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Ion collection by a sphere calculated over the entire range of charge-exchange collisionality

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Abstract

The ion collection by a sphere in a stationary plasma has been calculated accounting fully self-consistently for plasma shielding and charge-exchange collisions over the complete collisionality range from collisionless to collisional continuum-like. The computational results from the Specialized-Coordinate Electrostatic Particle and Thermals in Cell (SCEPTIC) code, agree to 2% accuracy or better with prior asymptotic approximations in the regimes of their applicability. They confirm the significant enhancement (by factors up to 5 when Debye length is large) of the ion collection flux by collisions at intermediate collisionality. An explicit analytic fit to the flux is given for easy evaluation.

1 Introduction

Plasma ion collection by an object idealized as a sphere is important for the operation of Langmuir probes, for spacecraft and other immersed objects, and especially for grains in dusty plasmas. The effect of collisions has long been known (e.g. [1, 2, 3]) to be important, and it has been hypothesised that their effect can be to *enhance* collection towards that of the ABR[4] (radial motion) model by the destruction of angular-momentum. This effect is counterbalanced by radial drag and eventually, where a continuum diffusive calculation of the ion collection[5] is appropriate, flux becomes inversely proportional to the collisionality, because increasing collisionality decreases the diffusivity.

Recently, a full-scale semi-analytical quantitative kinetic-theory calculation of the effects of charge-exchange collisions at low collisionality in spherical geometry has been published by Lampe et al[6] accurate to first order in the ratio of the thermal mean-free-path to the plasma shielding length.

The Particle-in-Cell code SCEPTIC[7, 8] has been upgraded to include constant-collision-frequency charge-exchange collisions. As part of the validation of this code for future flowing plasma work, extensive simulations of spherically symmetric cases have been performed to provide comprehensive self-consistent values of the ion flux. This paper is a summary of the results, recently published as [9].

2 Results

Fig 1 shows the SCEPTIC results compared with prior approximations for two values of the Debye length (relative to sphere size), plotted versus collisionality. The OML approximation, is obtained at very low collisionality ($\lesssim 10^{-4}$), but even moderate collisionality

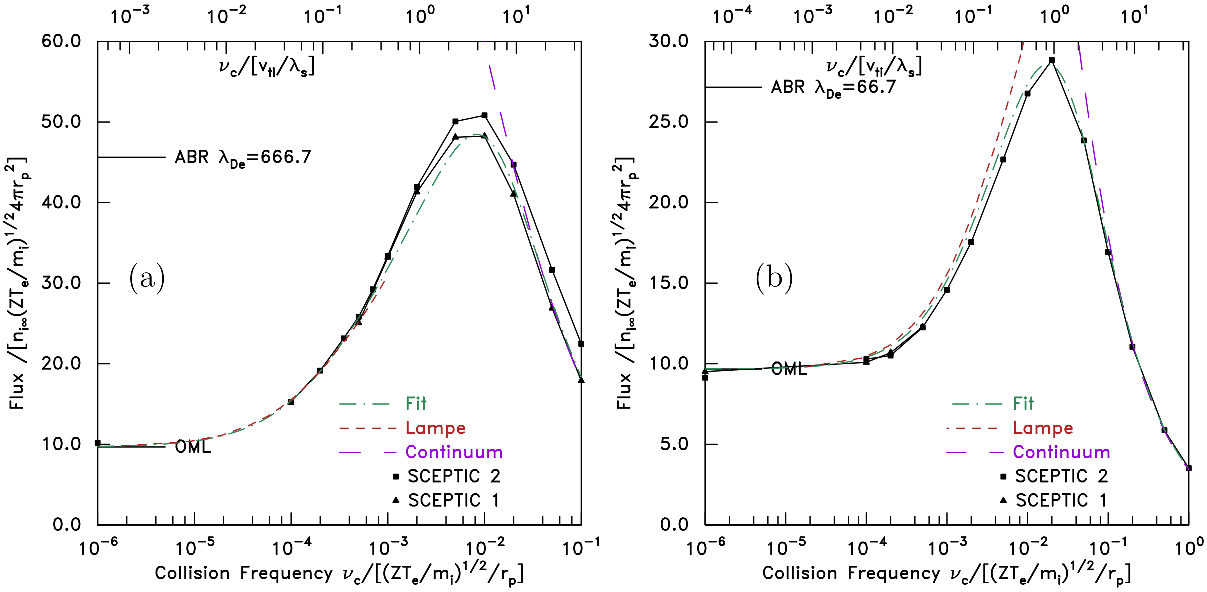

Figure 1: Ion collection flux for $\lambda_{De} = 666.7r_p$ (a), and $\lambda_{De} = 66.7r_p$ (b), $T_i = 0.01T_e$, to a floating sphere in Ar^+ plasma, as a function of (charge-exchange) collision frequency. Points SCEPTIC 1 and 2 correspond to alternative ion reinjection schemes, applied at the computational boundary ($r_b = 100r_p$ in this case). Scheme 1 is the most appropriate for high collisionality.

