Tomography on Lao-Hirschman Type of Equilibria using Mode Rotation.

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Abstract. A method is described to obtain the plasma perturbations from line-integrals distributed over the plasma cross-section. The mode apparent rotation is used either by mapping time to poloidal angle on realistic plasma surfaces or by taking the Fourier components and projecting these on the poloidal plane.

Tomographic reconstructions. Reconstructions of plasma instabilities are important to furthering their understanding. Tomography using mode rotation has the advantage of giving a higher poloidal resolution than methods that use two cameras. However, the rotation has to be mapped onto realistic flux surfaces. In general tokamak configurations, such as in JET, have high ellipticity and triangularity and can be well described by the following equations:

\[
R = R_n(x) + x \cos \vartheta + \sum_{k=2}^{N} R_k(x) \cos k \vartheta + \sum_{k=1}^{N} S_k(x) \sin k \vartheta \tag{1}
\]

\[
z = z_0(x) + E(x) (x \sin \vartheta - \sum_{k=2}^{N} R_k(x) \sin k \vartheta + \sum_{k=1}^{N} S_k(x) \cos k \vartheta) \tag{2}
\]

\( \vartheta \) is the angle from the surface centre to the surface crossing point and \( x \) the surface parameter. A simpler version is the Lao-Hirschman representation with \( N = 2 \). The tomography code described here only takes into account \( R_2 \) and \( R_4 \) terms and neglects all other \( R_k \) and \( S_k \) terms. Comparison with equilibria calculated using the EFIT code generally gives very good agreement. The geometry of the lines of sight can be calculated using the equilibrium reconstruction. The crossing points of the lines of sight with the plasma surfaces are obtained in only a few iterations since \( R_k \) and \( S_k \ll x \). In this way, all line segments and angles are calculated and a tomographic inversion can be made.

If only one set of cameras is used, only 2 independent modes \([1]\) can be determined at each surface for each toroidal mode number \( n \). With 2 cameras, 4 independent modes can be analysed. The attribution of the time to poloidal angle uses the phase-locking mechanism: the period of the modes is determined by their toroidal \( n \)-number. This means that an \( n=1 \) mode has half the frequency of an \( n=2 \) mode and one third of an \( n=3 \) mode.

Application to a highly shaped plasma. The discharge taken is JET 67896, with a plasma current of \(-1.5\) MA, a toroidal field of \(-2.3\) T and a diamagnetic energy of 3.5 MJ. Total power is 17 MW NBI and 3.5 MW ICRH with a brief period of LHCD prior to the main
heating pulse of around 4 seconds. During the main heating the normalised beta $\beta_N$ reaches a value of two. Some low amplitude MHD modes ($\delta B/B < 10^{-4}$) are seen with a magnetic signature of $n=1$, $n=2$ and $n=4$ at 7 to 8, around 15 and 23 kHz. The $n=1$ mode has been analysed at its highest amplitude, when also its higher harmonics visible. The modes rotate in the ion-diamagnetic direction. A reference signal is chosen in order to map time to poloidal angle, depending on the sense of rotation either in the negative direction (for JET the ion diamagnetic one) or positive direction poloidally. A symmetric grid is set up and the fluxes and oscillations are interpolated on semi-logarithmic and linear scale respectively.

After this the data can go through the tomographic inversion, which results in the fig.2. Shown on this figure are also the positions of the radii, which form the boundary between the various odd modes and those for the various even modes. From the centre to the plasma boundary one can see the $(1,1)/(3,1)$, the $(2,1)/(4,1)$, the $(3,1)/(5,1)$, the $(4,1)/(6,1)$ and finally the plasma boundary as given by EFIT.

**Application to a circular plasma.** Fig.2 shows an example of a tomogram on a FTU discharge 29479. This discharge at 5.2 T and 0.5 MA had MHD activity introduced by Laser Blow-Off.

**Fig.2** Cross-section of the JET plasma in soft-X-ray emission allowing for various odd and even modes in the plasma shells, indicated in the fig. Also given are in good approximation the line-of-sights of the KJ5 cameras. The position of the most outer 2,1 mode is close to where EFIT calculates the $q=2$ position. The inner $(2,1)$ suggests that there may even be some inversion of the $q$-profile in the plasma centre.

