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CRITERION FOR THE OUT-OF-PLANE INSTABILITY OF 2D-CRYSTALS

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In a typical experiment on Earth (under gravity), a plasma crystal is formed within a complex plasma when dust particles are levitated in the horizontal sheath region due to a balance between the gravitational and electrostatic force of a rf discharge [1]. The total external potential in this sheath region has been shown experimentally to approximate a parabolic potential well [1,2]

$$V_{ext} = \frac{1}{2}\alpha z^2. \tag{1}$$

Here z is the particle coordinate in vertical direction and α is the parabolic coefficient.

Under some conditions the external potential (1) is insufficient and the destruction of crystal plane leads to the structural phase transition, which can be observed.

To describe all possible interactions between dust grains in plasmas, we examine the out-of-plane instability of a lattice in general case, assuming only that the dust particle interaction is described by an arbitrary paired isotropous potential.

We start from the dispersion relation for the out-of-plane instability of a primitive lattice, when there is one particle in a primitive cell:

$$\omega^2(\mathrm{q}) + i\nu\omega(\mathrm{q}) = \frac{1}{m}\left(\alpha + \Omega(\mathrm{q})\right) \tag{2}$$

where

$$\Omega(\mathrm{q}) = \sum_{n\neq 0} \frac{V'\left(\xi_n^0\right)}{\xi_n^0}\left(1 - e^{i\mathrm{q}\mathrm{a}_n}\right). \tag{3}$$

Here $\xi_n^0 = |\mathrm{a}_n|$ and $\mathrm{a}_n = n_1\mathrm{a}_1 + n_2\mathrm{a}_2$, where $\mathrm{a}_1, \mathrm{a}_2 \in XY$ are primitive lattice vectors and $n = \{n_1, n_2\}$ is a multi-index numbering the primitive cells, and $n_1, n_2 = \mathbb{Z}$ are integer. The wave vector q is defined with the accuracy of an arbitrary reciprocal vector $\mathrm{b}_g = g_1\mathrm{b}_1 + g_2\mathrm{b}_2$ (b_1 and b_2 are the basic vectors of the reciprocal lattice). So we can consider it in a restricted region $-\pi \le \mathrm{q}\mathrm{a}_i \le \pi$ (*i=1,2*), that coincides with the definition of the first Brillouin zone (or the Wigner-Seitz cell).

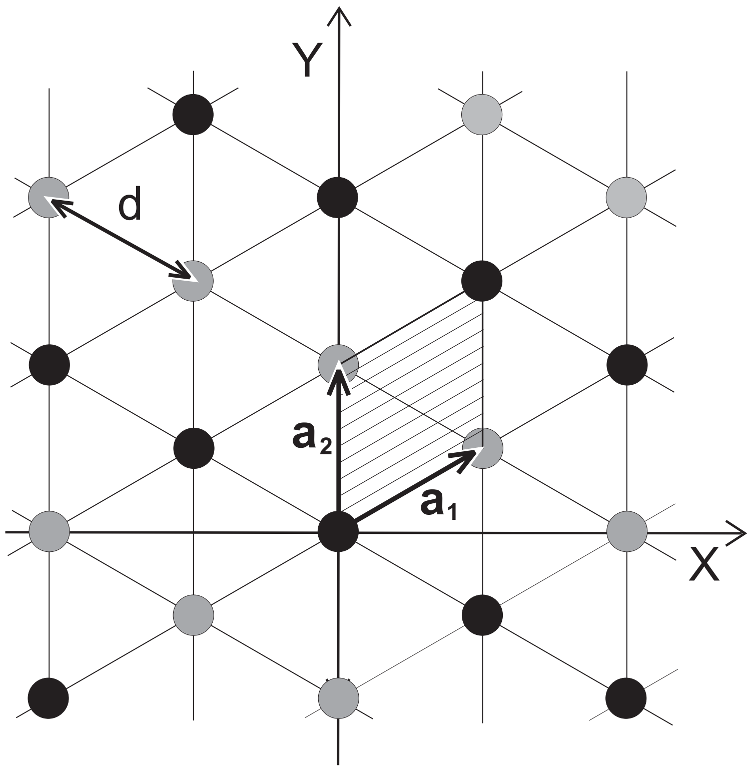

Figure 1: The hexagonal lattice is as example of primitive lattice. The lattice vectors are $a_1 = 1/2\, d\left(\sqrt{3},1,0\right)$ and $a_2 = d\left(0,1,0\right)$. The development of out-of-plane instability is shown by gray and black, which correspond to the different directions of particle displacement.

The out-of-plane instability begins developing as soon as the imaginary part of frequency ω is positive for the some value of q. This situation takes place for $\min\left\{\Omega(\mathrm{q})\right\} < -\alpha$ and does not depend on damping coefficient ν. In this case the image part of frequency gives the growth rate of the out-of-plane instability, which already depends on ν. In opposite case the lattice is stable against out-of-plane excitations.

