

Search for quasi-isodynamic configurations with diminished parallel current density

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Previous quasi-isodynamic configuration with poloidally closed contours of B

Quasi-isodynamic [1] (qi) configurations have been previously found by computational optimization with high stability β limit, good neoclassical confinement properties and excellent fast particle collisionless confinement for configurations with poloidally closed contours of the strength of the magnetic field B [2,3]. It was shown analytically [3] that the secondary parallel current density in qi configurations remains contained within each plasma field period, namely, between the cross-sections with maximal magnetic field strength B . In the qi configurations considered hitherto, the divergence of the current density perpendicular to the magnetic field lines changes sign only once along the magnetic field within one field period. From this it follows that the parallel current density cannot change sign along the magnetic field within one period. Thus, because of the vanishing net parallel current, the parallel current density exhibits a dipole component which impairs MHD stability at very high β in the qi situation considered here for configurations with shallow magnetic well in the associated vacuum magnetic field. (Note that a different possibility to eliminate this current density is quasi-helical symmetry [4].) The search for possible ways to diminish this current density in quasi-isodynamic configurations constitutes the subject of this work.

Search for a new qi configuration

For this search a two-staged approach has been taken. In a model investigation it is clarified that quasi-isodynamicity is compatible with vanishing dipole current density; then a configurational investigation establishes a geometry realizing the essential features of this model.

First, a near-axis model for the strength of B in magnetic coordinates s, θ, ϕ

$$\begin{aligned} B &= B_{0,0} - B_{0,1} \cos \phi + s^{\frac{1}{2}} B^{(1)} \\ &= 1 - b_{0,1} \cos \phi + s^{\frac{1}{2}} \sum_{-n}^n b_{1,n} \cos(\theta - n\phi) \end{aligned}$$

is investigated. Within such a model the condition for stationarity of the second adiabatic invariant \mathcal{J} reads [1, 5]

$$0 = \cos \iota_p \phi \sum_{-n}^n b_{1,n} \cos(n\phi) + \sin \iota_p \phi \sum_{-n}^n b_{1,n} \sin(n\phi)$$

with ι_p the rotational transform per period and the condition for vanishing dipole component of the parallel current density is $(1/B^2)_{1,0}$ to vanish. Computational minimization of the stationarity condition with this constraint shows that stationarity can be approximated well. The structure of B found suggests an initial condition in configurational space. Computational optimization in configurational space for stationary \mathcal{J} and vanishing $(1/B^2)_{1,0}$ finds a consistent geometry.

Results

The model investigation used a prescribed mirror coefficient $b_{0,1} = 0.3$ and a prescribed value of rotational transform per period, $t_p = \frac{1}{6}$. Since the period is required to have a dominant internal structure, the Fourier terms with $|n| = 2$ should yield the largest contributions (roughly of the order of the inverse toroidal aspect ratio). Fig. 1 shows a simple example of an optimization with $b_{1,2} = 0.02$ and only five additional variables which already leads to a linear reduction of the stationary condition by a factor of about 10. The resulting structure of B clearly shows the additional (to the regions of maximum and minimum B) two regions with $\partial B / \partial \theta \approx 0$.

The above information is sufficient for a starting point in configurational space characterized by a geometry with similar rotational transform and bumpiness as well as sufficient freedom in the geometry of the magnetic axis. Approach through optimization to the structure of B of Fig. 1, to stationarity of \mathcal{J} and vanishing $(1/B^2)_{1,0}$ then leads to the results shown in Figs. 3 - 5. Apart from bumpiness and ellipticity the geometry of the configuration is characterized by an extended nearly straight section enclosing the maximum of B in the equatorial plane and a nearly straight section enclosing the minimum of B inclined with respect to this plane (Figs. 3 and 5). Figure 4 shows the extent to which the stationarity of \mathcal{J} is realized.

