

# On the absolute value of the density fluctuation determined by fluctuation reflectometry: The role of the wavenumber spectrum

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## 1 Introduction

Progress in understanding turbulence and anomalous transport in a tokamak plasma requires localised measurements of plasma fluctuations, in particular the temporal dynamics and the typical length scales. On the Tore Supra tokamak and on the Wendelstein 7-AS (W7-AS) stellarator X-mode reflectometry was applied in order to measure profiles of the density and its fluctuations. Using a 1D approach the absolute value of the density fluctuation amplitude can be extracted if the Born approximation is valid [1], which connects the measured phase variation (O-mode) with the density fluctuation:

$$\langle \tilde{\phi} \rangle = k_0 \sqrt{\frac{\pi L}{k_{\text{eff}}}} \frac{\langle \tilde{n} \rangle}{n_{\text{cr}}}, \quad (1)$$

where  $\langle \rangle$  denotes the root mean square value,  $k_0$  and  $n_{\text{cr}}$  are the vacuum wavenumber and the cut-off density, respectively, for the launched frequency, and  $L$  is the gradient length of the squared index of refraction. In a turbulent plasma  $k_{\text{eff}}$  is an effective fluctuation wavenumber, which is related to the spectrum of the fluctuations. Equation (1) corresponds to an oscillation of the cut-off. In addition, local Bragg backscattering can occur far from the cut-off in the strongly fluctuating plasma edge. If the amplitude of the wave from the Bragg reflection is not negligible when compared to the amplitude of the incident wave, equation (1) is not applicable any more, i.e. the relation between  $\langle \tilde{\phi} \rangle$  and  $\langle \tilde{n} \rangle$  becomes nonlinear.

In the following we present a 1D numerical study of reflectometry in an inhomogeneous turbulent plasma. We use the Tore Supra and W7-AS plasma parameters. Starting with experimentally obtained values [2, 3] we vary the shape of the fluctuation wavenumber spectrum and the profile of the fluctuation amplitude. Finally, we discuss the validity domain of equation (1) for the analysis of the reflectometer data.

## 2 Setup of the Simulation

We use the profiles from a deuterium plasma of an Ohmic discharge in Tore Supra (figure 1) and from a hydrogen plasma heated by Neutral Beam Injection in W7-AS (figure 2). These choices are done in order to investigate the application of fluctuation reflectometry under two different conditions.

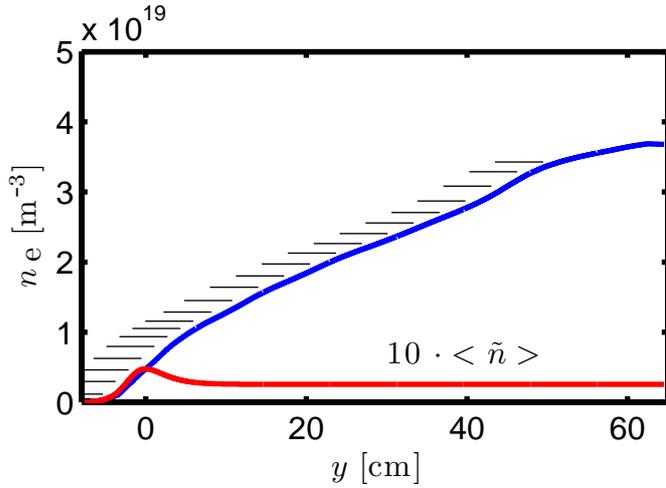


Figure 1: Density (blue) and density fluctuation (red) profile along the reflectometer line-of-sight in the Tore Supra tokamak [2]. The profile of the density fluctuation amplitude is magnified by a factor 10. The black dashes indicate the cut-off densities of the frequencies used in the simulation (frequency range 62...105 GHz in X-mode).

In the Tore Supra case the magnetic field increases from 2.2 T to 3 T along the  $y$  coordinate in figure 1, while in W7-AS the increment is from 2.2 T to 2.5 T.

