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MODELLING OF LOW-FREQUENCY ALFVÉNIC ACTIVITY IN WENDELSTEIN 7-AS

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Introduction. Fast-ion-driven Alfvén Instabilities (AI) with the frequencies ranging from a few tens of kilohertz to a half of megahertz were observed in stellarators, in particular, in W7-AS [1]. A part of them, having higher frequencies, is associated with the lack of the axial symmetry in stellarators and, thus, do not exist in tokamaks, see, e.g., [2]. In this work, we concentrate on AIs with Lower Frequencies (LF). We show that the physics of LF phenomena in stellarators has also some differences in comparison with that in tokamaks. This can be especially clearly demonstrated when the rotational transform, $\iota(r)$, is non-monotonic. Then in tokamaks there exist Alfvén Cascade Modes (ACM), which manifest themselves as instabilities with the frequencies chirping up [3]. In contrast to this, the instability of Non-conventional Global Alfvén Eigenmodes (NGAE) existing in currentless stellarators and having, like ACM, frequencies above the Alfvén Continuum (AC) can be either bursting, leading to strong thermal crashes [4], or steady-state, with no influence on the bulk plasma. The conditions of existence of NGAE differ from those for ACM in tokamaks, as will be shown in this work.

Basic equations and a general consideration. One can expect that plasma compressibility, $\zeta = \mathrm{div}\boldsymbol{\xi}$, with $\boldsymbol{\xi}$ the plasma displacement, will affect low frequency Alfvén Eigenmodes (AE). Therefore, we take it into account. We proceed from the equations derived for stellarators in Ref. [5]. Eliminating ζ, we obtain the following equation:

$$\frac{1}{r}\frac{d}{dr}r\delta_0\left(\frac{\omega^2-\omega_{G1}^2}{V_A^2}-k_{mn}^2\right)\frac{d\Phi_{mn}}{dr}-\left[\frac{m^2\delta_0}{r^2}\left(\frac{\omega^2-\omega_{G1}^2}{V_A^2}-k_{mn}^2\right)+\frac{k_{mn}}{r}\left(r\delta_0 k'_{mn}\right)'\right]\Phi_{mn}$$
$$+C^{(c)}(\varepsilon_g,\varepsilon_{B,}\varepsilon_G)+C^{(j)}=0, \qquad (1)$$

where Φ_{mn} is a Fourier harmonic of the scalar potential of the electromagnetic field, $\Phi=\sum_{mn}\Phi_{mn}(r)\exp(im\vartheta-in\varphi-i\omega t)$, r, ϑ, φ are the radial, poloidal, and toroidal coordinates, respectively; m and n are the mode numbers; $C^{(j)}$ takes into account a contribution of the plasma current; $C^{(c)}$ describes the coupling between the harmonics of Φ because of the inhomogeneity of the equilibrium magnetic field, B_0, the plasma shaping and compressibility;

$k_{mn} \equiv k_{\|}(m,n) = (m\iota - n)/R$ is the longitudinal wave number, with ι the rotational transform and R the major radius of the torus; ε_B, ε_g, and δ_0 are defined by

$$B_0 = \overline{B}\left(1+\frac{1}{2}\sum_{\mu\nu}\varepsilon_B^{(\mu\nu)}e^{i\mu\vartheta - i\nu N\varphi}\right),\quad g^{rr} = \delta_0(r)\left(1+\frac{1}{2}\sum_{\mu\nu}\varepsilon_g^{(\mu\nu)}e^{i\mu\vartheta - i\nu N\varphi}\right),\quad g^{\vartheta\vartheta} = \frac{\delta_0(r)}{r^2}\left(1+\frac{1}{2}\sum_{\mu\nu}\varepsilon_g^{(\mu\nu)}e^{i\mu\vartheta - i\nu N\varphi}\right), \quad (2)$$

N is the number of the field periods, g^{rr} and $g^{\vartheta\vartheta}$ are metric tensor components,

$$\omega_{G1}^2 = \frac{c_s^2}{\delta_0 r^2}\sum_{\mu\nu}\alpha_{mn}^{(\mu\nu)}\mu^2\left(\varepsilon_B^{(\mu\nu)}\right)^2,\ \omega_{G2}^2 = \frac{c_s^2}{\delta_0}\sum_{\mu\nu}\alpha_{mn}^{(\mu\nu)}\left(\frac{d\varepsilon_B^{(\mu\nu)}}{dr}\right)^2,\ \alpha_{mn}^{(\mu\nu)} = \frac{\omega^2}{\omega^2 - k_{m+\mu,n+\nu N}^2 c_s^2}. \quad (3)$$

