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On O-X mode conversion in spherical tokamaks

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Introduction

The O-X-B mode conversion scheme is paid much attention to nowadays. Its efficiency is determined mainly by ordinary (O) to extraordinary (X) mode conversion process near the O mode cut-off surface. The mode O-X process being general physical phenomena was examined a couple of decades ago for 1D inhomogeneous plasma. The first who examined 2D model of the O-X mode conversion was Weitzner [1]. His conclusion concerning total conversion of the O mode into the X mode has not been confirmed by later studies [2] and [3]. The theory developed in [3], accounting for dependence of the dielectric tensor on the poloidal angle and the finite but small poloidal magnetic field, and, to some extent, in [2], ignoring the poloidal component of the magnetic field, is valid with respect to stellarator and standard tokamak but, unfortunately, not applicable to the case of spherical tokamak, where the poloidal and toroidal components of the magnetic field are comparable. The goal of the present paper, filling in this theoretical gap, to examine the 2D O-X mode conversion of the incident Gaussian beam in the plasma with an arbitrary poloidal magnetic field.

Assumptions and basic equations in the O-X mode conversion region

Assuming high localization of the conversion region, we neglect both the curvature of the magnetic flux surfaces and the magnetic field shear. We suggest also the straight magnetic field lines. It is useful for our aims to introduce two Cartesian co-ordinate systems (x,ζ,ξ) and (x,y,z) with its common origin located on the O-mode cut-off surface. The co-ordinate x is the flux surface label, ζ and ξ are co-ordinates perpendicular and parallel to the magnetic field on the magnetic flux surface, y and z are co-ordinates imitate poloidal and toroidal angles. The first co-ordinate system seems to be natural to represent the cold dielectric tensor with components $\varepsilon_\pm = 1-\omega_{pe}^2/\left(\omega^2 \pm \omega\omega_{ce}\right)$ and $\eta = 1-\omega_{pe}^2/\omega^2$, where ω_{pe}, ω_{ce} are electron Langmuir and cyclotron frequencies, respectively. The second one, rotated against the first on an angle θ, is convenient for the 2D problem, $\omega_{pe}(x)$, $\omega_{ce}(x,y)$. The transformation from y, z components to ζ, ξ components is $y=\cos(\theta)\zeta+\sin(\theta)\xi$, $z=-\sin(\theta)\zeta+\cos(\theta)\xi$. We restrict ourselves to the case of not extremely strong plasma inhomogeneity, $\delta=\max\left[|\nabla\eta|,|\nabla\varepsilon_\pm|\right]\ll k_0$, $k_0=\omega/c$, when geometrical optics can be applied except possibly near cut-off surfaces. Since the O-X conversion occurs in a nearest vicinity of the O mode cutoff surface, where the perpendicular wave vector components are small $|\partial_{x,\zeta}| \ll k_0$, $\partial_{x,\zeta} \equiv \partial/\partial(x,\zeta)$ and the first component of the electric field $\mathbf{E} = \left(E_-, E_+, E_\xi\right)$, $E_\mp = (E_x \mp iE_\zeta)/\sqrt{2}$ is negligible compared to the last two $E_- \ll E_+, E_\xi$, we can represent Maxwell's equations for a monochromatic wave $\exp(-i\omega t)$ as

$$\begin{aligned} \sqrt{2}\left(\partial^2_{\xi\xi} + k_0^2\varepsilon_+(x,y)\right)E_+ - \left(\partial_x + i\partial_\zeta\right)\partial_\xi E_\xi = 0 \\ -\left(\partial_x - i\partial_\zeta\right)\partial_\xi E_+\sqrt{2} + k_0^2\eta(x)E_\xi = 0. \end{aligned} \tag{1}$$

