ALFVÉN RESONANCE HEATING IN STRAIGHT FIELD
LINE MIRROR

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The low electron temperature is a critical problem for the mirror confinement approach for controlled fusion. An increase of the electron temperature, combined with a special scheme to improve end confinement could be achieved with radio-frequency heating. Electron cyclotron heating is one option. Another possibility is Alfvén resonance heating where an advantageous feature is that the energy is delivered to the electrons with velocities lower than the thermal. Thus, superthermal electrons which are deficiently confined in mirrors are not generated.

In this report Alfvén resonance heating is theoretically examined for the straight field line mirror [1], a recently proposed single cell mirror device.

The Alfvén resonance phenomenon [2] is the physical base of this heating scheme. It appears as a result of coupling of the fast wave field to the shear Alfvén wave. In uniform plasma the shear Alfvén wave propagates along the magnetic field and

$$k_\parallel^2 = k_0^2 \varepsilon_\perp,$$

(1)

where $k_0 = \omega / c$, $\varepsilon_\perp = 1 - \sum_\alpha \omega^2_{pa} / (\omega^2 - \omega^2_{ba})$ is the diagonal element of the cold plasma dielectric tensor corresponding to the directions perpendicular to the magnetic field, $\omega_{pa}$ and $\omega_{ba}$ are the plasma and cyclotron frequencies and $\alpha$ denotes the sort of charged particles. In non-uniform plasma the region of shear Alfvén wave existence shrinks to a magnetic surface, and the condition (1) has to be met with varying $\varepsilon_\perp$. The fast wave, being excited externally, couples and delivers energy to the shear Alfvén wave so that the amount of power does not depend on the specific features of the wave damping processes in the Alfvén resonance vicinity. If no damping appears, the electromagnetic field would diverge on the Alfvén resonance surfaces. In a full-wave plasma description [3], near the Alfvén resonance, mode conversion takes place and the fast magnetosonic or fast Alfvén wave field converts to slow waves. In high temperature plasma ($|k_0 v_Te| > \omega$) the kinetic (modified) Alfvén wave emerges. It propagates toward higher plasma density, i.e. into the plasma core. In the opposite case
\( |k_z v_T| < \omega \), the slow quasi-electrostatic wave which propagates to the plasma edge is generated. The slow waves mentioned are damped via Landau or collisional mechanisms. Their damping is much stronger than the damping of the fast waves.

In the Alfvén resonance heating scenario, the antenna excites the fast wave field from the plasma column periphery. It reaches the Alfvén resonance layers and converts into slow waves. The slow waves deliver their power to bulk electrons. Normally, in the space between the antenna and the Alfvén resonance location the condition for fast wave propagation is not met, and the fast wave field does not perform oscillations across the plasma column. This means that the Alfvén resonances are excited by the antenna near-field.

In general, the antenna excites a full \( k_\parallel \) spectrum. Formula (1) shows that the smaller plasma densities at the surfaces of the Alfvén resonance correspond to smaller \( k_\parallel \). This means that small \( k_\parallel \) wave excitation would result in heating of the plasma periphery. On the other hand, high \( k_\parallel \) wave excitation is wasteful since no Alfvén resonance surface exists for such waves. Thus an antenna with a narrow \( k_\parallel \) spectrum is preferable. A narrow \( k_\parallel \) spectrum means that the antenna needs to be broad with a nearly periodical structure of the currents. A compromise solution is chosen (see [4]): a fragment of the periodical antenna consists of \( \pi \)-phased strap elements and the number of straps in the fragment is limited to 4 (see Fig.1). The antenna may occupy any position along the plasma column, and our choice is to place the antenna near the midplane of the open trap.