We assume that $\alpha_{mn}^{(\mu\nu)} \neq \infty$, i.e., we exclude from the consideration Alfvén-sound resonances because sound waves typically are strongly damped. It follows from Eq. (2) that the point of the local Alfvén resonance is determined by the equation $\omega^2 = k_{\|}^2 V_A^2 + \omega_{G1}^2$. This resonance for $\omega^2 << N^2 c_s^2 / R^2$ in the limit case of $k_{\|} = 0$ is reduced to $\omega = \omega_G$, where ω_G is the geodesic acoustic frequency, $\omega_G^2 \equiv \frac{2c_s^2}{R^2}\frac{\varepsilon_t^2}{\delta_0\varepsilon^2}\left(1+\frac{\iota^2}{2}\frac{\delta_0\varepsilon^2}{\varepsilon_t^2}\right)$, where $\varepsilon_t = -\varepsilon_B^{(10)} > 0$, $\varepsilon = r/R$. Note that this ω_G agrees with the result of Ref. [6] for circular tokamaks.

In the frequency range for which ω is not close to sound resonances, ω_{G1} and ω_{G2} weakly depend on ω. Due to this, when the modes are well localized around a point r_*, and the harmonic coupling is neglected, Eq. (1) can be written in the form of the Schrödinger equation, which has solutions provided that $g > 1/4$, where g is a function given by

$$g^S = \frac{2V_A^2}{(\omega_{A1}^2)''}\left\{\frac{mk_{\|}\iota'}{rR}\left[\delta_0\left(1+\frac{r\iota''}{\iota'}\right)+r\delta_0'\right]-\frac{m^2}{r^2}\frac{\omega_{G2}^2-\omega_{G1}^2}{V_A^2}\right\},\qquad g^T = -\frac{4V_A^2}{(\omega_{A1}^2)''}\frac{mk_{\|}\iota'}{rR}, \quad (4)$$

where g^T and g^S are relevant to tokamaks and currentless stellarators, respectively, $\omega_{A1}^2 \equiv k_{\|}^2 V_A^2 + \omega_{G1}^2$, prime denotes the radial derivative, and all the magnitudes are taken at the point r_* determined by $(\omega_{A1}^2)' = 0$. In the case when $\iota' \approx 0$ in the vicinity of the point r_* and effects of compressibility are not important, $g^S = \delta_0 > 1$. On the other hand, $g^T = 0$ for $\iota' \approx 0$. This means that the condition $g > 1/4$ is satisfied in stellarators but not in tokamaks. Therefore, in the considered approximation, GAE and NGAEs exist in stellarators with non-monotonic $\iota(r)$, whereas taking into account additional factors (the presence of the energetic ions, toroidicity, the plasma density gradient) is required to calculate ACMs in tokamaks [7].

Note that low frequency GAE/NGAE modes with $m >> 1$ are much more sensitive to the magnitude of the rotational transform than gap modes; in particular, they are more sensitive than TAEs by a factor of $2m$. One can see that when $\omega = k_{\|}V_A$,

$$\Delta\omega = m\ \Delta\iota\, V_A / R\,, \tag{5}$$

where $\Delta\omega$ is the change of ω or uncertainty of the GAE/NGAE mode frequency associated with $\Delta\iota$, which is the change of ι or the ι-error bar, and ι and V_A are taken at the point where the mode amplitude is maximum. For instance, $\Delta\iota = 0.001$ leads to $\Delta\omega = 3$ kHz in a hydrogen plasma with the density of $5\cdot10^{13}\,\mathrm{cm}^{-3}$, B_0 = 2.5 T, R = 200 cm, and m=5.

Analysis of W7-AS shots. We selected three W7-AS shots where a steady-state or quasi-steady state low frequency Alfvénic activity was observed:

Shot No.	B (T)	P_{inj} (MW)	$n_e(0)$ (cm^{-3})	$T_e(0)$ (eV)	ι_a	Species	ω, obs (kHz)	m obs.	Moment of time (s)
#39029	2.53	0.44	1.1×10^{14}	540	0.355	D-plasma H-beam	46 33, 35, 38	5 3	0.45
#40173	2.53	0.357	6.6×10^{13}	435	0.34	D-plasma D-beam	16 28	3	0.35
#43348	1.15	≤ 2.0	5×10^{13} 1×10^{14}	400 373	0.35 0.31	D-plasma H-beam	25 40		0.18 0.27