We may expand plasma parameters near $x=0$ and $y=0$ with result $\eta(x) \simeq -|\nabla\eta|x$, $\varepsilon_+(x,y) \simeq n^2_{opt} - |\nabla\varepsilon_+|(\cos(2\alpha)x - \sin(2\alpha)y)$, where $n_{opt} = \sqrt{q_0/(1+q_0)}$, $q_0 = \omega_{ce}(0,0)/\omega$ and 2α is an angle at which the O mode cut-off layer and the X mode cut-off layer are crossed in the poloidal cross-section. We focus on the possible general case for spherical tokamaks when this angle is small, $\sin(2\alpha) = (\mathbf{e}_y \cdot \nabla\varepsilon_+/|\nabla\varepsilon_+|) \approx 2\alpha$ and $|\nabla\eta|^{-1} \approx L_n$, $|\nabla\varepsilon_+|^{-1} \approx (1+q_0)L_n$, where L_n the scale on which the density profile varies. In this paper we suppose that 2α is counter clockwise, i.e. $2\alpha < 0$. We use further dimensionless co-ordinates $\mathbf{R} = \mathbf{r}/a$, where $a = (L_n/k_0)^{1/2}(q_0/2)^{1/4}$, $\mathbf{r} = (x,y,\xi,\zeta) \leftrightarrow \mathbf{R} = (X,Y,\tau,\rho)$ and new dependent variable $\tilde{E}_+ = -iE_+/\sqrt{1+q_0}$. Then (1) can be written as

$$\begin{aligned} \left(\partial^2_{\tau\tau} + (n^2_{opt} + X - 2\alpha Y)\right)\tilde{E}_+ - \partial_\tau\left(\partial X + i\partial_\rho\right)E_\xi = 0 \\ -\partial_\tau\left(\partial X - i\partial_\rho\right)\tilde{E}_+ + XE_\xi = 0 \end{aligned} \tag{2}$$

This is our basic system. An integral representation of solution to it is constructed in the next section.

An integral representation of solution

Due to tokamak symmetry we may assume that the wave fields vary as $\exp(ik_z z)$, where k_z being constant is large $k_z \gg k_0$. We look to develop an integral representation for required solution of Laplace integral type

$$\tilde{\mathbf{E}}(x,y,k_z) = \int_C \frac{dk_y}{2\pi}\exp(ik_y y)\tilde{\mathbf{E}}(x,k_y,k_z), \tag{3}$$

where $\tilde{\mathbf{E}}(x,k_y,k_z)$ is assumed analytic in some domain and the path of integration in the k_y plane is such that the integrand vanishes rapidly at the ends of the contour C or at infinity. As long as the O-X conversion is possible if both $|k_\zeta| \ll k_0$ and $|k_\xi - k_{opt}| \ll k_0$, it is not unreasonable to represent k_ξ and k_ζ as $k_\xi \simeq k_{opt} + \delta k_\xi$, $k_\zeta \simeq \delta k_\zeta$, where $\delta k_\xi, \delta k_\zeta \ll k_0$. In the another co-ordinate system these expansions could be represented as $k_{y,z} \simeq k^{opt}_{y,z} + \delta k_{y,z}$, where $k^{opt}_y = \sin(\theta)k_{opt}$, $k^{opt}_z = \cos(\theta)k_{opt}$. Omitting the mathematics, the integral representation of solution for a given $\delta k_z = 0$ is

$$E_+(x,y,z) = i\sqrt{1+q_0}\exp\left(ik_y^{opt}y + ik_z^{opt}z\right)\left(\hat{I}(u,v) + \hat{R}(u,v)\right) \quad (4)$$
$$E_\xi(x,y,z) = \exp\left(ik_y^{opt}y + ik_z^{opt}z\right)\left(-\hat{I}(u,v) + \hat{R}(u,v)\right),$$

where $u = (\cos(\alpha)X + \sin(\alpha)Y/\cos(\theta))$, $v = (\sin(\alpha)X - \cos(\alpha)Y/\cos(\theta))$ and $\hat{I}$ and $\hat{R}$ are transforms of given functions

$$\hat{I} = \sum_{p=0}^{\infty} C_p \int_{-\infty}^{\infty} dy'\, G\left(y-y'\right)\phi_p(v)\, D_{i\gamma_p/\pi}\left(\sqrt{2}\exp(i\pi/4)u\right),$$
$$\hat{R} = \sum_{p=0}^{\infty} C_p \int_{-\infty}^{\infty} dy'\, G\left(y-y'\right)\phi_{p+1}(v) D_{i\gamma_p/\pi-1}\left(\sqrt{2}\exp(i\pi/4)u\right), \quad (5)$$
$$\gamma_p = \pi\left(|\alpha|p + (|\alpha|-\alpha)/2\right)$$

with $G(y-y')$ being a filter function

$$G\left(y-y'\right) = \frac{\exp(-i\pi/4)}{\sqrt{2\pi}\lambda}\exp\left(i\frac{(y-y')^2}{2\lambda^2}\right),\ \lambda^2 = \frac{\sqrt{q_0(1+q_0)}L_n|\sin\theta|}{\alpha k_0} \quad (6)$$

