EFFECTS OF VORTEX-LIKE ELECTRON-POSITRON DISTRIBUTION ON SOLITARY WAVES IN PLASMA

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Abstract

Electron-positron plasmas are believed to exist in the early universe, in active galactic nuclei, and in pulsar magnetospheres. Most of the astrophysical plasmas usually contain ions as well in addition to electrons and positrons. In this work, the ion-acoustic solitons are investigated in three-component plasmas, whose constituents are inertial ions, vortex-like electrons and positron. The properties of stationary structures are studied by pseudo-potential approach, which is valid for large amplitude.

Introduction

The localized nonlinear structures of intense electromagnetic waves in two–electron-temperature electron-positron-ion plasmas have been investigated. It was found, that for stationary propagation of finite amplitude electromagnetic waves only supersonic solitons exist Shatashvili and Rao **[1]**. On the other hand, Popel e*t al*. **[2]** have investigated nonlinear dynamics of ion acoustic waves in plasma with cold ion fluid and hot isothermal electrons and positrons. They showed that the amplitude of ion acoustic solitons is reduced due to the presence of positrons in an electron-ion plasma. Alinejad *et al*. **[3]** employed a model comprising nonisothermal electrons with ion fluid and the presence of positrons leads to the possibility solitary waves. It is found that the effect of the positron density change the maximum value of the amplitude and Mach number for which solitary waves can exist. In this work, we study the effects of two-temperature nonisothermal electron that have vortex-like distribution and the presence of positron on electrostatic solitary structures. Hence, we investigate the proprieties of nonlinear ion-acoustic waves (IAW) by using the reductive perturbation method **[4]**.

Formulation

We consider an unmagnetized plasma consisting of singly charge positive ions nonisothermal distribution electrons and Boltzmannian positrons. The quasi-neutrality at equilibrium is written as, $N_{eh0} + N_{el0} = N_0 = N_{i0} + N_{p0}$ where, N_{eh0} (N_{el0}), N_{i0} and N_{p0} are the unperturbed high (low) temperature electron, ion and positron densities respectively. The nonlinear dynamics of one-dimensional low phase speed ion-acoustic waves in such plasma positron are described by,

$$\frac{\partial n_i}{\partial t} + \frac{\partial n_i u_i}{\partial x} = 0 \qquad (1)$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = \frac{\partial \Phi}{\partial x} \qquad (2)$$

$$\frac{\partial^2 \Phi}{\partial x^2} = n_{el} + n_{eh} - n_p - (1-p) n_i \qquad (3)$$

where, n_i, n_p, and $n_{eh(l)}$ are the ion, positron and high (low) temperature electron densities respectively, normalized by N_0 ; u_i is the ion fluid velocity normalized to the ion-acoustic speed $C_s = (T_{\text{eff}}/m_i)^{1/2}$, Φ is the electrostatic potential normalized by T_{eff}/e, $T_{\text{eff}} = T_{\text{efl}} T_{\text{efh}} [\delta_{el} T_{\text{efh}} + \delta_{eh} T_{\text{efl}}]^{-1}$, $\sigma_p = T_{\text{eff}}/T_p$, $\sigma_{el(h)} = T_{\text{eff}}/T_{\text{efl(h)}}$, $p = N_{p0}/N_0$, $\delta_{el(h)} = N_{el(h)}/N_0$. The space coordinate x and time t are normalized by the Debye length $\lambda_D = (T_{\text{eff}}/4\pi e^2 N_0)^{1/2}$ and the plasma period $\omega_p^{-1} = (m_i/4\pi e^2 N_0)^{1/2}$ respectively. The positrons are assumed to have Boltzmann distribution. Thus, we can express the positron density as

$$n_p = n_{p0} \exp(-\sigma_p \Phi) \qquad (4)$$

On the other hand, the electron number density n_{ej} in the presence of trapped particles can be expressed as **[5]**

$$n_{ej} = n_{ej0} \left[\exp(\sigma_{ej}\Phi) erfc\left(\sqrt{\sigma_{ej}\Phi}\right) + \frac{1}{\sqrt{\beta_j}} \begin{Bmatrix} \exp(\sigma_{ej}\beta_j\Phi) erf\left(\sqrt{\sigma_{ej}\beta_j\Phi}\right) & \beta_j \geq 0 \\ \frac{2}{\sqrt{\pi}} exp(\sigma_{ej}\beta_j\Phi) \int_0^{\sqrt{\sigma_{ej}\beta_j\Phi}} e^{y^2} dy & \beta_j < 0 \end{Bmatrix} \right]$$

