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Kinetic Alfvén waves in a Maxwellian dusty plasma

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The propagation and damping of kinetic Alfvén waves in a dusty plasma are discussed using a kinetic formulation which takes into account the charge variation of the dust particles. The dielectric tensor for a magnetized dusty plasma, homogeneous, fully ionized, with identical immobile dust particles and charge variable in time, can be written as $\varepsilon_{ij} = \varepsilon^C_{ij} + \varepsilon^N_{ij}$, where the ε^C_{ij} indicate the contribution which is formally identical to the "conventional" components, modified by the presence of the dust particles, and the ε^N_{ij} indicate the contribution which is entirely due to the presence of the dust [1, 2, 3]. The equations for the components of a wave electric field propagating in such a plasma can be formally written as $(N_iN_j - N^2\delta_{ij} + \varepsilon_{ij})E_j = 0$, where the N_i are the components of the wave refraction vector $\mathbf{N} = c\mathbf{k}/\omega$. The dispersion relation is obtained by setting the determinant of the coefficients of this equation equal to zero.

Let us consider a magnetized plasma with magnetic field along the z direction and wave vector lying on $x-z$ plane. Let us assume that electrons and ions are described by Maxwellian distribution function, in equilibrium. For evaluation of velocity integrals which appear in the components of the dielectric tensor, we approximate the velocity- dependent inelastic collision frequency between dusty particles and ions and electrons by the average value in momentum space, $\nu_\beta = \int d^3p\,\nu^0_{\beta d}(p) f_{\beta 0}/n_{\beta 0}$, and obtain, for instance,

$$\varepsilon^C_{xx} = 1 + \sum_\beta X_\beta \zeta^0_\beta \sum_{n=0}^{+\infty} \sum_{s=\pm 1} \frac{n^2 \Gamma_n(\lambda_\beta)}{\lambda_\beta} Z(\zeta^{s,n}_\beta), \tag{1}$$

$$\varepsilon^C_{zz} = 1 + 2\sum_\beta X_\beta \sum_{n=0}^{+\infty} \sum_{s=\pm 1} \Gamma_n(\lambda_\beta) \left\{ \zeta^0_\beta \zeta^{s,n}_\beta \left[1 + \zeta^{s,n}_\beta Z(\zeta^{s,n}_\beta) \right] \right\}, \tag{2}$$

where

$$\zeta^0_\beta \equiv \frac{\omega}{\sqrt{2} k_\parallel v_{T_\beta}}, \quad \zeta^{s,n}_\beta \equiv \frac{\omega - sn\Omega_\beta + i\nu_\beta}{\sqrt{2} k_\parallel v_{T_\beta}}, \quad \Gamma_n(\lambda_\beta) \equiv e^{-\lambda_\beta} I_n(\lambda_\beta), \quad \lambda_\beta = \frac{k_\perp^2 v_{T_\beta}^2}{\Omega_\beta^2} = k_\perp^2 r_{L_\beta}^2,$$

and $X_\beta = \omega^2_{p\beta}/\omega^2$, $\nu_i = 2\sqrt{2\pi} a^2 n_{d0} v_{Ti}(1+\chi_i)$, $\nu_e = 2\sqrt{2\pi} a^2 n_{d0} v_{Te} e^{\chi_e}$, $\chi_i \equiv Z_d e^2/(aT_i)$, $\chi_e \equiv -(T_i/T_e)\chi_i$, and $v_{T_\beta} = (T_\beta/m_\beta)^{1/2}$ [3]. In these expressions, $I_n(\lambda_\beta)$ is the modified Bessel function and $Z(\zeta^{s,n}_\beta)$ is the well known plasma dispersion function. Other ε^C_{ij} components can be

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similarly obtained, and are not shown here for the sake of economy of space. These expressions for the ε^C_{ij} are formally almost identical to the ε_{ij} expressions for conventional electrons-ion plasmas, with the only difference appearing in the argument $\zeta^{s,n}_\beta$ of the dispersion function, due to the presence of the average frequency of charging collisions. For the present application, we will assume that the dust density is sufficiently small, and we will neglect the contribution of the ε^N_{ij} components, since they vanish for vanishing dust density [3].

