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The relativistic quasi–plane wake waves excitation by laser pulses sequence with tuneable curvature

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The 2D3V PIC modelling of the relativistic wake waves excited by nonrelativistic laser pulses sequence propagation have been performed. The code SUR-CA, based on local-recursive/nonlocal-asynchronious algorithm is used. The observed wake wave edge curvature limits the accelerated field growth. The curvature is the dependence of the wave phase on the transverse coordinate. The relativistic increase of the plasma electron mass is the base cause of that. We propose to use pulses with tunable curvature of the leading edge. The distance between the leading edges depends on the transverse coordinate and the pulse number in sequence. The plane wake wave is taken on the first period after pulse sequence.

Introduction

The laser wake field acceleration of charged particles is based on excitation of longitudinal electric waves with the relativistic amplitudes by laser pulses penetrated through undercriticial plasma ($\omega_0 \gg \omega_\mathrm{p}$, here ω_0 and ω_p are the laser frequency and plasma frequency respectively, $\omega_\mathrm{p} = \sqrt{4\pi n_\mathrm{e} e^2/m_\mathrm{e}}$) [1]. The recent experimental result [2] that was obtained is the 1 GeV electron gain energy in the plasma canal by relativistic single pulse. The PIC modelling of the wake wave excitation by the pulse are being researched widely (see [3]). The acceleration of trapped electrons has been considered because copious amount of accelerated electrons have been observed when driving waves beyond the breaking threshold. To accelerate external electrons beam the wake field should be less than E_WB [4]:

$$eE_\mathrm{WB}/m_e c\omega_0 = (\omega_\mathrm{p}/\omega_0)\sqrt{2(\gamma_g - 1)}, \tag{1}$$

here $\gamma_g = (1 - V_\mathrm{ph}^2/c^2)^{-1/2} = \omega_0/\omega_\mathrm{p}$. In this paper we examine the modelling results of the relativistic wake waves excitation by non-relativistic laser pulse sequence. The excitation by pulse sequence have been proposed early. In [5] the one dimensional case of laser-plasma interaction have been considered. More, the pulses propagated to plasma layer haven't changed. In our work the self-consistent 2D3V model is considered. Thus, it is possible to consider the influence of the transversal effects on the wake waves propagation.

Problem statement

The numerical modelling of the laser pulses interaction with plasma is based on the Maxwell-Lorentz equations:

$$\frac{1}{c}\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}, \quad \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \nabla \times \vec{B} - \frac{4\pi}{c}\vec{j}, \quad \nabla \vec{B} = 0, \quad \nabla \vec{E} = 4\pi\rho. \tag{2}$$

Here the charge and current density are determined by the distribution function moments of the plasma particles ($\alpha = e, H^+$): $\rho = \sum_\alpha \int f_\alpha e_\alpha d\vec{p}, \vec{j} = \sum_\alpha \int \vec{v_\alpha} f_\alpha e_\alpha d\vec{p}$. The evolution of the distribution function is described by the Vlasov equation:

$$\frac{\partial f_\alpha}{\partial t} + \vec{v}_\alpha \frac{\partial f_\alpha}{\partial \vec{r}_\alpha} + e_\alpha \left(\frac{1}{c}\vec{v}_\alpha \times \vec{B} + \vec{E}\right)\frac{\partial f_\alpha}{\partial \vec{p}_\alpha} = 0. \tag{3}$$

