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Dispersion equation analysis for horseshoe and ring distribution functions cyclotron maser instabilities

I. Vorgul[1], B.J.Kellett[3], R.A. Cairns[1], R. Bingham[2,3], K. Ronald[2], D.C. Spiers[2], S.L.M^c^Conville, A.D.R. Phelps[2], V. Graffagnino[3]

1. School of Mathematics and Statistics, University of St Andrews, St Andrews, Fife, KY16 9SS, UK.
2. Department of Physics, University of Strathclyde, Glasgow, G4 0NG, UK.
3. STFC Rutherford Appleton Laboratory, Chilton, Didcot, Oxfordshire OX11 0QX, UK.

Introduction

The electron cyclotron maser instability is a powerful mechanism for producing non-thermal stimulated radiation in a plasma. When a beam of electrons moves into a converging magnetic field, the velocity distribution function takes on a horseshoe shape as a result of conservation of magnetic moment. A few years ago it was pointed out that such a distribution is unstable to a cyclotron maser type of instability and it was suggested that this instability might be the source of auroral kilometric radiation [1] and also of emission from certain types of stars [2]. Here we present a more detailed analysis of the dispersion relation than before, seeking a detailed understanding of the wave propagation properties. This is relevant to planetary radio emission, radiation from astrophysical shocks and stellar objects and to a laboratory experiment which we are currently carrying out [3]. The latter investigates radiation generation, exploiting the fact that the mechanism only depends on dimensionless ratios like the ratio of the wave frequency to the cyclotron and plasma frequencies and the factor by which the magnetic field varies along the path of the electon beam. Thus it is possible to scale it to laboratory size and to GHz and above frequencies. The analysis is also relevant to studying radiation propagation in a strong dipole magnetic field geometry. This is important for understanding the latest radio results for the 100% polarized, coherent radiation from the star CU Virginis.

Distribution function and dielectric tensor

An electron beam moving into a converging magnetic field changes its momentum distribution. For a drifting Maxwellian initial distribution the laws of conservation of magnetic moment and particle energy mean that we obtain the transformed distribution with a shape of a horseshoe in velocity space [1]

$$f(v_{||}, v_{\perp}) = Ae^{-\frac{1}{2v_{th}^2}\left[\left(\sqrt{v_{||}^2+(1-B/B_0)v_{\perp}^2}-v_0\right)^2+B/B_0\, v_{\perp}^2\right]}. \quad (1)$$

which is unstable to cyclotron radiation.

The dielectric tensor of this plasma is determined by

$$\hat{K}_{\perp} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \left(1 - \frac{\omega_{pe}^2}{\omega^2}\right)\hat{I} - \frac{\omega_{pe}^2}{\omega^2} \cdot \int \left\{ \frac{\hat{T}}{k_z v_{||} - \omega + \Omega\sqrt{1-(v_{||}^2+v_{\perp}^2)/c^2}} \cdot \left(-\frac{n\Omega}{v_{\perp}}\frac{\partial f}{\partial v_{\perp}} + k_z \frac{\partial f}{\partial v_{||}} \right) \frac{1}{n_0} \right\} d\vec{v}, \quad (2)$$

where the matrix $\hat{T}$ can be simplified for small Larmor radius as $\hat{T} = \frac{1}{4} v_{\perp}^2 \begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$, ω_{pe} and Ω are the plasma and cyclotron frequencies, respectively. The integral in (2) can be reduced to a single integral after changing the coordinates and using residues of the function. In our previous work [4] we considered an annular electron beam, forming a ring $R_0 \le r \le R_1$ of plasma inside a transverse cross section of a circular metallic tube with its radius equal to R. We took a perfectly conducting boundary, though other boundary conditions could be imposed. To investigate the radiation properties of the horseshoe distribution instability we looked at the growth rate of different modes, determined as the imaginary part of a complex-value k_z in the field propagator $e^{-i\omega t + i k_z z}$. The frequency was chosen to be just below the cyclotron frequency, in the range where the instability is expected to occur. Numerical analysis of the dispersion relations showed that a number of spatial modes have high growth rate.

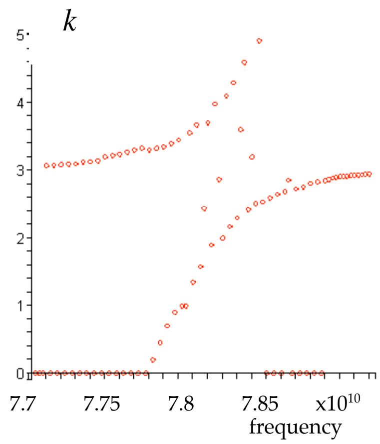

Fig.1. Dispersion of plasma with horseshoe type instability

To further understand the maser radiation dispersion and propagation in auroral cavities we analysed numerically the dispersion equation for unbounded plasma with the horseshoe instability described by the dielectric tensor (2). In Fig.1 there are two branches which bear an obvious similarity to the familiar cold plasma curves, but there are also points which suggest there is an extra branch joining them.