As it is known, the kinetic Alfvén wave is the result of coupling between the shear Alfvén mode and the ion acoustic mode [4]. The kinetic Alfvén waves can be considered the extension of the shear Alfvén mode in the range of small perpendicular wavelength. Let us illustrate this point with some limiting cases. For a dustless plasma and waves with perpendicular polarization, the dispersion relation may be written as $\varepsilon^C_{xx} - N^2_\parallel = 0$. For low frequency, $\omega \ll \Omega_\beta$ and long parallel wavelength, $k_\parallel v_{T_\beta} \ll \Omega_\beta$, $|\zeta^{s,n}_\beta| \gg 1$,, $Z(\zeta^{s,n}_\beta) \approx -1/\zeta^{s,n}_\beta$, and Eq. (1) can be written as

$$\varepsilon^C_{xx} = 1 + \sum_\beta \frac{\omega^2_{p\beta}}{\Omega^2_\beta}\left[\frac{1-\Gamma_0(\lambda_\beta)}{\lambda_\beta}\right] \cong 1 + \frac{c^2}{V^2_A}\left[\frac{1-\Gamma_0(\lambda_i)}{\lambda_i}\right],$$

where we have used $(\omega^2_{pe}/\Omega^2_e) \ll (\omega^2_{pi}/\Omega^2_i) = c^2/V^2_A$, and $2\sum_{n=1}^{+\infty}\Gamma_n(\lambda_\beta) = 1-\Gamma_0(\lambda_\beta)$, with V_A being the Alfvén speed. In addition, for small ion Larmor radius, $\lambda_i \ll 1$, we obtain, for $c^2/V^2_A \gg 1$, the usual dispersion relation for shear Alfvén waves, $\omega^2 = k^2_\parallel V^2_A$.

On the other hand, the dispersion relation for ion-acoustic waves is obtained when we consider electrostatic waves propagating along the ambient magnetic field, and becomes simply $\varepsilon^C_{zz} = 0$. For $|\zeta^{s,n}_\beta| \gg 1$, $\omega \ll \Omega_\beta$, $k^2_\parallel v^2_{T_\beta}/\Omega^2_\beta \ll 1$, the Z function for the ions may be expanded in the large argument limit, and therefore the dispersion relation becomes

$$\varepsilon^C_{zz} \simeq \frac{\Gamma_0(\lambda_e)}{k^2_\parallel \lambda^2_{D_e}}\left[1+\zeta^0_e Z(\zeta^0_e)\right] - \frac{\omega^2_{pi}}{\omega^2}\Gamma_0(\lambda_i) = 0, \;\Rightarrow\; \omega^2 = 1 + \frac{c^2_s k^2_\parallel \Gamma_0(\lambda_i)}{\Gamma_0(\lambda_e)\left[1+\zeta^0_e Z(\zeta^0_e)\right] + k^2_\parallel \lambda^2_{D_e}},$$

where we have used $\omega^2_{pi}\lambda^2_{D_e} = T_e/m_i = c^2_s$. Considering the small Larmor radius limit, $\lambda_\beta \ll 1$, and $\zeta^0_e \ll 1$, we obtain the familiar form for the dispersion relation for ion-acoustic waves, $\omega^2 = c^2_s k^2_\parallel/(1+k^2_\parallel \lambda^2_{D_e})$.