where, β_j represent the ratio of free electron temperature T_{efj} to the trapped electron temperature T_{etj}, and $j = l$ $(j = h)$ for low (high) temperature electrons. If we expand this n_{ej} for the small amplitude limit and keep the terms up to Φ^2, it is found that $n_{ej}$is the same density for both $\beta_j \geq 0$ and $\beta_j < 0$ and is finally given by

$$n_{ej} = n_{ej0} \left(1 + \Phi - b_j \sigma_{ej}^{3/2} \Phi^{3/2} + \Phi^2/2\right) \qquad (5)$$

where, $b_j = (1-\beta_j)/\sqrt{\pi}$. To study the dynamics of small-amplitude ion-acoustic (IA) solitary waves in the presence of trapped electrons of two different temperatures, we employ a reductive perturbation technique **[4]**.

We introduce the stretched coordinates $\xi = \varepsilon^{1/4}(x - v_0 t)$ and $\tau = \varepsilon^{3/4} t$, where ε is a small parameter and $v_0$is the solitary wave velocity normalized by C_s. The variation n_i, u_i and Φ are expanded as

$$\begin{cases} n_i = 1 + \varepsilon n_{i1} + \varepsilon^{3/2} n_{i2} + \cdots \\ u_i = \varepsilon u_{i1} + \varepsilon^{3/2} u_{i2} + \cdots \\ \Phi = \varepsilon \Phi_1 + \varepsilon^{3/2} \Phi_2 + \cdots \end{cases} \quad (\mathbf{6})$$

Now, substituting these expansions into Eqs.**(1)-(3)** and collecting the terms of different powers of ε, in the lowest order, we obtain

$$\Phi_1 = \upsilon_0^2 n_{i1} = \upsilon_0 u_{i1} \quad (\mathbf{7})$$

where, $\upsilon_0^2 = \left[(1-p)/\left(\delta_{el}\sigma_{\mathrm{efl}} + \delta_{eh}\sigma_{\mathrm{efh}} + p\sigma_{\mathrm{p}}\right)\right]$

For the next order in ε, yields a system of equations that leads to the modified Korteweg-de Vries (MKdV) equation,

$$\partial_\tau \Phi_1 + A\sqrt{\Phi_1}\partial_\xi \Phi_1 + B\,\partial_\xi^3 \Phi_1 = 0 \quad (\mathbf{8})$$

where, $A = 2B\left(\delta_{el} b_l \sigma_{\mathrm{efl}}^{3/2} + \delta_{eh} b_h \sigma_{\mathrm{efh}}^{3/2}\right)$ and $B = \upsilon_0^3/2(1-p)$.

In order to obtain the steady-state solution of this modified KdV equation, we transform the variable ξ, to impose the appropriate boundary conditions, namely

$$\Phi_1(\eta) \longrightarrow 0,\ d\Phi_1(\eta)/d\eta \longrightarrow 0,\ d^2\Phi_1(\eta)/d\eta^2 \longrightarrow 0 \text{ as} |\eta| \longrightarrow \pm\infty \quad \eta = \xi - M\tau \quad (\mathbf{9})$$

Thus, we can express the steady-state solution of this modified KdV equation as,

$$\Phi_1 = \Phi_{1m} sech^4\left(\frac{\xi - M\tau}{w_1}\right) \quad (\mathbf{10})$$

where amplitude Φ_{1m} and the width w are given by,

$$\Phi_{1m} = \frac{(15M/8)^2\left(\delta_{el}\sigma_{\mathrm{efl}} + \delta_{eh}\sigma_{\mathrm{efh}} + p\sigma_{\mathrm{p}}\right)^3}{(1-p)\left(\delta_{el} b_l \sigma_{\mathrm{efl}}^{3/2} + \delta_{eh} b_h \sigma_{\mathrm{efh}}^{3/2}\right)^2}, \text{and } w_1 = \sqrt{\frac{8(1-P)^{1/3}}{M\left(\delta_{el}\sigma_{\mathrm{efl}} + \delta_{eh}\sigma_{\mathrm{efh}} + p\sigma_{\mathrm{p}}\right)^{3/2}}}$$

