideal single mode antenna in linear approximation provided by the reciprocity theorem [6]

$$A_s(\omega_j) = \frac{i\omega_j\sqrt{P_i}}{16\pi}\int\frac{\delta n(r,\vartheta)}{n_c}E^2(\omega_j,r,\vartheta)rdrd\vartheta, \quad (3)$$

where $E^2(\omega_j,r,\vartheta)$ gives the unite power probing wave electric field distribution, P_i and $\left|A_s(\omega_j)\right|^2 = p_s(\omega_j)$ are correspondingly the probing and scattered power. The probing wave electric field distribution in the plasma volume is computed for $\delta n(r,\vartheta)=0$ using equation (1) under assumption that at the plasma edge it takes a form of Gaussian beam possessing plane phase front and half width y_a taking values from 1.5 to 8 cm.

The RCR cross-correlation function computations.

The normalized RCR cross-correlation function (CCF) was computed in the cylinder geometry for different density profiles, antennae diagrams and turbulence correlation length.

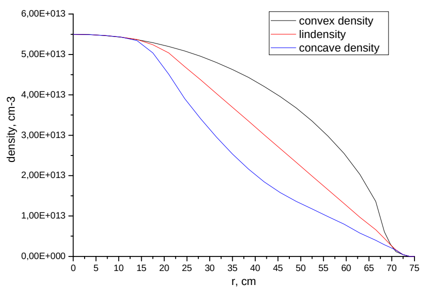

Figure 1. Density profiles.

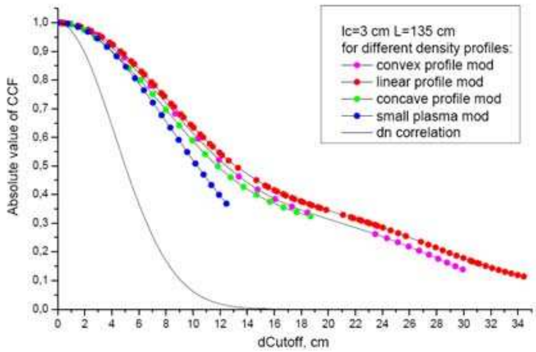

Figure 2. RCR CCF for different profiles.

The dependence of the CCF absolute value (coherence) on the cut off separation is demonstrated in Figure 2 for the three density profiles shown in Figure 1. It is important to note that all the dependencies practically coincide, being however substantially different from the density fluctuations correlation function, shown by black solid line in Figure 2. The shape of this universal function is not dependent on the antenna beam width variation leading to the transition of the cut off position from the near to the wave zone of the antenna (Figure 3). The influence of the turbulence correlation length on the CCF form is shown in Figure 4, where the CCF is plotted against the cut offs separation normalized to the turbulence correlation

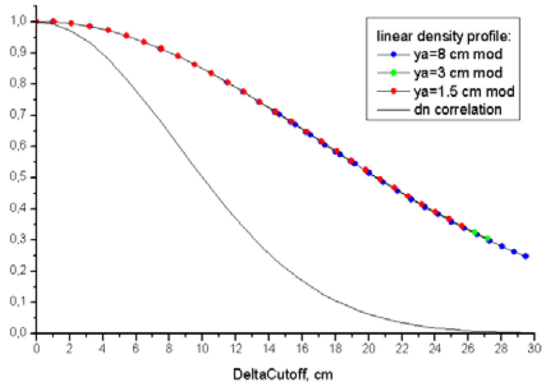

Figure 3. RCR CCF for different beam width.