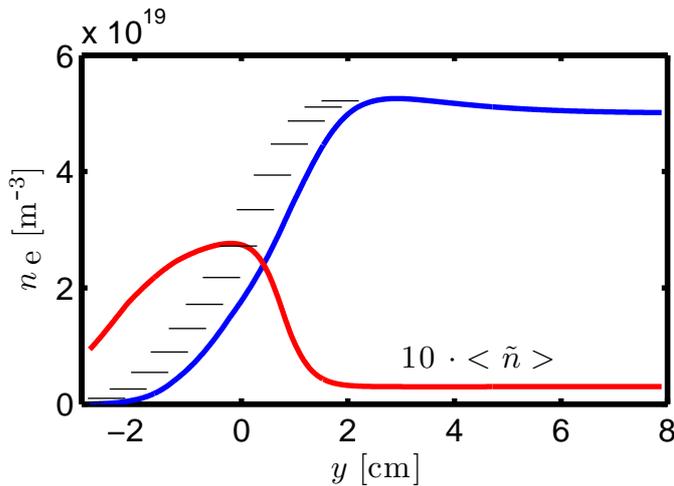


Figure 2: Density (blue) and density fluctuation (red) profile (magnified by 10) in the W7-AS [3]. Note that the plasma center is at  $y = 8$  cm. In the stellarator magnetic field,  $y$  and the effective radius from [3] are connected by a magnetic mapping technique. The fluctuation data for  $y < 0$  are based on probe measurements [4].

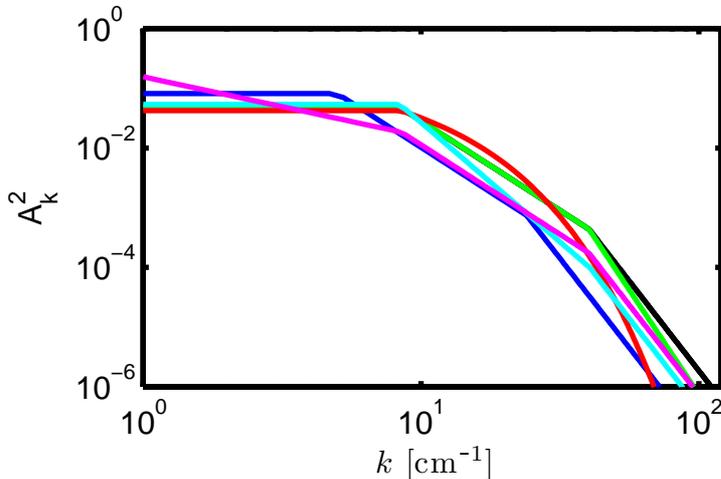


Figure 3: Input spectra to generate the plasma turbulence for W7-AS, based on the experimental data given in [5]. The location of the first shoulder ( $k \approx 8 \text{ cm}^{-1}$ ) and the spectral coefficients are varied to account for the uncertainty of the measurement. The spectrum plotted in magenta is used to demonstrate the impact of low wavenumbers.

We model the plasma fluctuations with a broadband spectrum in  $k$ -space (figure 3). Based on the investigations by Kendl [6] and Hennequin [5] we assume the location of the first shoulder at  $k\rho_i = 0.3$ , where  $\rho_i = \sqrt{m_i T_i}/q_i B$ . Hence, the spectrum can be scaled to the conditions of both machines: Assuming  $T_i = T_e$  we find in the edge plasma of Tore Supra  $\rho_i = 0.93 \text{ mm}$ , and in W7-AS  $\rho_i = 0.44 \text{ mm}$ .

Finally, the Helmholtz equation for the wave field of the launched microwave frequency is solved in the cold plasma approximation using a 4th order Numerov scheme [7].

### 3 Results

We investigate the influence of the fluctuation profile in two scans (figure 4). The first scan with flat profiles is used to determine the limit of the linear response. In the second scan we approach the experimental situation by introducing enhanced edge fluctuations.

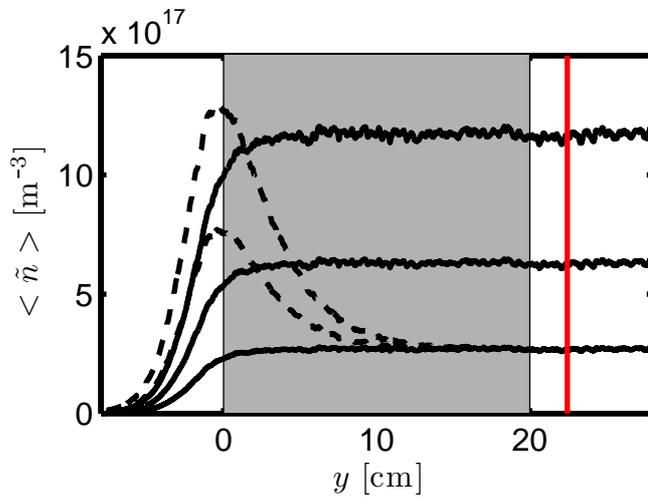


Figure 4: Variation of the fluctuation profile in Tore Supra. Solid lines: Scan of the central fluctuation level without edge fluctuations. Dashed lines: Scan of the edge fluctuation level. The location of the cut-off for the probing frequency 88.6 GHz is overplotted in red. In the grey shaded region the reflectometer measurement is nonlocal.