begins to enhance it. The curve of Lampe's calculation agrees extremely well with the SCEPTIC results for the parameters which he used (Fig. 1(a)), and quite well even for much shorter Debye length. Provided an appropriate ion reinjection scheme is used, SCEPTIC also agrees extremely well with the continuum calculation at high collisionality where it applies. In between, SCEPTIC shows that the flux peaks at approximately the ABR value (or slightly more). This observation holds good over the range of Debye lengths ($\lambda_{De} \gg r_p$) for which the ABR value substantially exceeds the OML value.

Figure 2 shows the radial ion density dependence is in good agreement with the Lampe profile at low collisionality, indicating the importance of trapped ions.

3 Analytic Fit

Results for a range different ion temperatures, masses, and Debye lengths have been fitted by an analytic approximation labelled "Fit" in Fig. 1, as follows.

The OML value[10] for ion collection current from a Maxwellian distribution with density n_∞ and thermal velocity $v_{ti} = \sqrt{2T_i/m_i}$ to a sphere of potential ϕ_p, can be written:

$$j_{OML} = (2\sqrt{\pi})^{-1}\, n_\infty v_{ti}\, (1 - e\phi_p/T_i)\,, \tag{1}$$

The self-consistent OML floating potential solution can be approximated to accuracy 2% for $0.003 \lesssim T_i/T_e \lesssim 1$ by

$$e\phi_p/T_e = -3.6 \ln\left(\frac{m_i T_i}{m_e T_e}\right) + 0.018 \ln\left(\frac{T_e}{10.T_i}\right) \ln\left(\frac{m_i}{80.m_H}\right), \tag{2}$$

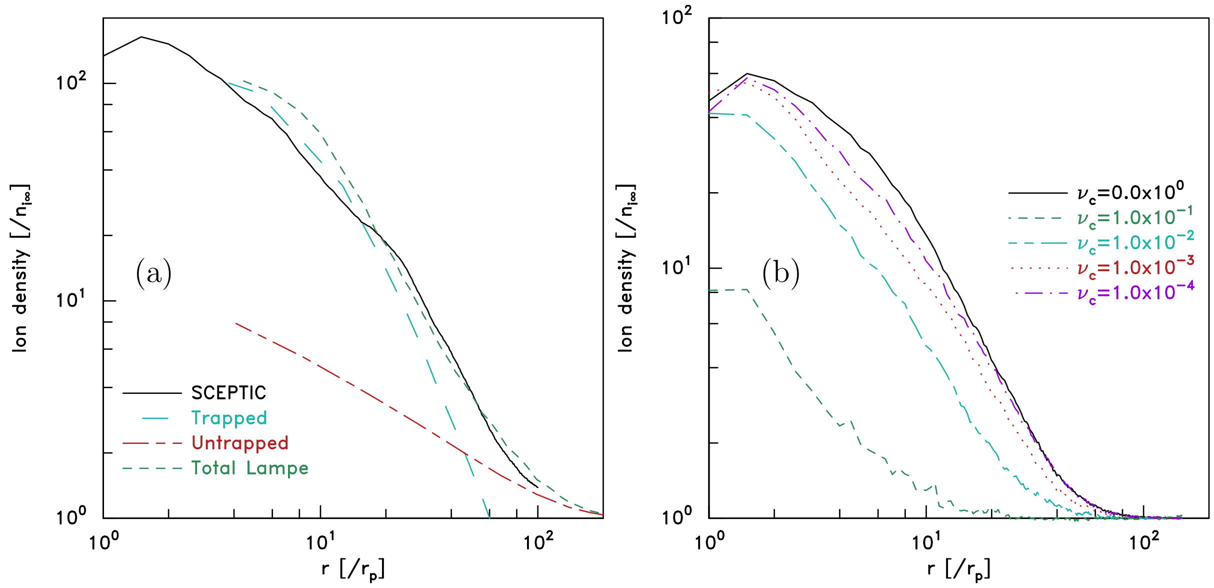

Figure 2: Radial profiles of ion density. (a) Total SCEPTIC density and corresponding analysis of Lampe; $\lambda_{De} = 666.7r_p$, $T_i = 0.01T_e$, $r_b = 100r_p$, Ar^+. (b) Variation with collisionality (ν_c in units of c_s/r_p; $c_s = \sqrt{T_e/m_i}$); $\lambda_{De} = 167r_p$, $T_i = 0.01T_e$, $r_b = 150r_p$, Ne^+.

where m_H is the proton mass and ions are assumed singly charged. The value of j_{OML} can be obtained directly from this floating potential.

