The pellet rocket acceleration caused by ∇B-induced drift

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1. Introduction.

The pellet acceleration towards the Low Field Side (LFS) was observed on ASDEX-Upgrade tokamak both for pellets injected from the LFS [1,2] and High Field Side (HFS) [2]. The reported acceleration was in the range of $(1-5)\cdot 10^5$ m/s^2, i.e. within the pellet lifetime its velocity changed significantly. The fact that the pellet is accelerated towards LFS indicates that the acceleration is connected with the ∇B-induced drift of plasma cloud surrounding a pellet [1,2].

In the present report neutral and plasma cloud parameters and their asymmetry are investigated in a manner similar to the approach suggested in [3]. It is demonstrated that due to ∇B-induced drift of plasma cloud its shielding properties at the HFS of a pellet becomes weaker, while at the LFS the shielding is stronger. This asymmetry leads to the enhanced ablation at the HFS of pellet and rocket acceleration towards the LFS.

2. Shielding properties of pellet ablation cloud.

Let us assume that (similarly to [3]) the pellet ablation cloud consists of spherically symmetric neutral part and cigar-like plasma cloud (Fig. 1) with typical densities, temperatures and expansion velocities n_N, T_N, $c_{sN} = \sqrt{T_N/m_i}$ and n_p, T_p, $c_{sp} = \sqrt{T_p/m_i}$ correspondingly, where m_i is the pellet atom/ion mass.

By r_p denoted is the pellet radius, n_{bg}, T_{bg} are the background plasma density and temperature, and l_i is the size of the plasma cloud in a direction transverse to $\vec{B}$. These quantities are related through integrated conservation laws for particles and energy [3] and for incident electrons energy depletion:

$$\dot{N} = n_N \cdot 4\pi r_p^2 \cdot c_{sN} = n_p \cdot 2\pi l_i^2 \cdot c_{sp} \qquad (1)$$

$$\beta \frac{n_{bg} T_{bg}^{3/2}}{m_e^{1/2} r_p} = \frac{n_N T_N^{3/2} c_{sN}}{r_p} \qquad (2)$$

$$(1-\beta)\frac{n_{bg} T_{bg}^{3/2}}{m_e^{1/2} l_z} = \frac{n_p T_p^{3/2} c_{sp}}{l_z} + \frac{n_p E_{ion}^{3/2} c_{sp}}{l_z} \qquad (3)$$

$$\alpha^4 T_{bg}^2 = 4\pi Z(Z+1)e^4 \Lambda (n_N r_p + n_p l_z) \qquad (4)$$

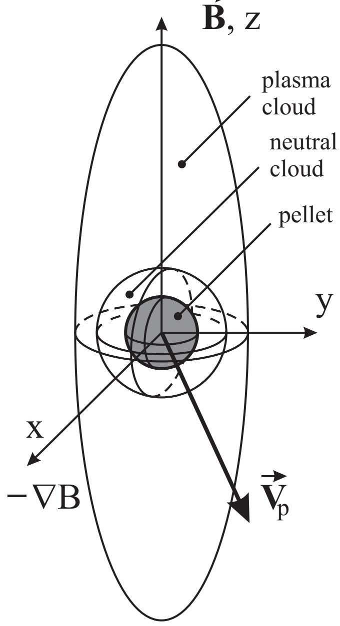

Figure 1. *Scheme of pellet ablation cloud.*

where l_z is the plasma cloud longitudinal length, e is a proton charge, $+Ze$ is the charge of pellet atom nuclei, Λ is the Coulomb

Logarithm, m_e is an electron mass, E_{ion} is the ionization energy and β is the fraction of incident electron energy flux reaching the neutral cloud. The Equation (4) is obtained from assumption that incident electrons deceleration inside the cloud is provided mainly by the Coulomb collisions

$$\frac{\partial V}{\partial t} = -\frac{4\pi Z(Z+1)e^4 n\Lambda}{m_e^2 V^3}\cdot V \qquad (5)$$

We replace $\frac{\partial}{\partial t} = V\frac{\partial}{\partial s}$ and intergrate Eq.(5) along the path of incident electrons inside the cloud from infinity to $s = s_0$. The integral $\int_{s_0}^{\infty} n\mathrm{d}s$ is estimated as $\int_{s_0}^{\infty} n\mathrm{d}s \approx n_N r_p + n_p l_z$, and for integration limits used are $V|_{s=s_0} = 0$ and $V|_{s=\infty} = \alpha\sqrt{2T_{bg}/m_i}$. This means that shielding is so strong that to penetrate up to the pellet surface electron should have velocity much larger than thermal one ($\alpha > 1$) and ablation is provided by negligibly small fraction of electrons which reach pellet with almost zero velocity.