Using AC calculated by the codes COBRAS and AEs calculated by the code BOA-fe, as well as VMEC-calculated equilibria, the $\iota(r)$-profile was reconstructed in the shots #39029 and #40173 (see Figs. 1-3). In addition, it was found that the fraction of hydrogen in the shot #39029 should be about 60 % to fit the experimental data. However, these results neglect the plasma rotation and the concomitant frequency Doppler shift because of the lack of data on the rotation. Therefore, the reconstruction of $\iota(r)$ for the shots #30029 and #40173 should be considered as demonstrating a possibility to use low frequency AIs for plasma diagnostics

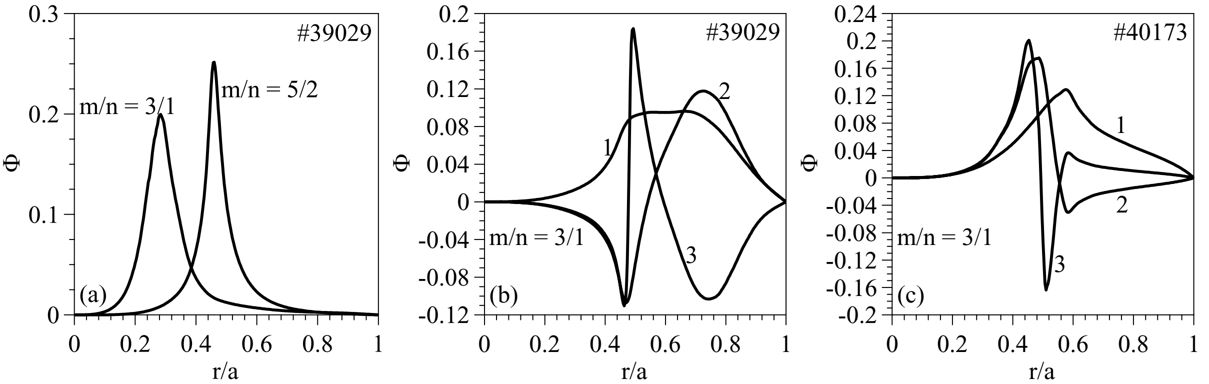
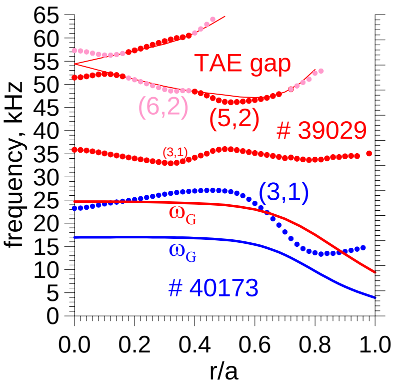


Fig. 1. Calculated AEs in the W7-AS shots #39029 and #40173: (a) GAE in AIs with the lowest frequency (33 kHz) and highest frequency (46kHz), respectively, in the shot #39029; (b) NGAE in AIs with intermediate frequencies of the shot #39029; curve 1, AI with the frequency of 38 kHz; curve 3, 35 kHz; (c) NGAEs relevant to AI in the shot #40173.

rather than finding the precise $\iota(r)$ in these shots. For the shot #43348, we used $\iota(r)$ calculated by VMEC because m and n were not known. We obtained higher frequency of the AEs at $t = 0.27$ s than at $t = 0.17$ s, which agrees with the experiment.



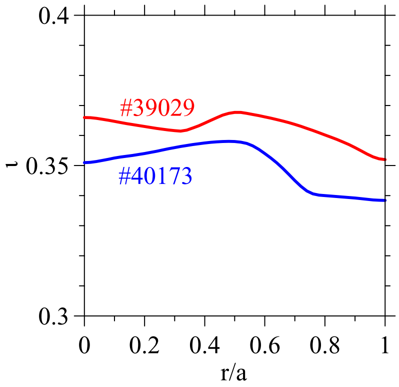


Fig. 2. *Continuum branches with given (m,n) and the geodesic frequency, ω_G.*

Fig. 3. *The reconstructed $\iota(r)$. These iotas are in ranges which are plausible given the VMEC calculation without currents and the expected effects due to the Ohmic current used to compensate bootstrap and NBI driven currents.*

Summary and conclusions.

- Conditions of existence of GAE/NGAE modes are obtained. It is found that they depend on the nature of the rotational transform. NGAE can be more easily observed in stellarators.
- Plasma compressibility affects the conditions of existence of the modes, the mode frequency and structure, gaps in AC.
- Experimental data on low-frequency Alfvénic activity can be used for the reconstruction of the profile of $\iota(r)$. Our work demonstrates this for particular shots of W7-AS. Furthermore, a possibility to use the observation of AI to determine the fraction of hydrogen in a plasma consisting of the mixture of deuterium and hydrogen is demonstrated.
- The mentioned results are obtained using the equations for GAE/NGAE modes derived in the paper, the code COBRAS and the new code BOA-fe.

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