In order to find the extremum value of the wave vector q, which corresponds to the minimal value of frequency in square, we estimate the sum (3) roughly by integral:

$$\Omega(\mathrm{q}) \approx \rho\int_{d^*}^{\infty}\int_{0}^{2\pi}\frac{V'(r)}{r}\left(1-e^{i\mathrm{qr}}\right)rdrd\varphi = \rho\int_{d^*}^{\infty}\left(1-J_0(qr)\right)V'(r)\,dr = 2\pi\rho\tilde{\Omega}\left(q\right) \qquad (4)$$

Here and below $\rho = 1/S_0$ is the averaged surface particle density in the layer; $d^* < d$ is a distance determined specifically both for each structure of lattice and for each potential. The integral estimation (4) does not depend on the direction of the vector q and is a negative (we don't consider the opposite case, because, on this occasion the lattice is stable against out-of-plane excitations) diminishing function of argument q. Indeed, the derivation of $\tilde{\Omega}$ is

$$\tilde{\Omega}'(q) = \frac{J_0(qd^*)}{q} V'\left(d^*\right) d^* + \frac{1}{q} \int_{d^*}^{\infty} J_0(qr)\left(rV'' + V'\right) dr \qquad (5)$$

The integral has approximately value $J_0(q\xi)\left(\xi V''(\xi) + V'(\xi)\right)\left(\mu_{0,1}/q - d^*\right)$, where $\xi \in \left[d^*, \mu_{0,1}/q\right]$ and $\mu_{0,1}$ is a first root of Bessel function $J_0(x)$. Therefore the second term will be less than the first. Thus, the sign of the derivation is determined by the first term, which is a negative until the inequalities $V'\left(d^*\right) < 0$ and $J_0\left(qd^*\right) > 0$ hold. This means that the frequency in square $\omega^2(q)$ amounts to the minimal value at $q = q_{\max}$, where $q_{\max}$ is a maximal value of wave vector within the first Brillouin zone, i.e. $q_{\max}$ corresponds to the vertexes of the square. Without loss of a generality we find $\mathrm{q}_{\max} = \pi\rho\left[\left(\mathrm{a}_1 + \mathrm{a}_2\right)\times \mathrm{e}_z\right]$. Then the *condition of the out-of-plane instability* of 2D primitive lattice takes the form

$$\sum_{n\neq 0} \frac{V'\left(\xi_n^0\right)}{\xi_n^0}\left(1 - (-1)^{n_1+n_2}\right) < -\alpha. \qquad (6)$$

It follows that the lattice plane is unstable in the first place against short-wavelength excitations. The instability begins developing as soon as $-\Omega\left(\mathrm{q}_{\max}\right)$ exceeds parabolic parameter α of confined potential. Instability breaks the crystal plane, turning it to the corrugated surface (like "washing-board"). The translation vector of perturbations is $\mathrm{a}_1 + \mathrm{a}_2$. Fig.1 shows by gray and black the different relative directions of particle displacement for beginning of the instability development in the case of hexagonal lattice.

Below we consider several examples and compare with results obtained early by numerical simulation. In all examples the hexagonal lattice is taken up.

(i) We compare our results with results obtained by molecular dynamics simulation for Yukawa system [2], when the interaction between grains is described by $V = Ae^{-kr}/r$. We introduce the dimensionless parameters used in paper [2]:

$$\zeta = kd\left(\frac{\sqrt{3}}{2\pi}\right)^{1/2}, \qquad \eta = \frac{\alpha}{4\pi A\rho^{3/2}}. \qquad (7)$$

The results represented in Fig.2 show a closed agreement between our result (line 1) and molecular dynamics simulation in Ref.[2] (dots).

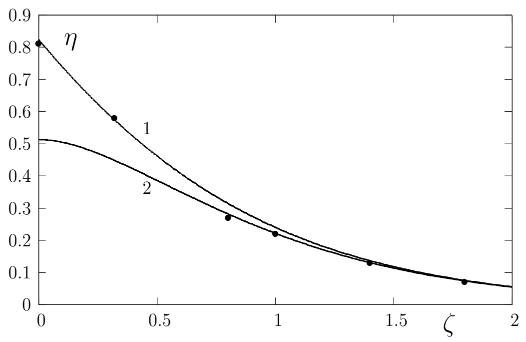

Figure 2: Lines and dots are the boundaries of the out-of-plane instability development, determined by calculation of sum (6) (line 1), by calculation of partial sum (line 2) and by molecular dynamics simulations in [2] (dots).

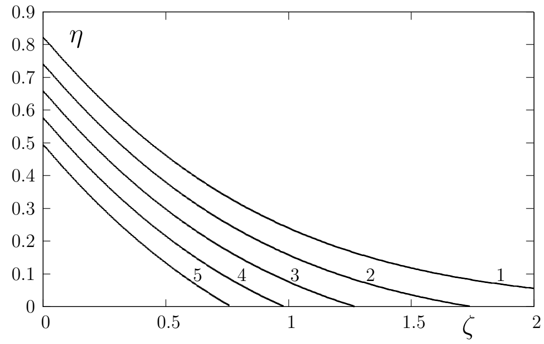

Figure 3: Lines are the boundaries of the out-of-plane instability determined for different values of parameter *h*:

h = 0 (line 1), *h* = 0:1 (line 2), *h* = 0:2 (line 3), *h* = 0:3 (line 4), *h* = 0:4 (line 5).

(ii) In case that charged dusty grains are immersed in plasmas and are exposed to flows of ions and neutral particles, one has to account not only Yukawa interaction but shadow attraction as well [1]. In the next example we consider the potential $V = A\left(e^{-kr} - h\right)/r$ which is a combination of repulsive Yukawa and shadow attraction. The results are presented in Fig.3 for different values of *h* and for a wide range of ζ values.

With parameter *h* increasing more and more, the boundary of the out-of-plane instability has shifted to the direction of smaller value of η. One can see, that critical value of ζ appeases, excess of which ($\zeta > \zeta_{crit}$) makes the lattice of being stable against out-of-plane excitations for $\eta = 0$. It is likely that there is a region of *h* and ζ parameters, in which the hexagonal 2D lattice will be stable against both out-of-plane and in-plane excitations. This question exceeds the bounds of our research, but it can be the subject of separate studies.

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References

[1] E. Fortov, A.G. Khrapak, and etc., Physics-Uspekhi, V.**47**, N.5, p.447 (2004)

[2] H. Totsuji, T. Kishimoto, and C. Totsuji, Phys. Rev. Lett., **78**, p.3113 (1997)