These equations should be supplemented by following estimating relations: $l_z = \sqrt{l_i R}$ (this equation comes from simple consideration of ∇B-induced drift of plasma cloud, R is the tokamak major radius), $n_N T_N = n_p T_p$ (absence of shock wave), and $T_p = const = 1-2$ eV (since ionization rates strongly depends on temperature). The electrostatic shielding may be taken into account by replacing $n_{bg} \to n_{bg}\cdot\exp(-e\Delta\Phi/T_{bg})$.

After some algebra one can derive an equation for β:

$$\frac{\pi\varepsilon_0^2\alpha^4 T_{bg}^{1/2} m_e^{1/2}(E_{ion})^2}{e^4\Lambda(Z(Z+1))r_p n_{bg} T_p} = \frac{(1-\beta)^2}{\beta}\left(\frac{1-\beta}{\beta}\frac{T_p}{E_{ion}} + \sqrt{\frac{R}{2r_p}}\cdot\left(\frac{\beta}{(1-\beta)}\frac{E_{ion}}{T_p}\right)^{3/4}\right) \qquad (6)$$

Solving (6) with $T_{bg} = 500$ eV, $T_p = 1$ eV, $Z = 1$, $r_p = 1$ mm, $n_0 = 3\cdot10^{19}$ m^{-3}, $\mu_I = 2$ ($\mu_I = m_i/m_p$, m_p is the proton mass), $R = 1.5$ m (typical parameters of modern tokamaks) one gets $\beta = 0.006$ and $\dot{N} = 2.1\cdot10^{23}$ s^{-1}, and with $T_{bg} = 5$ keV, $T_p = 1$ eV, $Z = 1$, $r_p = 5$ mm, $n_0 = 10^{20}$ m^{-3}, $\mu_I = 2$, $R = 6.2$ m (expected parameters for ITER) – $\beta = 0.029$ and $\dot{N} = 1.1\cdot10^{26}$ s^{-1} ($\alpha = 3$, $\Lambda = 10$ and $\Delta\Phi = 2T_{bg}/e$ were substituted). These results indicate that most of the incident electrons energy flux is deposited in plasma cloud, and in a limiting case $\beta << 1$ it is possible to get analytical scalings

$$\dot{N} = 4\pi\alpha^2\left(4\pi\varepsilon_0\sqrt{4\pi\Lambda Z(Z+1)}\Big/ e^2\sqrt[4]{m_e m_p}\right)\cdot r_p^{3/2}\cdot n_{bg}^{1/2}\cdot T_{bg}^{7/4}\cdot\mu_I^{-1/4}\cdot T_p^{1/4}\cdot E_{ion}^{-1/2}\cdot e^{-e\Delta\Phi/2T_{bg}} \qquad (7)$$

$$T_N = \left(e^2/4\pi\varepsilon_0\right)^2\cdot 4\pi\Lambda Z(Z+1)\cdot\left(m_p/m_e\right)^{1/2}\cdot\alpha^2\cdot r_p\cdot n_{bg}\cdot T_{bg}^{-1/2}\cdot\mu_I^{1/2}\cdot T_p^{1/2}\cdot E_{ion}^{-1}\cdot e^{-e\Delta\Phi/T_{bg}} \qquad (8)$$

$$n_N = \left(e^2/4\pi\varepsilon_0\right)^{-2}\cdot\left(4\pi\Lambda\cdot Z(Z+1)\right)^{-1}\cdot\alpha^4\cdot T_{bg}^2\cdot r_p^{-1} \qquad (9)$$

$$\beta = e^2/4\pi\varepsilon_0\cdot\sqrt{4\pi\Lambda\cdot Z(Z+1)m_p/m_e}\cdot\alpha^{-2}\cdot r_p^{1/2}\cdot n_{bg}^{1/2}\cdot T_{bg}^{-1/4}\cdot\mu_I^{1/4}\cdot T_p^{3/4}\cdot E_{ion}^{-3/2}\cdot e^{-e\Delta\Phi/2T_{bg}} \qquad (10)$$

3. Pellet rocket acceleration.

Pellet rocket acceleration is caused by the asymmetry of the plasma cloud, which drifts towards the LFS in the inhomogeneous tokamak magnetic field (see Fig. 2). The plasma shielding is weaker from the HFS of pellet, therefore the ablation is enhanced at HFS, and unbalanced momentum carrying out by evaporating neutrals causes the rocket acceleration.

