

Dynamics of positive radial electric field created by ECRH-pump out.

F. Castejón, S. Eguilior, I. Calvo, D. López-Bruna, and J. M. García-Regaña
*Laboratorio Nacional de Fusión.
 Asociación EURATOM/CIEMTA para Fusión. 28040-Madrid (Spain)*

1.- Introduction and Motivation

Previous works show the existence of an outward particle flux when Electron Cyclotron Resonance Heating (ECRH) is applied to plasmas confined in stellarators [1]. This flux is manifested experimentally through hollow density profiles that are usually accompanied by peaked temperature ones [2]. The resulting confinement regime is characterised by having a positive intense radial electric field and an improved electron heat confinement and is called CERC (Core Electron Root Confinement) by the Stellarator Profile Data Base collaboration group [3]. This extra flux, called pump-out, can be explained in terms of the increasing of particles that enter the loss cone in momentum space due to the enhancement of their perpendicular momentum. Those particles are lost in a short time scale giving an enhancement of outward flux, which causes an increase of the positive electric field. In this work, a simplified procedure to estimate the outward flux, including collisions and diffusion in momentum space, is presented and the dynamics of the electric field is estimated.

2.- The Langevin Equations

ECRH can be understood as particle diffusion in momentum space along the vector $\vec{d} = Y_s \hat{e}_\perp + N_{//} u_\perp \hat{e}_\parallel$, which is almost in the perpendicular direction ($Y_s \equiv s\omega_c/\omega$ and $\vec{u} \equiv \vec{p}/mc$). Therefore, the fraction of particles than enter the loss cone is increased, resulting in an enhancement of outward radial flux. The ambipolar condition implies the onset of a radial positive electric field that is able to stop the electron flux and to reduce the heat flux, appearing a peaked temperature profile. The exact estimation of this flux implies the resolution of the 5D kinetic equation (2D in momentum space and 3D in real space) [4]. The problem admits an alternative approach based on Langevin equations, which give the microscopic dynamics of particles in phase space [5]. The trajectories in momentum space of particles embedded in a wave field are given by $\partial \vec{u} / \partial t = \vec{d} \left[1/2 (\vec{d} \cdot \nabla) D_{cy} + \sqrt{2} D_{cy}^{1/2} (\vec{d} / |\vec{d}|) \cdot \vec{\xi} \right]$, with a deterministic part given by $1/2 d_i (\vec{d} \cdot \nabla) D_{cy}$, and a stochastic part, $\sqrt{2} D_{cy}^{1/2} |\vec{d}| d_i d_j \xi_j$, which contains a random vector that satisfies $\langle \xi_k(t) \rangle = 0$, $\langle \xi_i(t) \xi_k(t + \tau) \rangle = \delta_{ik} \delta(\tau)$. In all the expressions, the sum must be taken for repeated indexes that can take the values: $i, k = \perp, //$.

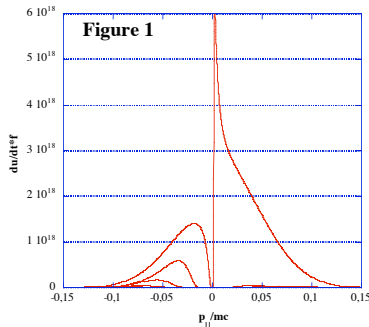
The coefficient D_{cy} comes from the quasi linear diffusion in momentum space and is proportional to the spectral density $\Gamma(N_{||})$, and to the wave power density w :

$$D_{cy}(\vec{u}) = \int dN_{||} \frac{w}{|u_{||}| \gamma} \left| \hat{e} \cdot \vec{\Pi} \right|^2 \delta(N_{||} - N_{||R}) \Gamma(N_{||}) = \frac{w}{|u_{||}| \gamma} \left| \hat{e} \cdot \vec{\Pi} \right|^2 \Gamma(N_{||R})$$

Here, $N_{||R} = (\gamma - Y_s)/u_{||}$ is the resonant refractive index, $\gamma = (1 + \vec{u}^2)^{1/2}$ is the Lorentz relativistic factor, \hat{e} is a unit vector proportional to the electric field, and $\vec{\Pi} = (sJ_s(\rho)/\rho, -iJ'_s(\rho), u_{||}J_s(\rho)/u_{\perp})$. The argument of Bessel functions is $\rho = N_{\perp} u_{\perp} \omega_c / \omega$.