This case study has been made at $\beta \approx 0$ and without higher-order shaping of the flux surfaces. Accordingly, the configuration has a magnetic hill and a minimum- \mathcal{J} structure. In still ongoing work it has been used as a starting point for MHD stable, high- β maximum- \mathcal{J} configurations.

Summary

In the context of quasi-isodynamic stellarators with poloidally closed contours of the magnetic field strength it is investigated whether their β -sensitivity can be reduced as compared to Ref. 1. A new structure of a period is introduced in which the B contours change their inclination (with respect to $\phi = const$) along a magnetic fieldline within half a period; this makes it possible to eliminate $j_{||1,0}$ for stationary \mathcal{J} . The associated near-axis geometry has been determined.

Acknowledgment

Part of the computations of this work has been performed on the NIFS LHD Numerical Analysis Computer SX-8.

References

- [1] S.Gori, W. Lotz, J. Nührenberg, Proc. Joint Varenna-Lausanne Int. Workshop on Theory of Fusion Plasmas (Bologna: Editrice Compositori, 1996) p.335.
- [2] M.I. Mikhailov et al, Nucl. Fusion **42** (2002) L23.
- [3] A.A. Subbotin et al, Nucl. Fusion **46** (2006) 921.
- [4] J. Nührenberg and R. Zille, Phys. Lett. **A129**(1988)113.
- [5] V.D. Shafranov, Plasma Phys. Control. Fusion **43**(2001)A1.

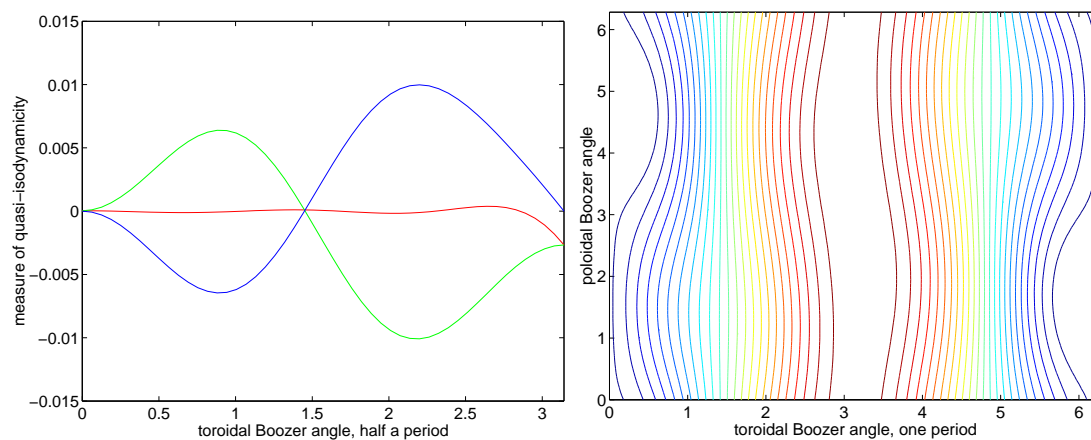


Figure 1: Left: For model of B : approach to stationarity of \mathcal{J} with the constraint $j_{||1,0} = 0$; first term of stationarity condition (green) and second term (blue) essentially cancelling to result (red). Right: the resulting structure of B (maximum (red), minimum (blue)) showing the change of inclination of the contours along a field line within half a period.