In figure 5 we exemplarily show the comparison of the two scans.

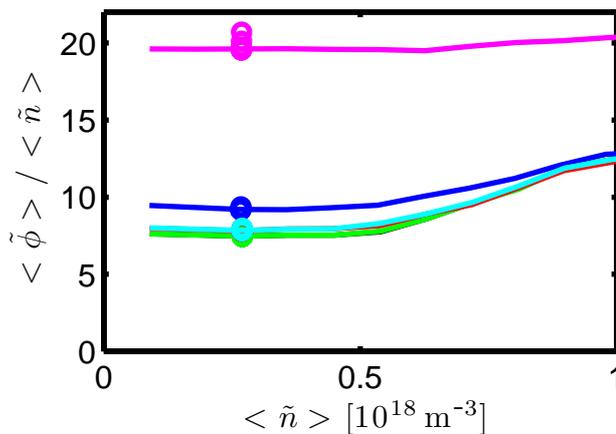


Figure 5: Ratio  $\langle \tilde{\phi} \rangle$  over  $\langle \tilde{n} \rangle$  (in  $10^{18} \text{ m}^{-3}$ ) for the situation from figure 4. Solid lines: Scan of the central fluctuation level. Circles: Scan of the edge fluctuation level. The different colors denote the different input turbulence spectra from figure 3.

Except for the two spectra with a power law of -1 below the shoulder, and with the shifted location of the shoulder, all other turbulence spectra yield basically the same result. There is a linear relation between  $\langle \tilde{\phi} \rangle$  and  $\langle \tilde{n} \rangle$  until  $\langle \tilde{n} \rangle = 5 \cdot 10^{17} \text{ m}^{-3}$ . If

the cut-off is located inside the plasma core of Tore Supra (cf. figure 4), both scans are in accordance, i.e. the measurement is local and equation (1) is valid. If the cut-off is located closer to the edge we find differences between the two scans: In the grey shaded region (figure 4) the deviations of  $\langle \tilde{\phi} \rangle / \langle \tilde{n} \rangle$  are  $> 1\%$  (given the nominal fluctuation profile from figure 1). This region is forbidden for the application of equation (1).

#### 4 Discussion and Conclusions

The simulations with different locations  $k_S$  of the shoulder in the wavenumber spectrum show that the linear relation between density and phase fluctuations follows the Born approximation, i.e. the constant of proportionality scales with  $(k_S)^{-1/2}$ . The validity range of equation (1) is from 0 to  $\langle \tilde{n} \rangle = 4 \cdot 10^{17} \text{m}^{-3}$  in the Tore Supra case and  $\langle \tilde{n} \rangle = 1.2 \cdot 10^{18} \text{m}^{-3}$  in the W7-AS case. This is robust for different frequencies.

The scan with different shapes of the fluctuation profile showed that equation (1) is not applicable in order to resolve a negative gradient of the fluctuation amplitude. The phase fluctuation is dominated by edge fluctuations and the measurement is non-local. With increasing frequency and increasing distance between the edge fluctuation peak and the cut-off position the situation improves. In the Tore Supra case a local measurement inside the plasma core and the application of equation (1) is possible (1% accuracy), if the ratio of the probing frequency relative to  $f_{\text{peak}}$  is larger than 1.25 ( $f_{\text{peak}}$  corresponds to the location of the cut-off at the peak of the fluctuation profile).

This behaviour can be explained by the Bragg backscattering condition  $k_F = 2 \cdot k(x_B)$  in conjunction with the shape of the wavenumber spectrum in plasma turbulence: The higher the launched frequency, the higher  $k_F$ . Since the spectral power decreases strongly towards high  $k$ , the nonlinear contribution from the Bragg reflection vanishes.

In W7-AS equation (1) is not applicable in the plasma core. The core fluctuation level is very small and the phase fluctuations are, therefore, always dominated by the edge. In order to recover the local fluctuation level in the region of the negative gradient, it seems to be necessary to complement the measurement with individual numerical simulations. The on-going development of the fast sweeping technique for the frequency can help to increase the accuracy of the input data on the wavenumber spectrum.

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