Numerical Evaluation of Ripple-Induced Diffusion of Suprathermal Ion in Tokamaks

H. Mimata¹, K. Tani², K. Tobita², H. Tsutsui¹, S. Tsuji-Iio¹, R. Shimada¹

¹ Tokyo Institute of Technology, Tokyo 152-8550, Japan
² Japan Atomic Energy Agency, Ibaraki 311-0193, Japan

Introduction

The confinement of fusion-produced alpha particles is important in tokamak reactors. In an axisymmetric field, alpha particles are well confined. In a practical system, however, a finite number of toroidal field (TF) coils cause TF ripples, which enhance the transport of alpha particles. Furthermore, particles can resonate with ripples, which also enhance diffusion coefficients. In this work, we evaluated the diffusion coefficients with the ripple resonance condition.

Banana Drift

In a nonaxisymmetric field, the toroidal canonical momentum $P_{\phi}$ is not conserved, and it has large drifts at banana tips. This drift is approximately given by [1]

$$\Delta P_{\phi} = \Delta P_{\phi}^* \sin(N\phi_b - \pi/4), \quad (1)$$

where $N$ is the number of toroidal field coils, $\phi_b$ is the toroidal angle of a banana tip and $\Delta P_{\phi}^*$ is the drift amplitude at the banana tip due to the ripple and is given by an integral over a unperturbed trapped orbit, most of the contribution coming near the banana tip.

Because of the $\phi_b$-dependence of the toroidal canonical momentum drift, banana particles can resonate with toroidal ripples. This ripple resonance condition is expressed as

$$\Delta \phi_b \equiv \phi_b^{i+1} - \phi_b^i = \frac{2k\pi}{N} \quad (k = \pm1, \pm2, \pm3, \cdots), \quad (2)$$

where $\phi_b^i$ denotes the toroidal angle of the $i$th banana tip. If a banana particle satisfies the ripple resonance condition, it bounces at the same toroidal angle normalized to $\phi_N = 2\pi/N$ at each
banana tip and it can make a large poincare surface in the $(\phi_b, P_\phi)$ phase space (Fig. 1). Then such particles, which are near the ripple resonance condition, can enhance diffusion.

**Ripple Diffusion**

We calculated the diffusion coefficients in the magnetic field which has ripple resonance points. Calculation parameters are major radius $R_0 = 6.0$ m, minor radius $a = 2.0$ m with a circular cross section, safety factor $q_s = 4.2$, magnetic field at plasma center $B_0 = 3.0$ T, the number of TF coils $N = 18$, and toroidal ripple at plasma surface $\gamma \sim 0.5\%$. Coulomb collisions were simulated by a Monte-Carlo method [2]. Particles start from $R = R_0 + 0.9a$, $Z = 0$, and the initial toroidal angle is randomly given. Particles have the same magnetic moment $\mu_m$ whose pitch angle $\zeta = 0.7 \times 2/\pi$ radians if ripple is zero. The trajectory of particles were calculated with the guiding center equations and diffusion coefficients were evaluated from 700 particles.

Results are shown in Fig. 2. Open triangles on abscissa mean ripple resonance energy in axisymmetric field. The diffusion coefficients are small at resonance energies and have peaks on both sides (M-shaped structure) and this M-shaped structure at higher energy is broader than that at lower energy. The peaks of the diffusion coefficients do not correspond to the peaks of the width of Poincaré surfaces.

**M-shaped structure**

To investigate the M-shaped structures of diffusion coefficients, we made a Poincaré map around a resonance energy. Figure 3 shows the variations of the Poincaré maps around a resonance energy, where the magnetic moment is constant and particles start from 25 initial toroidal angles.

Basically the Poincaré map gives open surfaces around closed surfaces as illustrated in Fig. 4 near the resonance energy. Since the changes in energy and magnetic moment by collisions are small (within about 5% and 1%, respectively), we consider particles move to adjacent surfaces in such Poincaré maps by collisions for simplicity. Particles on a closed surface spread to its
surface size at first, after that, however, diffusion by collisions is as small as that of particles on open surfaces. While the particles on open surfaces near the separatrix can jump over the large closed surface through the X-point by collisions and diffusion coefficients can be large. Hence diffusion coefficients exhibit M-shaped structures.

Diffusion coefficients were evaluated after particles spread to its surface size by ripple. In that time the standard deviation of the magnetic moment is about 0.5%. When particles jump from open surfaces to a large closed surfaces, large diffusion occurs. Therefore we investigated the ratio of open surfaces in Poincaré map when a separatrix exists including changes in the magnetic moment, and compared it with the diffusion coefficients. If the magnetic moment changes, the number of open surfaces is different especially at high energy (Fig. 5).

Figure 3: Variations of the Poincaré maps around a resonance energy of 2.9 MeV.

Figure 4: A simple picture of collisional diffusion in $(\phi_b, P_\phi)$ phase space
Figure 6 shows the ratio of open surfaces with the three values of the magnetic moment shown in Fig. 5, which is also the ratio of particles on open surfaces, has a similar dependence on energy as the diffusion coefficients. At large energy, the M-shaped structure has wide peaks because separatrix exists in a wide range of energy around a resonance energy.

**Conclusions**

In nonaxisymmetric field, some particles can resonate with toroidal ripple and it has a large change of toroidal canonical momentum. Then diffusion coefficients are large near resonance energies. Furthermore, the diffusion coefficients exhibit an M-shaped structure around a resonance energy because closed Poincaré surfaces alone do not contribute to the diffusion so much as open surfaces, and open surfaces just near separatrix enhance the diffusion. On the other hand, diffusion coefficients drop when most of particles are on closed surfaces and exhibit an M-shaped structure.

**References**