For a numerical analysis of the dispersion relation, we consider as basic parameters the ion charge number $Z_i = 1.0$ and the ion mass $m_i = m_p$, where m_p is the proton mass. We choose the ambient magnetic field $B_0 = 1.0 \times 10^{-4}$ T, ion density $n_{i0} = 1.0 \times 10^9$ cm^{-3}, ion temperature $T_i = 1.0 \times 10^4$ K, and electron temperature $T_e = T_i$. For the radius of the dust particles, we assume $a = 1.0 \times 10^{-4}$ cm. The electron density and the dust charge number Z_d are obtained from the quasi-neutrality condition. In Fig. 1(a) we show the real part of the normalized wave frequency, $z_r = \omega_r/\Omega_i$, as a function of $q_z = k_z v_A/\Omega_i$, for $q_x = k_x v_A/\Omega_i = 0.0$ (parallel propagation), and three values of the dust density, $\varepsilon = n_{d0}/n_{i0} = 0.0$ (dustless plasma), 1.0×10^{-6}, and 2.5×10^{-6}. The upper lines correspond to the *whistler* branch, and the lower lines to the branch of the *circularly polarized* waves. Of course, there are corresponding curves for opposite propagating waves, not shown in the figure. It is seen that the effect of the dust is the approximation of the curves for the two modes, which is most noticeable in the region of small q_z. Fig. 1(b) shows the corresponding values of $z_i = \omega_i/\Omega_i$, the imaginary part of the normalized wave frequency. It is seen that z_i is negligible in the absence of dust, for the range of q_z depicted in the figure. As seen in Fig. 1 of Ref. [3], conventional damping starts to become significant only for $q_z \simeq 1$, for the circularly polarized waves. Fig. 1(b) also shows that the presence of dust introduces significant damping, which increases with the increase of ε. It is seen that the values of z_i for the two modes are well separated for small q_z, becoming similar for increasing q_z, in the range appearing in the figure.

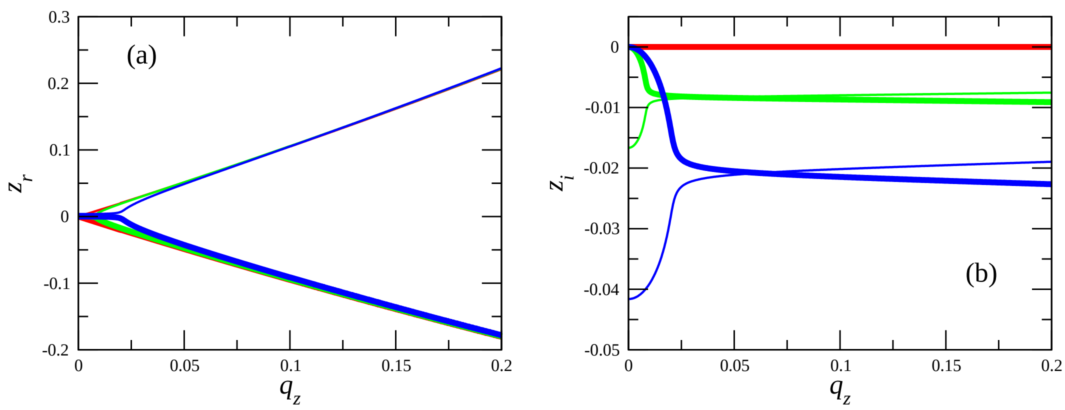

Figure 1: (a) z_r vs. q_z, for $q_x = 0.0$, with $\varepsilon = n_{d0}/n_{i0} = 0.0$ (red), 1.0×10^{-6} (green), and 2.5×10^{-6} (blue); (b) corresponding values of z_i.

In order to investigate the effect of q_x, we choose the value of $q_z = 0.06$, and show in Fig. 2(a) the values of z_r vs. q_x, for the same values of ε utilized for Fig. 1. Fig. 2(a) shows that z_r for the branch of circularly polarized waves (thick lines) is almost not affected by q_x, in the range considered, while z_r for the whistler waves (thin lines) grows with q_x, for the three values of dust density considered. Fig. 2(b) shows the corresponding values of z_i. The damping of the circularly polarized waves, caused by the dust, does not show any effect due to q_x. However, the whistler waves show the appearance of a damping which grows in magnitude with the increase of q_x, and which is added to the dust damping which was already present for $q_x = 0$. In panels (c) and (d) we show the results obtained if the effect of collisional charging is neglected in the dispersion relation by taking $\nu_\beta = 0.0$, such that the only effect of the dust is the charge