The analytic approximation to the SCEPTIC results combines currents for modified OML and collisional limits in the form

$$j_i = (1/j_{io}^w + 1/j_{ic}^w)^{-1/w}, \tag{3}$$

(with an ad hoc weighting: $w = [0.37 - 0.067\ln(T_i/T_e)]\ln(\lambda_{De}/r_p) - 1)$. Here the orbital-regime current is

$$j_{io} = j_{OML}[1 + \ln(1 + 17.\nu_s + 5.\nu_s^2)] \tag{4}$$

where $\nu_s \equiv (\nu_c\lambda_s/v_{ti})/[0.9+0.1(100T_i/T_e)^{1.5}]$, is the collision frequency scaled to screening length (λ_s), but with the factor $[0.9 + 0.1(100T_i/T_e)^{1.5}]$ used empirically to match higher T_i SCEPTIC results. The expression for the continuum current (for floating sphere) is

$$j_{ic} = \frac{n_\infty T_e}{m_i r_p \nu_c} 0.52 \left(\ln(1.1 + 30.\sqrt{m_i/m_H}\,\nu_c r_p/c_s)\right)^{1.15}. \tag{5}$$

4 Comparison to other results and Summary

Figure 3 shows some representative SCEPTIC results for the floating potential and corresponding flux. These are chosen to compare with the earlier molecular-dynamics results of Zobnin et al[11]. Rather satisfactory agreement is obtained. Zobin's results have recently been fitted empirically by Vaulina et al[12] but their fit does not match even the Zobnin results as well as the fit presented here, in part because Vaulina et al use an incorrect expression for the collisional limit, and therefore they do not reproduce the correct values there.

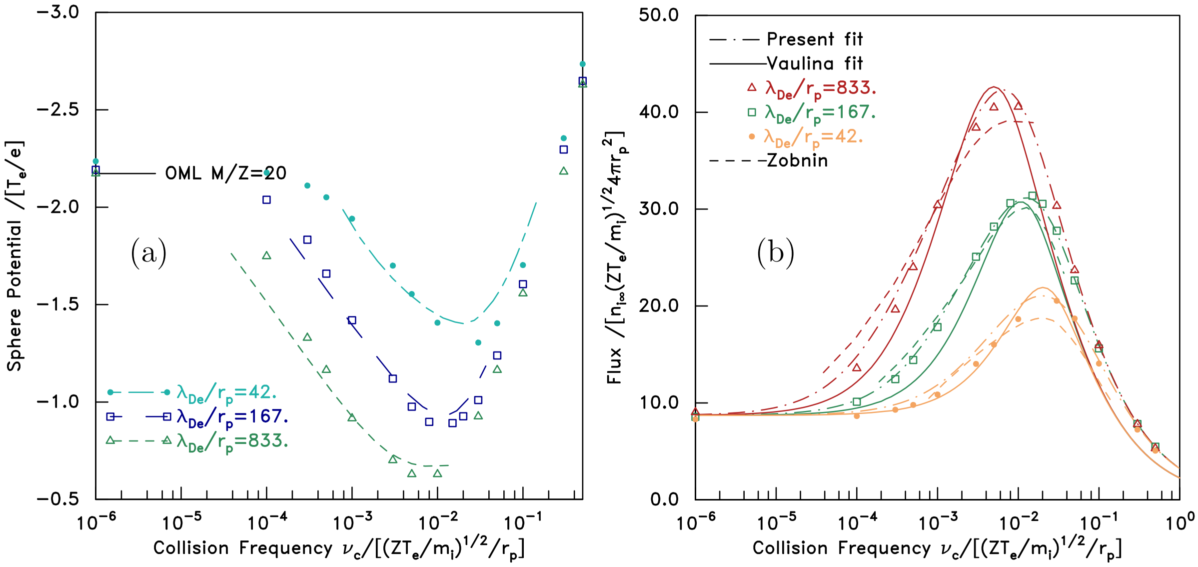

Figure 3: (a) Floating potential for $\lambda_{De} = 833, 167, 42r_p$, $T_i = 0.01T_e$, Ne^+ from SCEPTIC (points). Compared with the molecular modelling of Zobnin et al[11] (lines). (b) Corresponding Ion collection flux. Comparison of the SCEPTIC results (points) with the present fit (Eq 3, dot-dash lines), with the fit of Vaulina et al[12] (solid lines), and with the calculations of Zobnin et al[11] (dashed lines).

In summary, we have calculated fully self-consistent values of the ion collection by a sphere over the entire range of collisionality from the OML to the continuum limits, in a stationary plasma. We thus resolve in principle this long-standing quantitative question, to which much prior qualitative speculation and partial evaluation has been devoted. Collisions do enhance collection, but quantitatively the flux reaches the ABR value (actually slightly more) only in a relatively narrow range of intermediate collisionality.

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