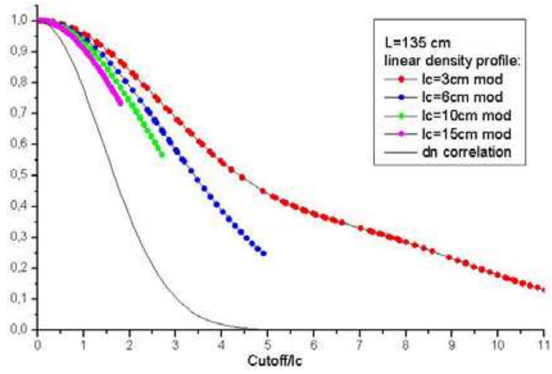

Figure 4. RCR CCF for different turbulence correlation lengths.

length. As it is seen, the slowly decaying CCF tail is decreased with the growing turbulence correlation length in agreement with predictions of analytical 2D theory [3]. Nevertheless even for the correlation length comparable to the distance from cut off to the edge, the RCR CCF is significantly different from that of the turbulence.

Wave number spectrum reconstruction algorithm.

The CCF dependencies obtained numerically in the previous section for cylinder experiment geometry possess features similar to those demonstrated analytically in 1D and 2D slab model. Universal dependence on normalized cut offs separation, different from that of the turbulence CCF, and slow coherency decay are among these essential features. These universality and difference are explained in analytics based on the WKB approximation [2, 4] by enhanced contribution of long radial scales to the reflectometry signal. Mathematically this enhancement is described by the following 1D theory [2] expression:

$$CCF_{12}(\Delta) \sim \int \frac{d\kappa}{2\pi} \frac{\tilde{n}_\kappa^2}{|\kappa|} e^{i\kappa\Delta} \left| F\left[\sqrt{\kappa x_c}\right] \right|^2 \tag{4}$$

where Δ is the cut off separation; $\tilde{n}_\kappa^2$ is the fluctuation radial wave number spectrum and $F[s] = \int_0^s \exp(i\varsigma^2) d\varsigma$ is the Fresnel integral. Strictly speaking, the model developed in [2, 4] is only applicable to long scale turbulence possessing the correlation length exceeding the so called Airy scale (providing the size of the probing wave electric field lobe at the cut off) $l_c \gg \left(c^2 x_c/\omega^2\right)^{1/3}$. Nevertheless, on one hand, this condition is satisfied in majority of reflectometry experiments and, on the other hand, the 2D theory developed in [3] in slab plasma model for linear density profile, but arbitrary turbulence scales, also confirms the conclusion on the enhanced contribution of long scales into the RCR signal and, as a consequence, into the CCF. The possible way of the turbulence wave number spectrum reconstruction, provided the CCF is measured in detail as a function of cut offs separation, is to perform its inverse Fourier transform. As a result, the turbulence wave number spectrum is reconstructed in the form

$$\tilde{n}_{\kappa}^{2} \sim \frac{|\kappa|}{\left|F\left[\sqrt{\kappa x_c}\right]\right|^2} \int e^{-i\kappa\Delta} CCF_{12}(\Delta)\, d\Delta \tag{5}$$

The proposed procedure allows accounting for the enhanced sensitivity of the reflectometry diagnostics to the long radial scales caused by the small angle scattering and provides a chance of the turbulence spectrum reconstruction based on the RCR data. It should be mentioned that the similar procedure is used for reconstruction of the turbulence spectrum from measurements performed with another localized microwave scattering diagnostic – enhanced scattering, which possesses enhanced sensitivity to small scale fluctuations [7].

Coming to discussion of feasibility of the proposed reconstruction scheme one should first of all mention the danger of small scale noise amplification due to the CCF spectrum multiplication by the radial wave number. The role of this effect may be studied during numerical testing of the proposed procedure in 1D and 2D geometry for different density profiles, turbulence spectra and accuracy of the CCF measurements.

Conclusions.

Summarizing the paper results we would like to stress that a 2D numerical study of the RCR performed in cylinder geometry have shown that the RCR CCF is a universal function of the cut off separation only weakly sensitive to the details of reflectometry experiment and plasma parameters, however different from the turbulence CCF. This difference is explained by enhancement of reflectometry scattering efficiency at small radial wave numbers. It is concluded that analytical expressions for the RCR CCF obtained in different approximations possess a typical form of Fourier transform. Based on this simple and universal representation a procedure for the turbulence spectrum reconstruction from the RCR experimental data is proposed which should be checked in future numerical modeling.

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