In a large region around the midplane the plasma column is almost cylindrical and, if the antenna size is much less than the open trap length, a bounded plasma cylinder model with a radially non-uniform plasma is appropriate to describe the Alfvén resonance heating. In the numerical model the electron Landau damping is accounted for as well as the finite ion Larmor radius corrections to the dielectric tensor. The Maxwell's equations are discretized in the radial direction using a uniform finite element method. Fourier expansions are used for the azimuthal and longitudinal coordinates. Calculations are made for the following reactor-scale open trap plasma parameters [1] near the midplane: Plasma density in its maximum is \( n_{p0} = 10^{14} \text{cm}^{-3} \), plasma density radial profile is parabolic with a small scrape-off layer, electron and ion temperatures \( T_e = T_i = 10 \text{keV} \) are chosen equal as well as the deuterium and tritium concentrations. The magnetic field \( B_0 = B_{z0} = 1.5 \text{T} \) is uniform, the plasma radius is \( r_p = 150 \text{cm} \) and the metallic wall radius is \( r_w = 200 \text{cm} \).
The azimuthal size of the antenna is chosen from the requirement of efficient excitation of \( m = \pm 1 \) azimuthal modes that make a major contribution to the Alfvén resonance heating. This means that the antenna azimuthal size should be close to \( \varphi_{\text{ant}} \approx \pi \), and \( \varphi_{\text{ant}} = 2 \) is chosen. The distance between neighboring straps is \( l_z = 320 \text{ cm} \). Frequency scan results are shown in Figs. 2 and 3. To characterize the antenna an estimated antenna Q is introduced by

\[
Q^* = \frac{\omega \mu_0 r_{\text{ant}} \varphi_{\text{ant}}}{r_{\text{RF}}},
\]

where \( r_{\text{ant}} \) is the antenna radius, \( r_{\text{RF}} \) is the antenna loading resistance per strap. Fig. 2 shows that \( Q^* \) has a broad minimum around the frequencies for which the main component of the antenna spectrum \( k_{\parallel} = \pi / l_z \) excites the Alfvén resonance at the plasma core. The optimum frequency \( \omega_{\text{opt}} \approx 3.2 \cdot 10^6 \text{ s}^{-1} \) is much smaller than the ion cyclotron frequency. The average radius of power deposition \( r_{\text{pow}} = \int r_p r_{\text{RF}} dV / \int p_{\text{RF}} dV \) is a measure of the power deposition centralness (\( p_{\text{RF}} \) is the deposited power density). Never being small, it ramps slowly with a frequency increase (Fig.3). The power deposition profile (Fig. 4) is always hollow. However, there is never any strong power deposition at the plasma periphery.

![Fig. 1. Sketch of antenna system. Arrows indicate phasing of RF current.](image1)

![Fig. 2. Antenna Q* as a function of frequency (dashed line corresponds to double antenna).](image2)

To find the optimum distance between neighbouring straps, \( l_z \) is varied simultaneously with the frequency. The variation of the frequency is necessary to keep the Alfvén resonance at the same location in the plasma column for a specific antenna characteristic parallel wavenumber \( k_{\parallel} \). Since the frequency is low, this can be achieved if \( \omega l_z = \text{const} \). During this numerical experiment the average radius of power deposition \( r_{\text{pow}} \) does indeed change very slightly. The dependence \( Q^*(l_z) \) in Fig. 5 shows a broad minimum at \( l_z = 320 \text{ cm} \) (see [5]). When \( k_{\parallel} \) is small the amplitude of the electric field excited by the antenna increases as \( k_{\parallel}^2 \).
When \( k_\parallel \sim m/r_p \) this increase saturates and, with a further increase of \( k_\parallel \), the exponential decay of the antenna-excited field with the distance from the antenna comes into play.

Fig.3. Average radius of power deposition as a function of frequency.

Fig.4. Power deposition profiles 1) for \( \omega = 2.4 \cdot 10^8 \text{s}^{-1} \) and 2) for \( \omega = 3.2 \cdot 10^8 \text{s}^{-1} \).

Fig. 5. Antenna \( Q^* \) as a function of distance between neighbouring straps.

The antenna can be improved by adding an identical array at the opposite side of the vacuum chamber. This new array is actually the initial array turned by \( \varphi = \pi \) in azimuth and \( \pi \)-phased. In this double array the excitation of even azimuthal harmonics is suppressed. The double array antenna has almost the same optima as the single array (see Fig.2), but the optimum \( Q^* \) value is reduced by a factor of 1.7.

Discussion

Partial optimization of a four-strap phased antenna array for Alfvén resonance heating results in a 10 m long antenna occupying a relatively small part of a 100 m reactor-scale straight field line mirror. The frequency \( f \approx 500\text{kHz} \) is low which reduces the voltage on the antenna. No sensitive dependence on the plasma parameters is found and heating of the plasma periphery is suppressed. However, the power deposition profile is hollow and there is almost no mean to control it. For the single array antenna, the antenna Q is marginally high even in the optimum regime. It is less for double array antenna, but still higher than typical values for ion cyclotron heating.

References