As schematically shown in Fig. 2, the plasma cloud thickness l_z depends on x – the radial coordinate, and this dependence may be expressed as $l_z(x)=\sqrt{R(l_i+x)}$. Consequently the β – fraction of incident electron energy flux reaching the neutral cloud – is also x-dependent, and therefore $n_N=n_N(x)$ and $T_N=T_N(x)$. Since we are interested in calculation of asymmetry on a scale $x\le r_p << l_i$, we may keep only the first term of Taylor series expansion with regard to small parameter r_p/l_i.

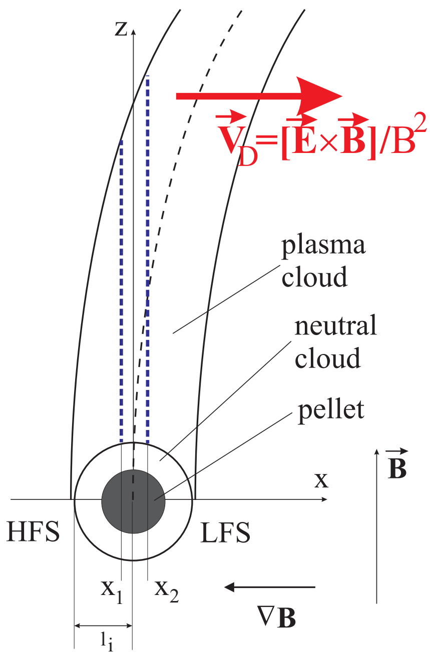

Figure 2. *Cross-section of drifting cloud by xz-plane. Blue dash lines show plasma cloud thickness at different x.*

For $l_z(x)$ we have (here and below subscript 0 denotes the value at $x=0$).

$$l_z(x)=l_{z0}\left(1+\chi_{l_z}\cdot x/r_p\right),\qquad \chi_{l_z}=1/2 \tag{11}$$

Since pellet evaporation energy ε_{vap} is small, the incident electrons flux to the pellet surface, which depends on $\int n\,\mathrm{d}s$, is also small, and one can treat it as zero and independent of x. Consequently from $\int_{s_0}^{\infty} nds \approx n_N(x)r_p+n_p l_z(x)=const$ one gets

$$n_N(x)=n_{N0}\left(1+\chi_{n_N}\cdot x/r_p\right)\qquad\qquad \chi_{n_N}=n_p l_{z0}/2n_{N0}r_p\,. \tag{12}$$

The β may be defined as $\beta=\left(2n_{bg}T_{bg}\sqrt{\dfrac{T_{bg}}{2\pi m_e}}\right)^{-1}\int_{-\infty}^{+\infty}\mathrm{d}w_x\int_{-\infty}^{+\infty}\mathrm{d}w_y\int_{0}^{+\infty}\mathrm{d}w_z w_z\dfrac{m_e w^2}{2}F(\vec{w})$,

where $F(\vec{w})$ is incident electrons distribution function at the boundary between plasma and neutral clouds. Assuming roughly that electrons with mean free path larger that l_z penetrate through the plasma cloud without energy losses and other electrons stop inside it, one obtains

$$\beta=\left(2n_{bg}T_{bg}\sqrt{\frac{T_{bg}}{2\pi m_e}}\right)^{-1}\int_{-\infty}^{+\infty}\mathrm{d}V_x\int_{-\infty}^{+\infty}\mathrm{d}V_y\int_{V_C}^{+\infty}\mathrm{d}V_z V_z\frac{m_e V^2}{2}\left(\frac{m_e}{2\pi T_{bg}}\right)^{\frac{3}{2}}\exp\left(-\frac{m_e V^2}{2T_{bg}}\right)=\frac{1}{\sqrt{3}}\int_{\sqrt[4]{l_{z0}/\lambda_{mfp}}}^{+\infty}u^3e^{-u^2}\mathrm{d}u\,,$$

where $V_C=m_e^2 l_{z0}/4T_{bg}^2\lambda_{mfp}$ and $\lambda_{mfp}=4\pi\varepsilon_0^2T_{bg}^2/Z(Z+1)e^4\Lambda n_p$ are defined with help of

Eq. (5), and used is the estimate $V_z^2 \approx V^2/3$. Consequently

$$\beta(x) = \beta_0\left(1 + \chi_\beta \cdot x/r_p\right), \qquad \chi_\beta = -n_p l_{z0} / 8\sqrt{3}\lambda_{mfp}\beta_0 \exp\left(-\sqrt{l_{z0}/\lambda_{mfp}}\right) \tag{13}$$

