NUMERICAL DESCRIPTION OF DUST PARTICLES
IN PLASMA SHEATH

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Introduction

The interface between plasma and its surrounding surfaces (walls, electrodes, substrates) is formed by a self-organizing structure, called the plasma-sheath. To gain additional insight, micro-sized particles can be used as electrostatic probes. Due to charge carrier fluxes these particles acquire a negative surface charge, allowing for trapping them within the plasma sheath. The particles attain an equilibrium position, where the sum of all acting forces vanishes. In our case, the system is dominated by gravitational force, electrostatic force and ion drag, while neutral drag, thermophoresis and photophoresis are of minor importance. The approach of suitable electrostatic particle probes has been successfully demonstrated in front of the powered electrode of a capacitively coupled rf discharge [1]. In the present work we focus on the behaviour of micro-particles of different sizes in front of a grounded or additional biased adaptive electrode, which is not the powered electrode.

Experimental

The experimental setup is shown in figure 1. A typical asymmetric, capacitively coupled rf plasma (13.56 MHz) in argon (1-10 Pa) is employed to charge the particles which are spherical melamine-formaldehyde (MF) particles of different diameter. The upper electrode is rf driven with a power of 10 W. The lower electrode is the so called adaptive electrode (AE). It consists of 101 identical square segments surrounded by 4 larger segments and an outer ring electrode. Each segment can be biased independently with dc and/or ac voltage. This arrangement allows distinct local manipulations of the plasma sheath to create different static or time dependent shapes of horizontal confinement of suspended particles. In dependence on the discharge conditions we measured electron densities of $10^9 - 10^{11}$ cm$^{-3}$, electron temperatures of 0.8 – 2.8 eV, and plasma potentials with respect to ground of 20 - 30 V for the pristine plasma [2]. The injected particles were illuminated using a laser at 532 nm, position, motion as well as the total plasma emission were monitored with a CCD camera.
For the evaluation we measured the relation between resonance frequency $f_0$ and equilibrium position $x_0$ of single MF-particles of different diameters at different positions above the central segment of the AE as shown in figure 2.

Sheath model for adaptive electrode

The equilibrium position of a negatively charged particle ($x_0$) is given by $q(x_0)E(x_0) = mg$, if drag forces and phoretic effects are neglected. Here $m$ denotes particle’s mass, $E(x)$ the electric field strength at position $x$, and $g$ the acceleration of gravity. A sinusoidal variation of the bias voltage at the central segment of the AE induces the particle to vertically oscillate around its equilibrium position. For small amplitudes this oscillation is harmonic and the particle’s charge is approximately constant. The resonance frequency of the particle at position $x_0$ is given by $\omega_o^2(x_0) = [-q(x_0)/m \ (dE/dx)]_{x_0}$. Combining this equation with the equilibrium condition, $m$ can be eliminated and the resulting simple differential equation can be solved by separation to determine the electric field strength. Formal integration yields:
Equating the negative integral over the electric field across the sheath with the sheath voltage fixes the value of $E(0)$ at the surface of AE.

The potential $U$ in the sheath is given by Poisson’s equation:

$$\frac{\partial^2 U(t,x)}{\partial x^2} = -\frac{e\left[n_i(x) - n_e(x,t)\right]}{\varepsilon_0}$$

with boundary conditions $U(t,0)=U_0(t)$ and $U(t,s)=0$, where $n_i$ and $n_e$ are local ion and electron number densities, respectively. The coordinate $x$ is oriented upward from the AE to the plasma. We relate the zero potential to the sheath edge at $x=s$. Potential $U_0(t)$ of the electrode will be specified later. The electrons are assumed to be Maxwellian, their density is given by the Boltzmann relation: $n_e(t,x) = n_e \exp\left(\frac{eU(t,x)}{kT_e}\right)$

where $n_e$ is the electron density at the sheath edge. The ions are considered cold, their behavior in the sheath is described by ion continuity and ion motion:

$$n_i(x)v_i(x) = \text{const.} \quad m_i v_i \frac{d v_i}{d x} = \left(-e \frac{\partial U}{\partial x}\right) + \frac{1}{\lambda_i} m_i v_i^2$$

with boundary conditions $n_i(s) = n_i$, $v_i(s) = -v_B$. Here $\lambda_i$ is the ion mean free path and $v_B = \sqrt{kT_e/m_i}$ is the Bohm velocity. The sheath thickness $s$ is determined by additional boundary condition:

$$\left\langle \frac{\partial U(t,s)}{\partial x} \right\rangle = \frac{kT_e}{e\lambda_i}$$

The total current density at the electrode consists of electron and ion part: $J_0(t) = J_e(t) - J_i$.

$$J_e(t) = \frac{1}{4} e n_e(t,0) \sqrt{\frac{8kT_e}{m_e}}, \quad J_i = e n_i v_B.$$  

The electron density $n_e$ at the electrode is determined from the Boltzmann distribution function. In the case of capacitively coupled rf discharge the average of total current has to be zero but additional voltage brought to the electrode pixel violates this condition.

For the pixel voltage $U_0(t)$ and current density $J_0(t)$ we take their average values from measurement:

$$<U_0(t)> = U_{dc}, \quad <J_0(t)> = J_{dc}.$$  

Neglecting higher harmonics we approximate the time behavior of $U_0(t)$ by sinusoidal: $U_0(t) = A_0 + A_1 \sin(\omega t)$, with coefficients $A_0, A_1$ uniquely determined by eq.(7).

We get immediately $A_0 = U_{dc}$. Coefficient $A_1$ is obtained by eq.(8) ($I_0$: Bessel function):
\[
\ln I_0 \left( \frac{A_i}{kT_e} \right) = \frac{1}{2} \ln \left( \frac{2\pi m_e}{m_i} \right) - \ln \left( 1 + \frac{J_{dc}}{J_i} \right) - \frac{eU_{dc}}{kT_e} \quad (8)
\]

For \( J_{dc} = 0 \) the above equation reduces to the standard condition for capacitive rf discharges.

Results and discussion

In figure 3 experimental and theoretical results for the electric field strength as a function of the position in the plasma sheath above the AE are presented. The small MF particle of 1.1 \( \mu m \) diameter is levitated at positions of low electric field near the sheath edge and the heavy particle (9.62 \( \mu m \)) at positions around the middle of the sheath.

The experimental results as well as the theoretical curves demonstrate the expected expansion of the plasma sheath with increasing negative dc bias voltage. As can be seen, the predicted expansion of the calculations is obviously smaller than the values obtained from experiments. The theoretical curve for dc bias of -60 \( V \) appears approximately at the same position as the experimental values for -40 \( V \). However, each of both data sets shows nearly an equidistant shift of position with increasing negative bias of constant steps and the shape of the curves remains almost unchanged.

**Fig. 3: Electric field strength from experiments (symbols) for different negative dc bias voltages together with the corresponding calculated curves (dotted) above the AE.**

Furthermore, the experiment shows that the electric field strength at different positions of the particles of same size is nearly constant. We can conclude that the particle charge is unchanged and the charged particle simply moves with increasing negative bias voltage to new position where the electric field satisfies the equilibrium condition.

References