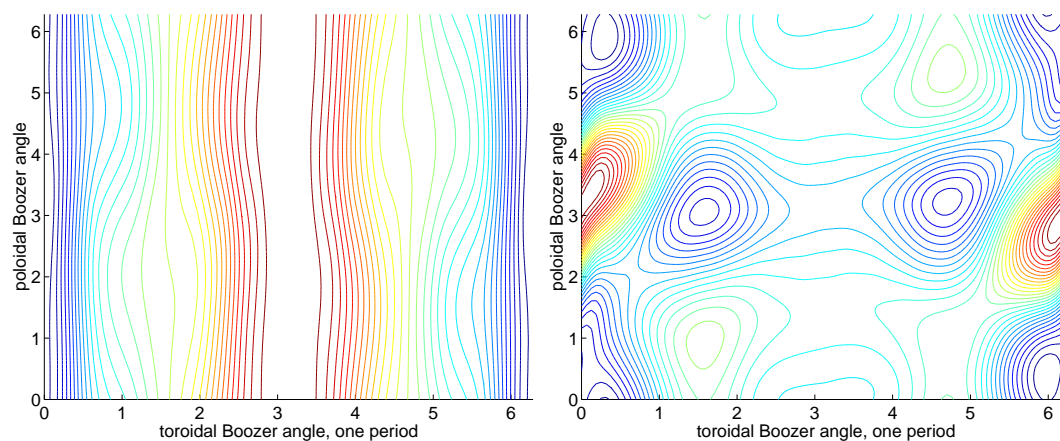


Figure 2: Left: similar to fig. 1 (right), but here the structure of B in the geometry seen in Figs. 3 and 5 obtained via configuration optimization. Right: the associated structure of the parallel current density ($j_{||}/B$) showing $j_{||1,0} = 0$.

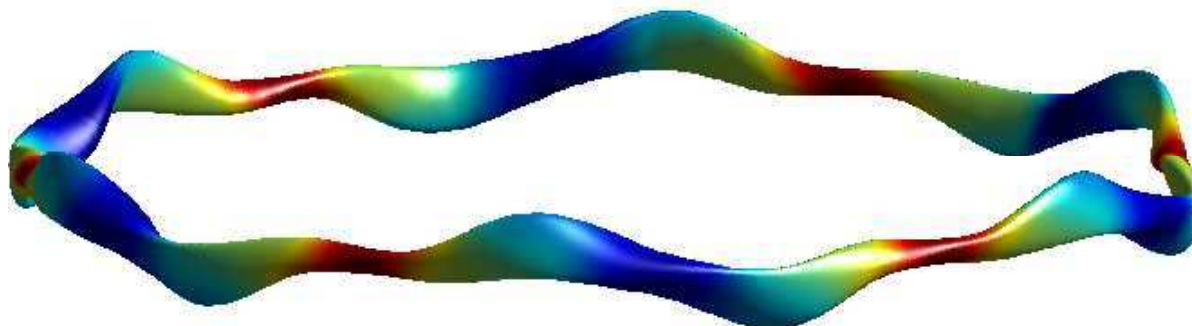


Figure 3: Boundary magnetic surface showing the magnetic topography.