The MKdV equation is, therefore, inadequate, and one has to find another equation in order to study the nonlinear properties of IA waves, use the new stretched coordinates $\xi = \varepsilon^{1/2}(x - \upsilon_0 t)$ and $\tau = \varepsilon^{3/2} t$, and follow the same procedure used before. Accordingly, for the lowest order of ε we obtain the relations,

$$\Phi_1 = \upsilon_0^2 n_{i1} = \upsilon_0 u_{i1} \quad (\mathbf{11})$$

and for next order of ε we get,

$$\Phi_2 = \upsilon_0^2 n_{i2} = \upsilon_0 u_{i2} \quad (\mathbf{12})$$

$$\left(\delta_{el}\sigma_{\mathrm{efl}} + \delta_{eh}\sigma_{\mathrm{efh}} + p\sigma_p - (1-p)/\upsilon_0^2\right)\Phi_2 = (4/3)\left(b_l \delta_{el} \sigma_{\mathrm{efl}}^{\frac{3}{2}} + b_h \delta_{eh} \sigma_{\mathrm{efh}}^{\frac{3}{2}}\right)\Phi_2^{\frac{3}{2}} \quad (\mathbf{13})$$

For the next order in ε, we obtain a set of equations, which after making use of Eqs. **(11)-(13)**, yields,

$$\partial_\tau \Phi_1 + \acute{A}\partial_\xi\left(\Phi_2\sqrt{\Phi_1}\right) + B\,\partial_\xi^3 \Phi_1 + \mathrm{C}\Phi_1 \partial_\xi \Phi_1 = 0 \quad (\mathbf{14})$$

where, $\acute{A} = 4A/\sqrt{3}$ and $C = -(\upsilon_0^3/2)[(\delta_{el}\sigma_{\mathrm{efl}} + \delta_{eh}\sigma_{\mathrm{efh}})/(1-P) + (3/\upsilon_0^4)]$

The one-soliton solution of Eq. **(13),** $A = 0$, is given by

$$\Phi_1 = \Phi_{2m} sech^2\left(\frac{\xi - M\tau}{w_2}\right) \tag{15}$$

where amplitude Φ_{2m} and the width w_2 are given by $\Phi_{2m} = 3M/C$, and $w_2 = \sqrt{4B/M}$, Eq. **(15)** clearly indicates that both rarefactive and compressive solitons exist. One can observe that the inclusion of the second trapped electron species admits and existence of the two kinds solitons. On the other hand, when $\acute{A}\Phi_2 \longrightarrow 2D\Phi_1/3$, Eq. **(13)** would reduce to

$$\partial_\tau\Phi_1 + D\sqrt{\Phi_1}\partial_\xi\Phi_1 + B\,\partial_\xi^3\Phi_1 + \mathrm{C}\Phi_1\partial_\xi\Phi_1 = 0 \tag{16}$$

Now, substituting $\eta = \xi - M\tau$ in Eq. **(16)** and integrating twice, using (14), we get

$$\frac{1}{2}\left(d_\eta\Phi_1\right)^2 = \frac{M}{2B}\Phi_1^2 - \frac{4D}{15B}\Phi_1^{5/2} - \frac{C}{6B}\Phi_1^3 = -V(\Phi_1, M) \tag{17}$$

$$V(\Phi_1, M) = -\frac{M}{2B}\Phi_1^2 + \frac{4D}{15B}\Phi_1^{5/2} + \frac{C}{6B}\Phi_1^3 \tag{18}$$

For the formation of double layers, we must satisfy the following conditions

$$V(\Phi_m, M) = 0,\ (dV/d\Phi_1)_{\Phi_1=\Phi_m} \text{ and } (d^2V/d\Phi_1^2)_{\Phi_1=\Phi_m} < 0 \tag{19}$$

Produce the double layer solution

$$\Phi_1 = \frac{1}{2}\Phi_m[1 - \tanh((\xi - M\tau)/w)] \tag{20}$$

where, $\Phi_m = 4D/5C$, $M = -16D^2/75C$ and $w = (5/D)\sqrt{-3BC}$. The double layer solution exists only if < 0 .

Summery

In this work, we have studies the effects of the vortex-like distributed electron on solitary waves in four-component plasma. For small but finite amplitude is used the reductive perturbation method **[4]** and the modified K-dV equation is obtained. Our investigation reveals that such a model can support the existence of compressive as well as rarefactive solitons. Additionally, a double layer is obtained.

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