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# Dependence of the resistive wall mode growth rate on the wall thickness

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## 1. Introduction

The resistive wall mode (RWM) growth rate dependence on the wall thickness is analyzed within the single mode model [1] which is extended to incorporate the finite thickness of the wall, as briefly described in [2]. Recent discussion of this problem [3] was based on the numerical results obtained with the ideal-magnetohydrodynamic (MHD) adaptive shooting code AEGIS for the toroidal axisymmetric noncircular plasmas. The method included the use of the energy principle with the solutions of the Euler-Lagrange equations to minimize the energies in various regions.

Our approach is completely analytical up to the point where it gives us the dispersion relation in the explicit form $f(\gamma, d/w) = \Gamma_m$ with $\gamma$ the growth rate, $d/w$ the ratio of the wall thickness to the wall minor radius, and $\Gamma_m$ the quantity determined by the equilibrium plasma characteristics. The model is based on cylindrical approximation, which simplifies the problem allowing the mode separation. In other respects our approach is more general than the modeling [3]. We assume a linear response of the plasma to the external perturbation with the response coefficient (or $\Gamma_m$) as a parameter. This allows to complete the calculations without restrictions on the plasma pressure and current distributions.

## 2. Theoretical model

In the wall with uniform conductivity $\sigma$ and magnetic permeability $\mu$, we have

$$\mu\sigma \frac{\partial \mathbf{b}}{\partial t} = \nabla^2 \mathbf{b}. \tag{1}$$

The boundary conditions for the magnetic perturbation $\mathbf{b}$ at the two wall-vacuum interfaces are

$$\langle \mathbf{n} \cdot \mathbf{b} \rangle = 0, \qquad \langle \mathbf{n} \times \mathbf{b} / \mu \rangle = 0, \tag{2}$$

the brackets mean the jump across the surface. In the plasma-wall vacuum gap and behind the wall, $\mathbf{b} = \nabla\varphi$ and $\nabla^2\varphi = 0$. For $\mathbf{b} = \nabla\psi \times \mathbf{e}_z$ with $\psi = \psi_{mn}(r,t)\exp(im\theta - in\zeta)$, in the cylindrical approximation, these equations are reduced to (prime shows the radial derivative)

$$r(r\psi'_{mn})' - (m^2 + n^2 r^2 / R^2)\psi_{mn} = \sigma\mu \frac{\partial \psi_{mn}}{\partial t} r^2, \tag{3}$$

$$\langle \psi_{mn} \rangle = 0, \qquad \langle \psi'_{mn} / \mu \rangle = 0 . \tag{4}$$

Assuming low $m$ and $n$ and $r/R << 1$ (large aspect ratio), we disregard $n^2 r^2 / R^2$ in (3).

In the vacuum regions ($\sigma = 0$) on the both sides of the wall

$$\psi_{mn} = g r^{m} + h r^{-m} , \tag{5}$$

where $m > 0$, $g$ and $h$ are the time-dependent constants. Behind the wall ($r > r_{out} = w + d$, $w$ is the wall inner radius, and $d$ its thickness) we have $g = 0$, if there are no sources of $\mathbf{b}$. Then equations (4) give us for the outer side of the wall with $\mu = \mu_0$ (permeability of vacuum):

$$(r\psi'_{mn} + m\psi_{mn})_{wall} = 2mgr_{out}^{m} = 0 . \tag{6}$$

Integration of (3) through the wall with condition (6) at $r = r_{out}$ yields [2]

$$\Gamma_m = W_m , \tag{7}$$

$$\Gamma_m \equiv -\left( \frac{r\psi'_{mn}}{\psi_{mn}} + m \right)\Bigg|_{w+0} = -\left( \frac{w\psi'_{mn}}{\psi_{in}} + m \right)\Bigg|_{vac} , \quad W_m \equiv \frac{\tau_\infty}{\psi_{in}} \frac{\partial}{\partial t} \int_{in}^{out} \psi_{mn} x^{-m+1} dx , \tag{8}$$

where $\psi_{in} = \psi_{mn}(w)$, $x \equiv r / w$, '*in*' and '*out*' denote the inner and outer sides of the wall, and

$$\tau_\infty \equiv \mu_0 \sigma w^2 . \tag{9}$$

## 3. Dispersion relation

The $\Gamma_m$ in (7) comes from the boundary conditions (8) at the inner side of the wall. Therefore, $\Gamma_m$ is determined by the solution for $\psi_{mn}$ in the region $r < w$, which includes the vacuum gap and the plasma. When the plasma response to external perturbation is linear, the quantity $\Gamma_m$ is determined by the equilibrium plasma parameters [1].

The right hand side in (7) depends on the solution in the wall, which is, for $\psi_{mn} \propto \exp(\gamma t)$,

$$\psi_{mn} = g I_m(y) + h K_m(y) , \tag{10}$$

where $I_m$ and $K_m$ stand for the standard modified Bessel functions, and

$$y = \sqrt{\gamma \tau_\infty}\, x . \tag{11}$$

This form is convenient for real $\gamma > 0$, though (10) allows arbitrary complex $\gamma$.

