Dust acoustic solitons and shocks in a charge varying dusty plasma

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Abstract

A theoretical investigation has been made to analyze the propagation of dust acoustic solitary waves in an unmagnetized charge varying dusty plasma. The charge variation is found to increase the pulse amplitude while its width decreases, i.e., the dust charge fluctuation makes the solitary structure more spiky. On the other hand, it has been shown that the dust charge fluctuations lead to a dissipation which is responsible for non collisional shock waves formation the nature of which depends sensitively on the equilibrium dust density.

I- Introduction

The dust grains immersed in a plasma are highly charged, and the dust charge on the dust grains also fluctuates due to the fluctuating electrostatic force. This is an important characteristic of dusty plasma, which distinguishes it from a three component plasma. The charging processes and grain charge fluctuations are also interesting phenomena and have been recently investigated. In this paper, we study the properties of nonlinear dust acoustic solitons and shocks by incorporating the effects of the dust charge fluctuation.

II- Basic equation

Let us consider an unmagnetized dusty plasma composed of Boltzmann distributed electrons, ions, and variable charge dust grains of density \( n_e, n_i \) and \( n_d \), respectively. We assume for simplicity that all the grains have the same charge, equal to \( q_d = -eZ_d \), with \( Z_d \) positive for negatively charged dust and negative for positively charged dust. All the dust grains are assumed to be spheres of radius \( r_d \). In the presence of the dust grains, the electrons and ions may be considered as point particles. Moreover, the electrons and ions are in a local thermodynamic equilibrium and their densities are given by the Maxwell-Boltzmann distribution

\[
n_s = n_{s0} \exp\left(-\frac{q_s \Phi}{T_s}\right)
\]

Let us begin to look at a very simple case, where each cold species is a beam of particles, each particle of species \( s \) having the same speed at a given position. Thus, we choose [1]
\[
\tilde{v}_d = v_{d0} \left( 1 - \frac{2}{m_d v_{d0}} \int_0^\phi q_d d\phi \right)
\]

where

\[
\tilde{v}_d = v_{d0} \left( 1 - \frac{2}{m_q v_{d0}} \int_0^\phi q_d d\phi \right)^{1/2}
\]

Integrating the dust distribution function \( f_d \) over all velocity space, we find

\[
N_d = \frac{n_d}{n_{d0}} = \frac{v_{d0}}{\tilde{v}_d} = \frac{1}{(1-\alpha \chi)^{1/2}}
\]

where

\[
\alpha = \frac{2 r_d \sigma T_e^2}{e^2 m_d v_{d0}^2}
\]

and

\[
\chi = \int_0^\Psi Q_d d\Psi
\]

Poisson’s equation can be expressed as

\[
\frac{d^2 \Psi}{dX^2} = \exp(\sigma \Psi) - f \exp(-\Psi) - (1 - f) \frac{Q_d}{Q_{d0}} N_d
\]

where \( f = n_{i0}/n_{e0} \) and \( \sigma = T_i/T_e \). The following normalized quantities: \( \psi = e\phi/T_i, \chi = \chi/\lambda_{Dm}, \)
\( Q_d = q_d/r_d T_e \) are introduced and \( \lambda_{Dm} = (T_i/4\pi n_{e0}e^2)^{1/2} \).

Charge neutrality at equilibrium requires

\[
f = 1 - \left( \frac{r_d T_e Q_{d0}}{e^2} \right) n_{d0} / n_{e0}
\]

The system is closed by the normalized charging equation

\[
\frac{dQ_d}{dX} = \frac{K}{(1-\chi \xi)^{1/2}} \left[ -\sqrt{\frac{m_i}{m_e \sigma}} \exp(\sigma \Psi) \exp(Q_d) + f \exp(-\Psi) \left( 1 - \frac{Q_d}{\sigma} \right) \right]
\]

with \( K \) given by
Equation (9) is the additional dynamical equation that is coupled to the plasma equations through the plasma currents. The dust electric charge becomes a dynamical variable which is coupled self-consistently to the other dynamical variables such as density and potential. Initially, in the absence of any perturbation ($\psi=0$), equation (9) yields

$$f = \sqrt{\frac{m_i}{m_\sigma}} \left( \frac{\exp(Q_{d0})}{1 - Q_{d0}/\sigma} \right)$$

In the following, the value of $f$ is deduced from the above relation while the remaining other parameters are given first.

Next, equations 6, 7 and 9 are integrated numerically. In figure 1, we study the effect of the dust charge fluctuation on the soliton profiles. It is seen that the variation of the dust charge produces a reduction of the dust acoustic soliton width as well as an increase of its amplitude.

In figure 2, it can be seen that as $K$ decreases, the wave amplitude suffers the well known anomalous damping. In figure 3, we study the effect of the parameter $v_{d0}$ (the dust speed) on the dust acoustic shock when the charge of the dust grain is variable. It is found that greater values of $v_{d0}$ favor the development of dissipative structures. In figure 4, it can be seen that when $r$ (the dust radius) increases, the shock wave has a monotonic profile (dissipation dominant).

References

Constant and variable charge soliton

Collisional shock wave profile

Monotonic and oscillatory shock profile

Effect of dust radius on shock profile