Solution (4) deserve few comments. First, the transform of the type (5) can be viewed as a result of k_ξ variation on the magnetic surface due to the poloidal inhomogeneity. This effect is important when $\lambda \geq \rho$, where ρ is a beam radius. If the poloidal magnetic field is small so that $\lambda \ll \rho$ we return to the electric field components obtained earlier in [3]. Second, the first term of solution (4), $\hat{I}$, is associated with the O wave launched by antenna from the low plasma density side and the X wave propagating towards the plasma density increase. The second term of (4), $\hat{R}$, corresponds to the partially reflected O wave. Third, the coefficients C_p in (5) are arbitrary constants, which could be chosen so that the solution found fits the incident WKB ordinary wave beam outside the conversion layer at $u = u_b$, where WKB approximation is valid. Being the orthogonal basis of the solution, ϕ_p allow us to represent C_p in the following way

$$C_p = \int_{-\infty}^{\infty} dv\phi_p(v)\exp\left(i\frac{u_b^2}{2} - i\frac{\gamma_p}{\pi}\ln\left(\sqrt{2}u_b\right)\right)\tilde{A}\left(u_b, v, k_z^{opt}\right) \quad (7)$$

$$\tilde{A}(x,y,k_z) = \int\int_{-\infty}^{\infty} dzdy' G^*\left(y-y'\right)\exp\left(-i(k_y^{opt}y' + k_z^{opt}z)\right)A\left(x,y',z\right)$$

where * is the complex conjugation, $A(\mathbf{r})$ is a distribution of the incident O wave beam at $u = u_b$.

Conversion coefficient

Let us introduce the "partial" conversion coefficient T_{OX} corresponding to a separate p-th term in the sum (4) over the basis functions. These coefficient can be obtained from the asymptotics of parabolic cylinder functions. Due to the orthogonality of the basis functions ϕ_p, there is no interference between different terms in the sum (4) when calculating the conversed wave intensity. Each p-th term is converted independently. As result

$$T_{OX} = \sum_{p=0}^{\infty} \left|C_{2p}\right|^2 \exp\left(-2\gamma_p\right) \tag{8}$$

Above expressions are similar to that obtained for the plasma with the small poloidal magnetic field but with coefficients C_p defined in a different way than in [3].

Consider now the case $\alpha \to 0$. If we now use the Mehler's formula we obtain the O-X mode conversion coefficient as in 1D theory

$$T_{OX} = \int_{-\infty}^{\infty} \frac{d\delta k_\zeta}{2\pi k_0} \left|C_p\left(\delta k_\zeta\right)\right|^2 \exp\left(-\pi L_n \frac{q_0^{1/2}}{2^{1/2} k_0}\left(\delta k_\zeta^2 + 2\delta k_\xi^2\left(1+q_0\right)\right)\right). \tag{9}$$

It could be shown that the following relation between the conversion coefficients for the O mode to the X mode and the X mode to the O mode conversion (for propagation to and from a dense plasma) occurs

$$T_{OX}\left(\mathbf{B}\right) = T_{XO}\left(-\mathbf{B}\right), \tag{10}$$

where **B** is the full magnetic field in plasma. This symmetry rules are due to plasma gyrotropy. It were mentioned first in [2] for plasma without the poloidal magnetic field.

Conclusion

The mode conversion of the O mode to the X mode has been examined for plasma with a cold plasma dielectric tensor and with an arbitrary poloidal magnetic field.

Expanding wave equations in a layer near intersection of the O and the X mode cut-off surfaces, one finds reduced wave equations appropriate for the O-X conversion problem. We seek required solutions to the set of equations so that it match to the WKB solutions outside the mode conversion region. It has been found an integral representation of required solution (4) describing k_ξ variations on the magnetic surface.

The O-X conversion coefficient has been obtained explicitly. It is similar to that obtained in [3] but with different C_p. We have shown that the conversion coefficients obey the reciprocity relations (10) arising from plasma gyrotropy.

References

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