From first of Eq. (2) (energy balance in neutral cloud) one gets

$$T_N(x) = T_{N0}\left(1 + \chi_{T_N} \cdot x/r_p\right), \quad \chi_{T_N} = 2\left(\beta_0 n_{bg} T_{bg}^{3/2} \mu_I^{1/2} \sqrt{m_p/m_e}\,\chi_\beta \Big/ n_{N0} T_{N0}^{3/2} - \chi_{n_N}\right)\Big/3 \tag{14}$$

Assuming that particle balance (first of Eq (1)) takes place also locally, i.e.

$$\dot{N}(\theta,\varphi)\,\mathrm{d}\Omega \big/ 4\pi r_p^2 = n_N(\theta,\varphi) c_{sN}(\theta,\varphi), \tag{15}$$

where everything depends on spherical angles through $x = r_p \sin\theta \cos\varphi$, one obtains

$$\dot{N}(x) = \dot{N}_0\left(1 + \chi_{\dot{N}} \cdot x/r_p\right), \qquad \chi_{\dot{N}} = \chi_{n_N} + \chi_{T_N}/2 \tag{17}$$

and may express the pellet rocket acceleration as

$$a = -\frac{3}{4\pi r_p^2 \rho_p}\int_0^\pi \mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\varphi\; p_0 \sin\theta\cos\varphi\; \dot{N}(\theta,\varphi) = -\frac{\sqrt{2\varepsilon_{vap}\mu_I m_p}\,\dot{N}_0\;\chi_{\dot{N}}}{\rho_p l_i r_p^2} = a_1 + a_2, \tag{18}$$

where ρ_p is the pellet material density, $p_0 = \sqrt{3\varepsilon_{vap} m_i}$ is the momentum carried out by evaporating neutrals, and a_1 and a_2 correspond to the first and second terms in Eq. (18).

4. Results and conclusions.

If $\beta << 1$ results may be expressed (using (7)) in a form of scalings

$$a_1 = \frac{4\pi}{3\sqrt[4]{2}} \frac{\sqrt{2\varepsilon_{vap} R}}{\rho_p \alpha^{5/2}} \left(\frac{e^2}{4\pi\varepsilon_0}\right)^{\frac{5}{4}} (4\pi\Lambda)^{\frac{5}{8}} \cdot \left(\frac{m_p}{m_e}\right)^{\frac{13}{16}} \left(Z(Z+1)\right)^{\frac{5}{8}} \cdot r_p^{-\frac{7}{8}} \cdot n_{bg}^{\frac{13}{8}} \cdot T_{bg}^{\frac{19}{16}} \cdot \mu_I^{\frac{13}{16}} \cdot T_p^{-\frac{5}{16}} \cdot E_{ion}^{-\frac{13}{18}} \cdot \exp\left(-\frac{13 e\Delta\Phi}{8T_{bg}}\right)$$

$$a_2 = \frac{4\pi}{24}\sqrt[4]{\frac{2}{9}} \frac{\sqrt{2\varepsilon_{vap} R}}{\rho_p} \left(\frac{e^2}{4\pi\varepsilon_0}\right)^{\frac{1}{4}} (4\pi\Lambda)^{\frac{1}{8}} \cdot \left(\frac{m_p}{m_e}\right)^{\frac{9}{16}} \alpha^{\frac{7}{2}} \left(Z(Z+1)\right)^{\frac{1}{8}} \cdot r_p^{-\frac{11}{8}} \cdot n_{bg}^{\frac{9}{8}} \cdot T_{bg}^{\frac{23}{16}} \cdot \mu_I^{\frac{9}{16}} \cdot T_p^{-\frac{17}{16}} \cdot E_{ion}^{-\frac{1}{18}} \cdot \exp\left(-\sqrt{\frac{l_z}{\lambda_{mfp}}} - \frac{9 e\Delta\Phi}{8T_{bg}}\right)$$

With parameters of modern tokamaks (see section 2) and $\varepsilon_{vap} = 0.005\,\mathrm{eV}$, $\rho_p = 204\,\mathrm{kg/m^3}$ one gets $a = 2.8\cdot 10^5\ \mathrm{m/s^2}$, which is in a good agreement with [1,2]. For parameters of ITER, specified in section 2, with the same values of ε_{vap} and ρ_p, the acceleration is $a = 2.4\cdot 10^6\ \mathrm{m/s^2}$.

The effect of pellet rocket acceleration towards LFS may change the pellet velocity significantly during the pellet lifetime, and may be responsible for the larger pellet penetration depth in the case of HFS injection.

References

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