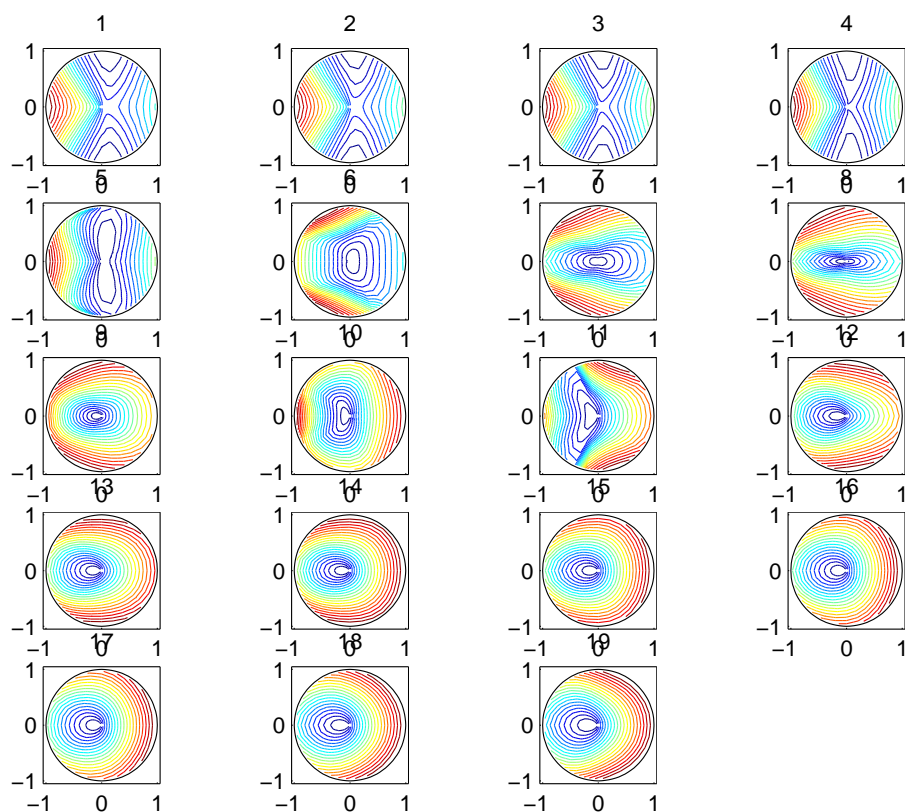


Figure 4: Structure of \mathcal{J} in the configuration obtained for different values of B , B_{ref} , at which trapped particles are reflected; 1 - near the minimum of B , 19 - near the maximum of B .

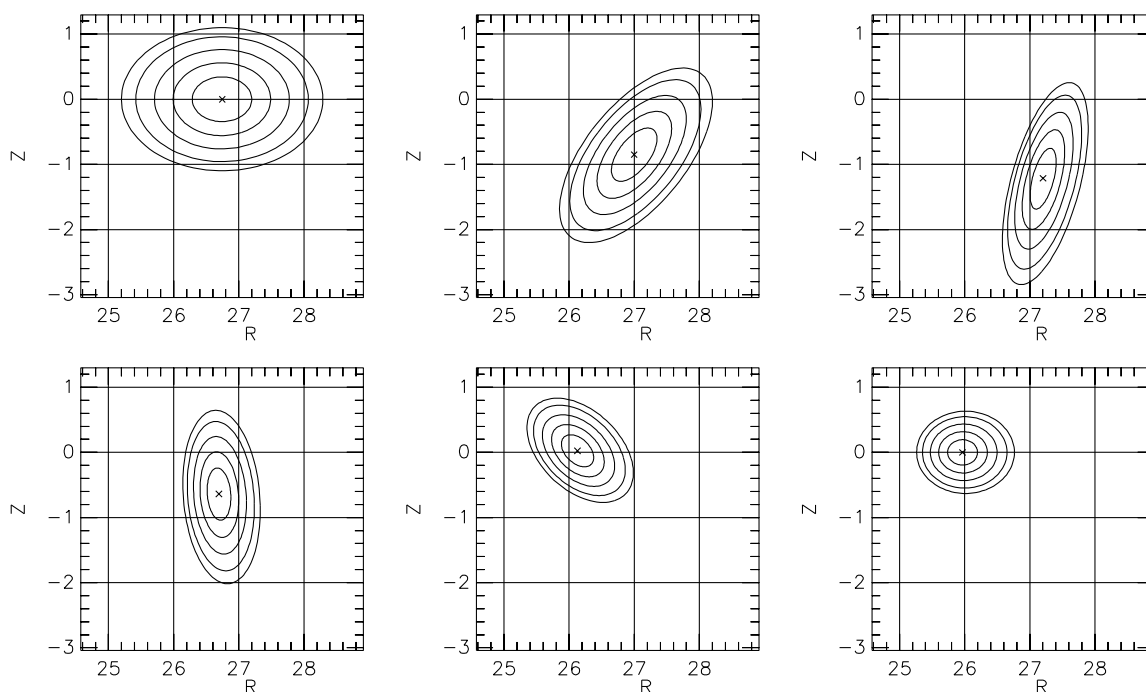


Figure 5: Cross sections of magnetic surfaces of the configuration found along half a period beginning with the minimum of B and ending at the maximum of B .