Such a branch actually appears in the cold plasma dispersion relation for a ring distribution, though it is not shown in the review paper on the subject by Chu [5]. How energy gets from the lower frequency branch to the upper branch which connects to the vacuum is a long-standing problem in the theory of AKR. The existence of the branch connecting the two cold plasma branches may have some bearing on the problem, but this needs further investigation.

Astrophysical applications

Pulsars were discovered nearly forty years ago yet, despite intensive study, the mechanism for generating their radio emission remains an unsolved puzzle. We know the radio emission from pulsars is highly nonthermal, covers a broad range of radio frequencies, originates from a very compact region with a superstrong magnetic field and involves both electrons and positrons. However, all these factors also make pulsars extremely difficult to probe for details that might elucidate the emission mechanism. The extreme brightness temperature of pulsar radio emission requires a coherent generation process which undoubtedly involves a plasma instability of some sort. We have suggested previously[2] that the mechanism described above is relevant to several types of astrophysical sources, including stellar radio sources.

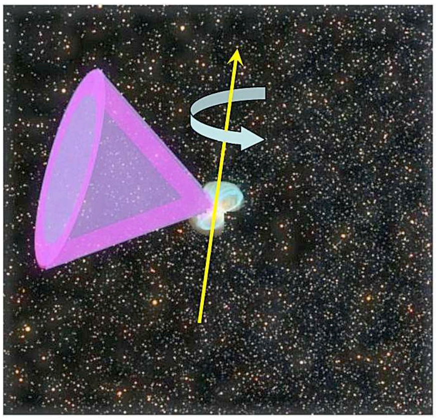

Fig.2. Pulsar geometry

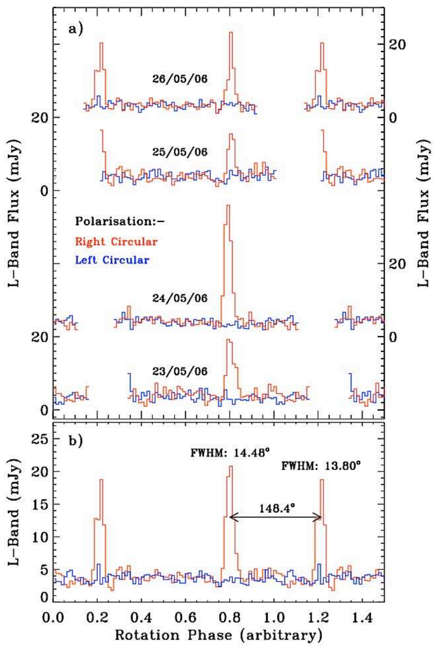

Fig.3.Radio emission observation results

The basic requirements for this mechanism are a dipole magnetic field and an electron beam directed into regions of converging field lines near the poles. Here we report radio observations of the star CU Virginis that reveal strikingly pulsar-like behaviour that could indicate we have found a new laboratory for studying the radio generation mechanism associated with pulsars. Four nights of MERLIN radio observations of the star have for the first time provided a complete radio light curve and demonstrate that the main emission features of the source are persistent and recurring as shown in Fig 3. The most striking emission features are two strong, narrow, 100% right-hand polarised, emission peaks that are 6-10 times stronger than the unpolarised background emission. These would seem to be explicable in terms of radiation emitted from around the magnetic poles of the star rotating around an axis not aligned with the magnetic dipole, as in Fig. 2. CU Vir also exhibits extremely rapid (for a star) rotation period changes (spin-down).

Conclusions

Our analysis indicates that the horseshoe distribution produces a rapidly growing instability at frequencies just below the electron cyclotron frequency. Depending on the proportions of the cavity, a variety of different modes may be unstable, with large growth rates. We suggest that this model is closer to explaining such phenomenon as auroral kilometric radiation than the loss-cone instability which has been proposed in the past. Our more recent analysis of the dispersion curves shows that it they have a more complicated structure than the cold plasma curves. Curves which are separate in a cold plasma are joined, a fact which may have some bearing on the long standing problem of how energy gets on to the branch which connects to vacuum propagation. Finally we have shown some new observations of stellar radio emission which may present another possible area of application of this theory.

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