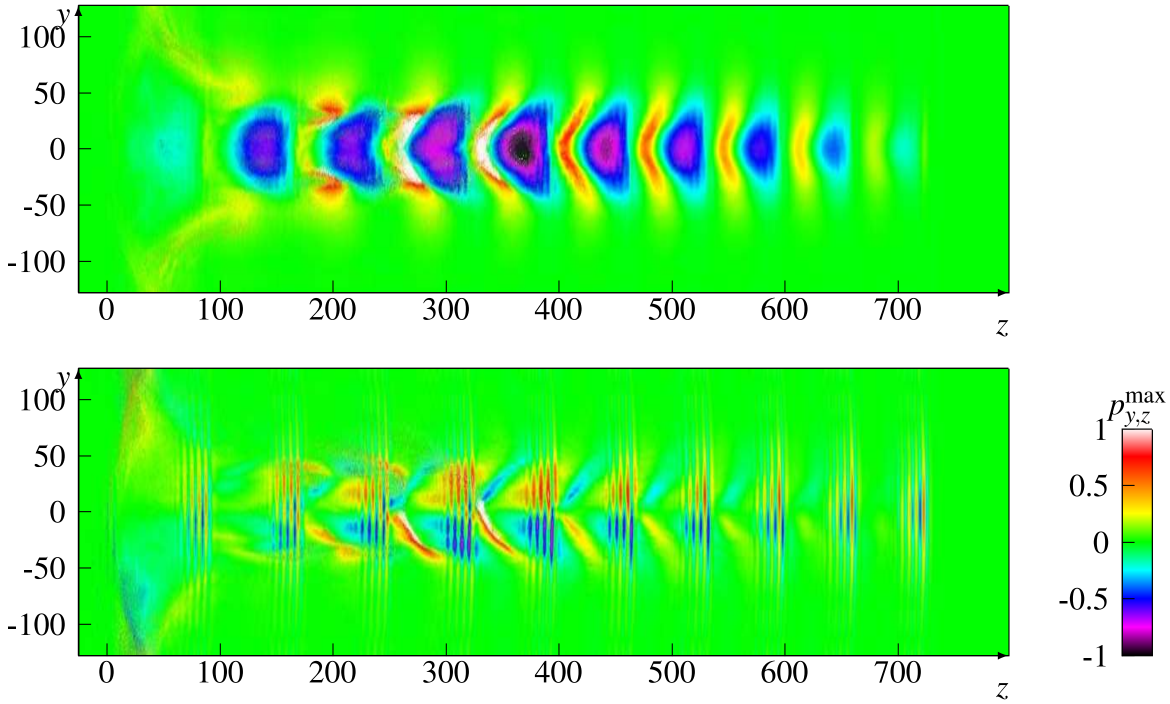

Figure 1: Initial conditions are $a_0 = 0.5$, $\vec{g} = \vec{e}_x + i\vec{e}_y$, $L_\perp = 20\pi$, $L_\parallel^{\mathrm{front}} = \pi$ $L_\parallel^{\mathrm{back}} = 9\pi$, $L^{\mathrm{pl}} = 2000$, $N_{\mathrm{pulses}} = 16$, $d = 2\pi c/\omega_{0\mathrm{p}}$. Spatial distribution of the maximal longitudinal (p_z/m_0c above) and transversal (p_y/m_0c below) electrons momentum in first wave break moment $t\omega_0 = 739$

The equations (2)-(3) are nonlinear self-consistent multidimensional system.
The dimensionless values are used in our work: $[v] = c$, $[t] = 1/\omega_0$, $[m] = m_\mathrm{e}$, $[q] = e$, here ω_0 is the laser pulse frequency. The other values can be shown through the basic values ones: $[r] = \frac{c}{\omega_o}$, $[n] = \frac{m_\mathrm{e}\omega_o^2}{4\pi e^2}$, $\frac{e[E,B]}{m_\mathrm{e}\omega_o c} = 1$, $\frac{e[A]}{m_\mathrm{e}c^2} = 1$. The collisionless plasma are considered. Thus the particle-in-cell method is used. The large particles motion is described by:

$$\frac{d\vec{p}_j}{dt} = e_\alpha \left(\frac{\vec{v}_j}{c} \times \vec{B}_i + \vec{E}_i \right), \; \frac{d\vec{r}_j}{dt} = \frac{\vec{p}_j/m_j}{\gamma_j}, \qquad (4)$$

here $\gamma_j = \sqrt{1 + (\vec{p}_j/m_j c)^2}$, indexes j and i are particles and grid cell indexes respectively. The macro particles have shape factor with the finite width. The special weighing of the elementary charge and the current on the nearest cells and sides of the cell is applied for calculating the complete current density. The problem phase space is five-dimensional $(y, z, \vec{p})$. The electromagnetic fields $(\vec{E}, \vec{B})$ have three components each (2D3V model). The 3D3V PIC code SUR-CA, based on local-recursive/nonlocal-asynchronious algorithm [6] is used for modelling processes of laser radiation interaction with plasma.
The basic parameters of the pulses in vacuum are the wave vector $\vec{k}_0 = \vec{e}_z c/\omega_0$, amplitude of dimensionless vector potential a_0, the polarization vector $\vec{g} \perp \vec{k}_0$, half-width $L_\perp$ and the lengths $L_\parallel^{\mathrm{front}}$, $L_\parallel^{\mathrm{back}}$ of the front and the back. The sequence parameters are the pulses quantity Npulses and the distance between the next pulses edges d. The plasma layer with the initial density n_p occupies region $0<z<L^{\mathrm{pl}}$. The initial particles distribution corresponds to latent start model with the temperature of zero. Plasma is full ionized. The pulses fall normal to the bound.