In our case, the boundary condition (6) at $r = r_{out}$ is satisfied by

$$g / h = K_{m-1}(y_e) / I_{m-1}(y_e) . \tag{12}$$

Then, with (10)–(12) and $\psi_{mn} \propto \exp(\gamma t)$, we obtain from (8)

$$W_m = y_i F(y_i, y_e) . \tag{13}$$

Here $y_i = \sqrt{\gamma\tau_\infty}$ and $y_e = y_i(1 + d/w)$ represent the inner and outer sides of the wall, and

$$F = \left. \frac{hK_{m-1}(y) - gI_{m-1}(y)}{hK_m(y) + gI_m(y)} \right|_{in} \tag{14}$$

with $g/h$ given by (12). The function $W_m$ depends on the normalized growth rate $\gamma\tau_\infty$ and the ratio $d/w$. For known $\Gamma_m$, equation (7), valid for the wall of arbitrary thickness, with $W_m$ given by (13) turns into equation for $\gamma$. First, we consider its asymptotic solutions.

**4. Asymptotic behaviour**

For a geometrically thin wall, $d/w << 1$, and assuming, in addition, $\psi_{mn} \approx \text{const}$ in the wall (so-called constant $\psi$ approximation valid for $y_i << w/d$), one obtains

$$W_m = \frac{\tau_w}{\psi_{mn}} \frac{\partial \psi_{mn}}{\partial t}, \tag{15}$$

where

$$\tau_w \equiv \tau_\infty d/w = \mu_0 \sigma r_w d\ . \tag{16}$$

Then, for $\psi_{mn} \propto \exp(\gamma t)$, equation (7) is reduced to the standard thin-wall relation [1]

$$\gamma\tau_w = \Gamma_m\,. \tag{17}$$

In the opposite limit, when $\psi_{mn}(y_e)/\psi_{mn}(y_i) \to 0$, $W_m$ can be approximated by

$$W_\infty(y_i) \equiv y_i \frac{K_{m-1}(y_i)}{K_m(y_i)}, \tag{18}$$

which corresponds to $g/h \to 0$ in (10) while keeping $I_m/K_m$ finite at $y = y_i$. More precisely,

$$\frac{I_{m-1}(y_i)}{K_{m-1}(y_i)} << \frac{I_{m-1}(y_e)}{K_{m-1}(y_e)}. \tag{19}$$

A finite difference $y_e - y_i = y_i d/w \geq O(1)$ is needed here. Therefore, for a wall with $d/w << 1$, which is typical for tokamaks, this inequality can be satisfied at very large $y_i$ only.

With $W = W_\infty(y_i)$ the dispersion relation (7) turns into

$$\Gamma_m = y_i \frac{K_{m-1}(y_i)}{K_m(y_i)} = \frac{y_i^2}{2(m-1)} \left[ 1 - \frac{K_{m-2}(y_i)}{K_m(y_i)} \right]. \tag{20}$$

The second expression is convenient for $y_i << 1$ ($\gamma\tau_w << d/w$, which may be of interest for a geometrically thick wall only), where it gives

$$\Gamma_m \approx 0.5\gamma\tau_\infty/(m-1), \tag{21}$$

which is equivalent to the thin-wall result (17) for the wall with a thickness

$$d_{eff} = 0.5w/(m-1)\,. \tag{22}$$

For a geometrically thin wall, at $y_i \geq O(w/d)$ equation (20) is reduced to

$$\Gamma_m \approx y_i = \sqrt{\gamma\tau_\infty}\,. \tag{23}$$

This gives larger $\gamma$ than one obtains from (17) for the same $\Gamma_m$. Larger $\gamma$ means that only a part of the wall plays a role in the process, which should be taken into account in RWM problems.

## 5. Thick versus the thin approximation

The asymptotic behavior of $\gamma$ at small and large wall thickness is described by simple formulas, while for the intermediate range of $d/r_w$ we find $\gamma$ from the dispersion relation (7) with (13) and (14) solved numerically as the equation for the lines of constant $\Gamma_m$. The latter task is much easier that the original one considered in [3], while we find the same decrease of the growth rate of RWM with increasing wall thickness, see Fig. 1.

In a wide range of parameters, our results show excellent agreement, with the numerical results [3] for specified equilibrium configuration. This proves that the single mode model can be a reliable tool for analysis of the wall effects on the plasma stability in tokamaks. In particular, it can be used in the cases when the traditional thin-wall theory is not longer valid. This may be the case when $\gamma$ becomes much larger than $\tau_w^{-1}$.

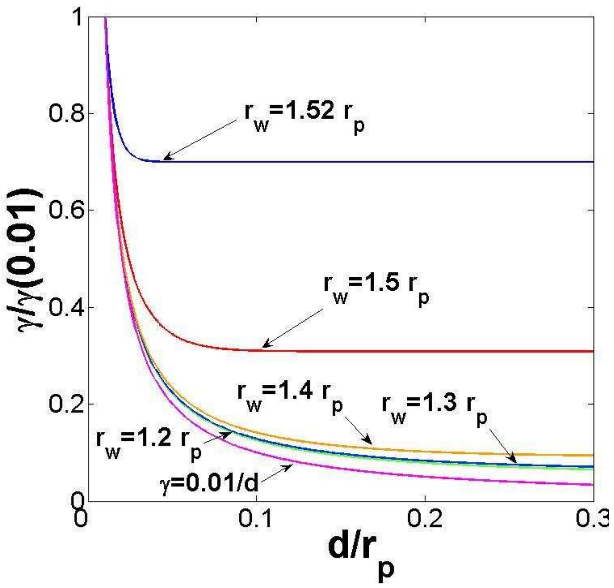


*Fig. 1. RWM growth rate $\gamma$ vs the wall thickness $d$ at different positions $r_w$ of the wall, as described by Eq. (7) with $W_m$ given by (13). Here $r_p$ is the plasma radius, $\gamma(0.01)$ is, in each case, the growth rate at $d/r_p = 0.01$. The lowest curve shows the 'thin-wall' $\gamma$ as described by (17).*

## References

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