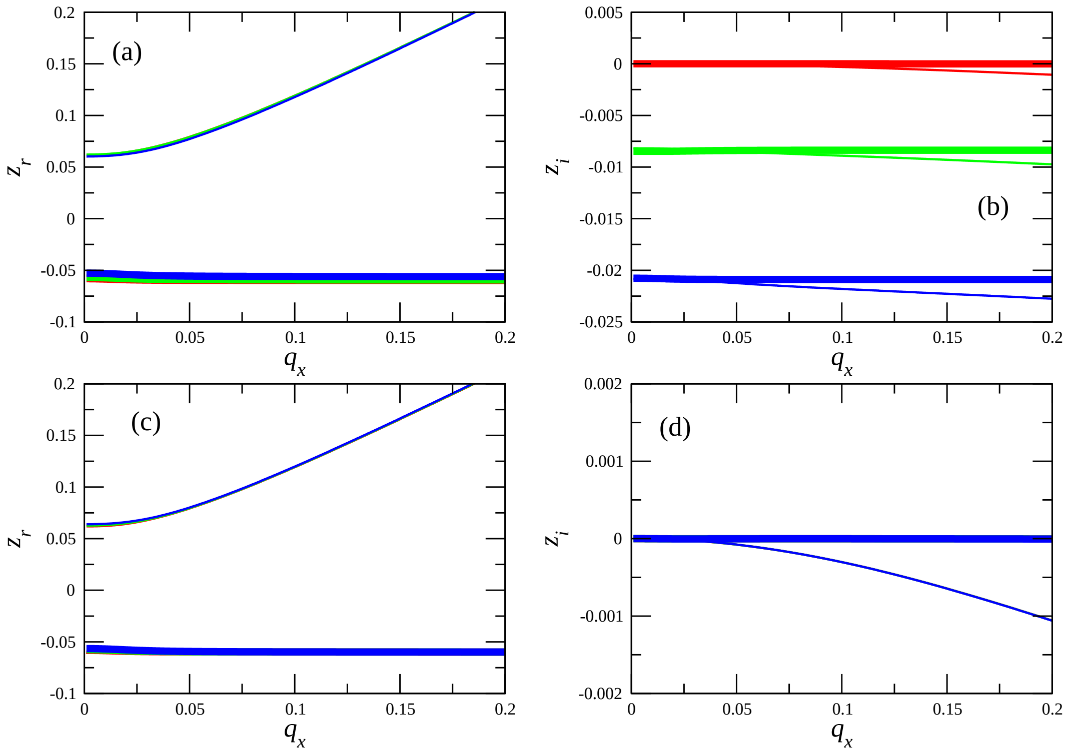

Figure 2: (a) z_r vs. q_x, for $q_z = 0.06$, with $\varepsilon = n_{d0}/n_{i0} = 0.0$ (red), 1.0×10^{-6} (green), and 2.5×10^{-6} (blue); (b) corresponding values of z_i; (c) z_r vs. q_x, for $q_z = 0.06$ and $\nu_e = \nu_i = 0$, with $\varepsilon = n_{d0}/n_{i0} = 0.0$ (red), 1.0×10^{-6} (green), and 2.5×10^{-6} (blue); (d) corresponding values of z_i.

imbalance between ions and electrons, an approximation frequently found in the literature. The results appearing in Fig. 2(c) are very similar to those of Fig. 2(a), indicating that z_r is very little affected by the collisional charging. However, the situation is different in the case of z_i. Fig. 2(d) shows that if the effect of ν_β is neglected, the values of z_i are very little affected by the small dust population which we have considered, as compared to Fig. 2(b). The effect of the collisional charging of the dust particles therefore appears to be fundamental for correct evaluation of the damping of both obliquely and parallel propagating Alfvén waves.

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