The modelling results

The distance between the next pulses edges should be equal λ_p to maximal wake field growth. The modelling results of the propagating the pulses sequence with the constant distance are shown in Fig. 1.

The wake field has curvature, i.e. the wave phase depends on the transversal coordinate. The wake wave curvature is the cause of the essential wave breaking limit drop. On our opinion the curvature is caused by relativistic increase of the plasma electron mass as follow: $m = m_0\sqrt{1+p^2/m_0^2c^2}$, here m_0 is the rest mass, p is the particle momentum. The momentum value in the circularly polarized electromagnetic wave:

$$p(r) = \sqrt{\left(E_x(r)^2 + E_y(r)^2\right)/\omega_0{}^2 + E_z(r)^2/\omega_p{}^2}. \quad (5)$$

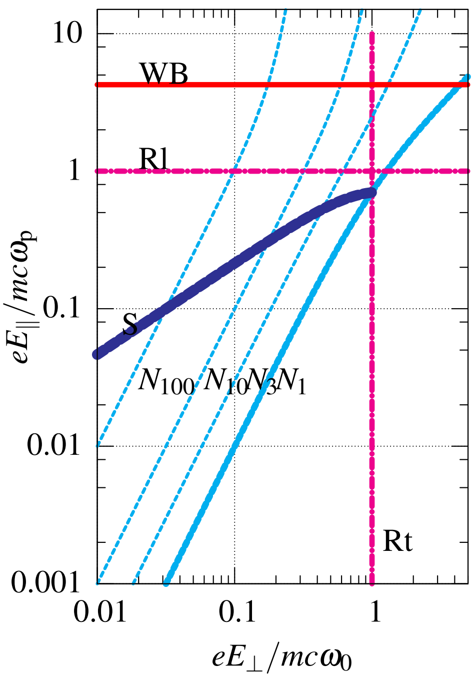

Figure 2. Dependence the wake field on the own pulse field with $\omega_0/\omega_p = 10$. N_n is optimized pulse sequence with n pulses, WB is wave break limit (1), Rl (Rt) is relativistic electrons in the longitudinal (transversal) field, S is the bound of the curvature pulses (7)

The own pulse field depends on transversal coordinate. i.e. this effect is two-dimentional. The longitudinal (wake) field depends on the transversal coordinate too. The plasma frequency increases from the axis to the periphery and the wave length decreases. The necessary condition of the wake field amplitude gain by the pulse sequence is $d_n(r) = \lambda_p(r)$ for $-L_\perp < r < L_\perp$ and every n, here $d_n(r)$ is the distance between n and $(n-1)$ pulses in the sequence. The condition applies for every weak-relativistic pulse automatically. But the phase difference accumulates in the every plasma wave period for the wake waves excited by the pulse sequence. Followed pulses should reinforce wake field excited by previous pulses. That's why the summary phase difference between edge transverse coordinates at fixed longitudinal coordinate must be less than $\pi/2$. One can see, that:

$$\sum_{n=1}^{N}\left(\sqrt[4]{1+\frac{((1+a_0{}^2)^n-1)^2}{(1+a_0^2)^n}+a_0^2}-1\right) \leq \frac{1}{4}. \quad (6)$$

The solution of (6) is:

$$a_0 \simeq N^{-\frac{3}{4}}. \quad (7)$$

The dependence the wake field on the own pulse field with $\omega_0/\omega_p = 10$ is shown in Fig.2.

One can see that region fit to particles acceleration is limited from above on the small accelerated field values. Thus that effect is considerable.

We propose to change the distance between the pulses in dependence on the pulses number in sequence and transversal coordinate. The pulses change the shape. The pulses curvature is fited to the distance between the edge next pulses in any longitudinal section is equal $\lambda_p(r)$. The results of modelling see in Fig. 3. One can see that the plane wake wave is excited on the first period after pulse sequence.

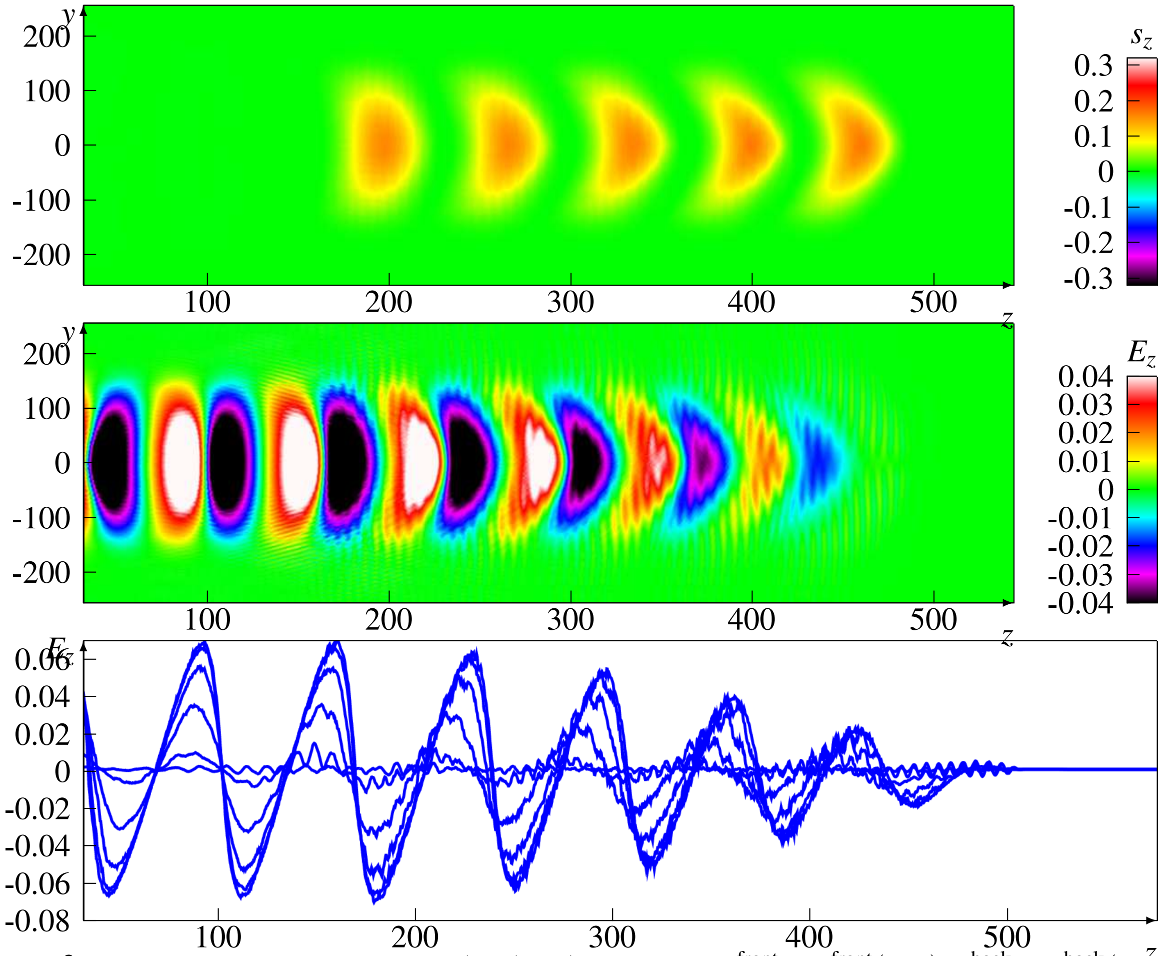

Figure 3: Initial conditions are $a_0 = 0.5$, $\vec{g} = \vec{e}_x + i\vec{e}_y$, $L_\perp = 20\pi$, $L_\parallel^{\text{front}} = L_\parallel^{\text{front}}(N,r)$, $L_\parallel^{\text{back}} = L_\parallel^{\text{back}}(N,r)$, $L^{\text{pl}} = 2000$, $N_{\text{pulses}} = 5$, $d = d(N,r)$. Spatial distribution of laser field $S_z = E_x B_y - E_y B_x$ (a) and E_z(b), cross-section of E_z at y=0,30, 60, 100, 150, 200 on the time moment $t\omega_0 = 640$

Conclusion

The interaction of the laser pulses sequence with plasma layer was researched by full-scale numerical modelling. The wake wave curvature was found. On our opinion the curvature basic reason is the relativistic increase of plasma electron mass in the wake wave. We have proposed to change the shape of each pulses in sequence. The plane wake wave is taken on the first period after pulse sequence.

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