**Fig.2** Starting at the plasma edge a $(4,1)$ mode at 0.27 m is present with an island width of around 1.5 cm. Further in at 0.24 m is a $(3,1)$ mode with a width of 1 cm and at 0.18 m is a 2 cm island of the $(2,1)$. At 0.06 m is an $(1,1)$ island of 1 cm width. The rings are the boundaries of the various odd and the various even modes e.g. the boundary between the $(1,1)$ and $(3,1)$ modes has been chosen based on information of the $q$-profile at 0.17 m.
This MHD activity normally leads to a major current disruption, but an ERCH power of 1MW applied at the right location [2] stabilises these modes. For this particular case modes of moderate amplitude remain present in an otherwise stable discharge.

**Tomography on Fourier components.**

In principle tomography solves a set of linear equations and hence can also be carried out on individual Fourier components. In this case, the restriction of phase-locking of the modes is no longer required and tomography can be carried out on a more general situation in which completely independent modes, like NTMs, are present. However, the modes must be present for a time longer than a single oscillating period. The measured Fourier components (marked by $i$ in the equations) are complex vectors, consisting of amplitude ($g_i$) and phase information ($\varphi_i$). The local measurements (line-integrals minus contributions of the outer surfaces) can be written as for N measurements on a particular magnetic surface:

$$g_i e^{i\varphi_i} = \sum_k \frac{\sin(m_k \Delta \vartheta_i)}{m_k \Delta \vartheta_i} f_k(\omega) e^{im_k \vartheta_i} \quad \text{with} \quad i = 1, N$$

For 2 measured fluxes and 2 modes and $\Delta \vartheta_i=0$ one readily gets the individual modes as:

$$f_k(\omega) = C_{1k} g_1(\omega) e^{-im_k \vartheta_1} + C_{2k} g_2(\omega) e^{-im_k \vartheta_2}$$

with $C_{11}=C_{22}$ and $C_{21}=C_{12}$ and the latter the complex conjugate of the former.

$$C_{11} = \frac{1}{2} \frac{1 - e^{-i\alpha}}{1 - \cos \alpha}, \quad \text{with} \quad \alpha = (m_1 - m_2)(\vartheta_1 - \vartheta_2). \quad \text{m}_1 \text{ and m}_2 \text{ are the odd and even mode pair at a given magnetic surface with tangent line-of-sights at angles } \vartheta_1 \text{ and } \vartheta_2. \quad \text{For } \vartheta_1 - \vartheta_2 = \pi \text{ the coefficients } C_{ij} \text{'s are real and equal to one half for any pair of odd and even modes. This procedure is about one hundred times faster then the period based tomogram and hence many time points can be analysed.}$$
This has been done for a discharge of FTU with 1.5 MW of LHCD and ECRH where a fairly sized (3,2) mode has been observed during the heating period. Fig. 3 and 4 show the mode amplitude and phase as function of radius and time.

Note that the large dynamics in amplitude. This is caused by the Fourier technique, which reduces the background noise. The $\pi$-shift is an indication that this mode is a resistive island with the rational q value of 1.5 at that location. It is expected that the radial variation of the position of the $\pi$-shift are indeed linked to slow radial variations in the q-profile over time.

Fig. 5. Plasma cross-section obtained by the period based tomogram at $t=0.82$ s (one of the maxima in time of the (3,2) mode activity).

Good agreement with mode position and phase inversion can be seen when comparing figures 3, 4 and 5. With the Fourier based technique one obtains the full time development.

Summary and discussion.

Using the moments-equations (Lao-Hirschman type) of the equilibrium allows for an accurate inversion of MHD structures observed by soft-X-ray cameras at high temporal resolution (one oscillating period) in a correct magnetic surface frame. With a vertical and horizontal camera one can determine 2 pairs of odd and even modes. In this way mode amplitudes, displacements and structures (resistive versus ideal modes) can be determined and of course the positions of the rational q surfaces connected with these modes.

The recently developed Fourier-based tomographic technique [3] shows promising potential of obtaining the local mode activity and structure possibly on-line. So that this technique could be used in mode feed back loops.

References: