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Nonlinear dynamics of a two-particle complex plasma in vertical alignment

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Two dust particles suspended in the sheath engage in a symmetrical Debye-type interaction as well as an asymmetric attractive interaction due to ion focusing of the vertically streaming plasma [1, 3, 2]. In addition to the linear vertical sheath electric field, which stabilises the particles against the force of gravity, a linear radial electric field is typically applied to trap the particles within the centre of the discharge. The combination of perpendicular electric fields breaks the spherical symmetry of the plasma potential to cylindrical symmetry about the vertical axis with a perpendicular plane of reflection.

It has been experimentally shown that a system of two dust particles can have stable configurations with the particles aligned either parallel or perpendicular with respect to the direction of plasma flow [4, 11, 6], with stability determined by the ratio of the confinement strengths in the horizontal and vertical directions and the interparticle forces [5]. The asymmetric charge polarization around the dust particles results in qualitatively different dynamics in each alignment [7]. The horizontal alignment corresponds to a regime dominated by Hamiltonian dynamics characterized by symmetric particle interactions [8]. In vertical alignment, the lower particle strongly interacts with the non-linear, asymmetric wake of the upper particle resulting in regime dominated by non-Hamiltonian dynamics. In this paper, we describe modeling of the non-linear system in the non-Hamiltonian regime. The model is used to explain the observed behaviour in rf-discharge experiments conducted with two particles.

Consider two vertically aligned particles in the sheath. In this configuration, the lower particle interacts strongly with the wake of the second (upper) particle due to its close proximity to the focus of positive ions deflected by the upper particle. Conversely, it may be assumed that the upper particle does not engage in any attractive interactions with the lower particle as they will screened out over the large separation. This non-reciprocal attractive force breaks the symmetry of the otherwise repulsive interparticle Debye interaction. Exactly where the lower particle lies in relation to the region of enhanced positive ion density will depend on the relative magnitudes of the Debye repulsion, wake attraction and the particle interaction with the vertical confinement well.

We assume that the particle interaction can be separated into a symmetric Debye interaction $\Phi_{\mathrm{D}} = Q_{\mathrm{d}}/(4\pi\varepsilon_0)\mathrm{e}^{-\kappa r}r^{-1}$ and an asymmetric attractive potential which we model by a linearly shielded, ellipsoidal Gaussian ion-wake distribution. The choice of ellipsoidal gaussian distribution was made on the basis of three-dimensional particle-in-cell and molecular dynamics simulations [9, 10] which reveal a well-defined region of enhanced positive ion density behind the dust particles in streaming plasma. Fourier transforming the Poisson equation along the cylindrical symmetry axis and inverting the solution to the ordinary differential equation in ρ gives the following expression for the wake-potential

$$\Phi_{\mathrm{w}}(\rho,z)=\frac{\pi}{\sqrt{8\beta}}\frac{\rho_{\mathrm{i(w)0}}}{\varepsilon_0}\int_{-\infty}^{\infty}\mathrm{d}k_z\,\mathrm{e}^{-k_z^2/4\beta-\mathrm{i}k_z z}J_0(\mathrm{i}K\rho)\int_0^{\rho}\mathrm{d}\rho'\,\mathrm{e}^{-\alpha\rho'^2}\rho' Y_0(-\mathrm{i}K\rho')$$

where α and β are parameters characterizing the decay of the ion-wake number density in the radial and vertical directions, respectively. In a collisionless plasma, the distribution elongates in the direction of the vertical ion flow. Due to collisional charge exchange however, the unperturbed ion distribution becomes approximately spherical. In this case $\alpha=\beta$ and the potential is given by

$$\Phi_{\mathrm{w}}(\rho,z)=\frac{Q_{\mathrm{w}}}{8\pi\varepsilon_0}\exp\left(\frac{\kappa^2}{4\alpha}\right)\frac{1}{r}\left[\mathrm{e}^{-r\kappa}-\mathrm{e}^{r\kappa}+\mathrm{erf}\left(\frac{2\alpha r-\kappa}{2\sqrt{\alpha}}\right)\mathrm{e}^{-r\kappa}+\mathrm{erf}\left(\frac{2\alpha r+\kappa}{2\sqrt{\alpha}}\right)\mathrm{e}^{r\kappa}\right]$$

where $r^2=\rho^2+(z+\ell)^2$.

Then assuming that the particle charge gradient in the vertical direction is small, the total particle interaction is described by the pair of dynamical equations

$$\ddot{z}_1=-\gamma\dot{z}_1-\frac{Q_{\mathrm{d}}}{m}\frac{\partial}{\partial z_1}(\Phi_{\mathrm{D}}(x_1-x_2,z_1-z_2)+V_{\mathrm{conf}}(x_1,z_1,x_2,z_2))$$
$$\ddot{z}_2=-\gamma\dot{z}_2-\frac{Q_{\mathrm{d}}}{m}\frac{\partial}{\partial z_2}(\Phi_{\mathrm{D}}(x_1-x_2,z_1-z_2)+\Phi_{\mathrm{w}}(x_1-x_2,z_1-z_2)+V_{\mathrm{conf}}(x_1,z_1,x_2,z_2))$$

where the confinement potential V_{conf} is assumed to obey the the parabolic approximation $V_{\mathrm{conf}}(x_1,z_1,x_2,z_2)=(1/2)M\omega_\rho^2(x_1^2+x_2^2)+(1/2)M\omega_z^2(z_1^2+z_2^2)$ and the particles are of equal mass M. If we assume that the particles remain aligned along the cylindrical symmetry axis, then the dust particle system moves in a 4-dimensional phase space $(z_1,z_2\,|\,\dot{z}_1,\dot{z}_2)$.

We found subsets of the parameter space permitting bi-stability with two vertically stable attractors and an unstable fixed point in-between. Horizontal stability depends on factors such as the radial confinement strength, the stabilising effect of the wake and the transverse perturbing force from the upper particle.

In general, the vertical alignment can be classified into two sub-regimes. On examining Fig. 1, we notice that at low confinement strengths, the wake dominates and there is just one fixed point corresponding to the situation when the lower particle sits directly inside the ion focus. At the other extreme of strong confinement, the wake-induced equilibrium disappears and the particle lies in the minimum of the confinement potential well. At moderate confinement strengths, two stable fixed points exist, separated necessarily by a third, unstable fixed point. Sweeping the control parameter ω_z in the reverse direction can cause the the stable wake and unstable attractor to collide and annihilate each other in a global saddle node bifurcation.

In two-particle dust experiments [11], continuous upward motion of the lower particle was observed to correlate with increasing characteristic angular frequency of vertical oscillations ω_z. This is explained by the shifting position of the confinement in which the lower particle resides. The continuous upward motion stage was followed by a discontinuous transition when the confinement strength reached a critical value, at which point, the lower particle jumped directly to the horizontal plane. In experiments performed at The University of Sydney, discontinuous vertical motion has been observed prior to the second stage of the transition which may be accounted for in terms of the model by the existence of two stable fixed points in vertical alignment. If we assume that the lower particle is initially inside the wake of the upper particle, then the transition to horizontal alignment can be expected to occur when the system loses stability with respect to horizontal linearized oscillations. Note that increasing of radial confinement strength can do nothing but to further stabilise horizontal oscillations. Increasing vertical confinement has the effect of moving the minimum of the confinement potential upwards, thereby lowering horizontal stability by increasing the transverse perturbing force from the upper particle at that position. If, however, the wake does not lose horizontal stability before it collides with the unstable fixed point, then the particle will jump to the upper equilibrium before the wake equilibrium is able to locally bifurcate. Stability is determined from the sign of the real of part of the eigenvalues of the stability matrix $\lambda = -\gamma/2 \pm \sqrt{(\gamma/2)^2 - \Omega_\pm^2} \approx -\gamma/2 \pm i\Omega_\pm$ where $\Omega_\pm$ denotes the oscillation mode frequencies with zero dampening. Numerics over a range of parameters holding $\gamma = 0$ indicate that the imaginary part is of the order $\Im[\lambda] \sim 10^{-6}$ whereas the magnitude of the real part is of order at most $|\Re[\lambda]| \lesssim 10^{-13}$. We therefore deem the wake to be stable since under realistic conditions

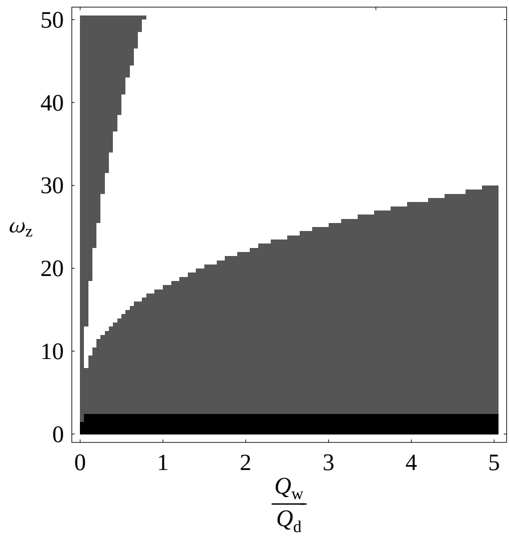

Figure 1: Density plot showing the number of vertically stable equilibria as a function of (Q_w, ω_z) at $\ell/\lambda_D = 3.5$. The grey and white regions correspond to combinations of parameters resulting in one and two equilibria, respectively.

the dampening term will kill off any small oscillations. Thus, since horizontal destabilisation of the wake is infeasible, then the particle must first depart the wake by moving vertically into the minimum of the confinement well. From this position, horizontal stability will be affected by the ratio of the confinement strengths in the horizontal and vertical directions.

Starting from elementary principles, we have developed and tested a new model for dust particle interactions in the plasma sheath. The model has revealed qualitatively new features on the static behaviour of the vertical dust particles in terms of existence of previously unknown equilibria and also on the dynamics of the vertical to horizontal transition.

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