3.- Transport estimates: Linear approximation.

The outward particle flux due to the pushing of electrons into loss cone is related to the flux in momentum space through the expression: $\vec{\nabla} \cdot \vec{\Gamma}_e^{ECH} = (\partial n / \partial t)_{ECH} = \int_{\delta} f(\vec{u})(d\vec{u}/dt) \cdot d\vec{S}$. δ is the border of loss cone in momentum space and f is the electron distribution function. Here we assume that all the electrons that enter into loss cone are lost immediately. In the former expression, it is necessary to know the distribution function and the structure of loss cone, which is equivalent to having solved the problem. Nevertheless, we can introduce some approximations in order to do a quick calculation that allows us to extract the main properties of the ECRH-induced particle flux. First, we assume that the electron distribution function is Maxwellian, i.e., the deformation of the distribution function is small (we perform a linearization of the problem); second, we assume that all the particles that enter the loss cone escape from the magnetic surface; and third, the structure of the loss cone is given by a cone



and does not change. Of course, the distribution function will be modified by the wave-particle interaction and by the escaping electrons, and the structure of loss cone is modified by the electric field. All these approximations mean that we are overestimating the flux.

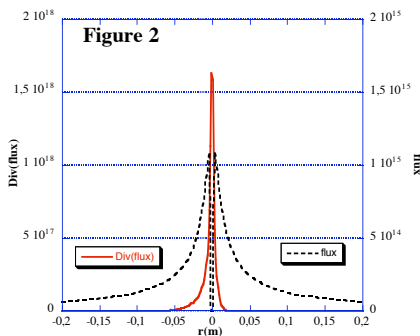


Figure 1 shows the structure of the flux in momentum space versus the parallel momentum for several radial positions. Assuming a temperature, density, and magnetic field profiles, the absorbed power density can be estimated in the weakly relativistic approximation and, considering also a ripple profile, the total flux is calculated: $\Gamma^{ECH} = 1/r \int r' dr' (\nabla \Gamma)_{ECH}$.

Figure 2 shows the divergence of the flux that gives the local contribution to the integrated flux, which is also plotted. The most important contribution to the total outward flux comes from the plasma core, where the absorbed power is maximum. The ambipolar condition will imply that a radial positive electric field must be created to keep the plasma quasi-neutrality.

4.- The electric field dynamics.

We propose a simple set of first order differential equations describing the behaviour of the radial electric field at short time scales. Our simplifications are based on the conditions prevailing in the heating power deposition zone of typical TJ-II ECH discharges: (i) the ion flux remains roughly unperturbed, (ii) the density profile is almost flat or has a negligible gradient scale length in comparison with the electron temperature one; and (iii) the slowing down time for electron-ion collisions (taking 1 keV electrons in a background of 0.1 keV ions) is large ($\sim 10^{-4}$ s) in comparison with the typical time-scales for the evolution of E_r . From these conditions, the equilibrium radial force balance imposes $\mathbf{E} \times \mathbf{B}$ rotation for the ion species and null rotation speed for the electrons, which can only be attained with the electrons being affected by a diamagnetic rotation exactly opposed to the electric field drift. For the conditions described above, this implies a radial electric field $E_r \approx -\nabla T_e / e$ as found in the experiments [6]. These constraints are equivalent to consider that the ions are frozen, so that the dynamics at the time scales of interest is governed by the changes in the radial electron flux, Γ_r . In addition, we assume that $E_\theta = E_\phi = 0$, $\Gamma_\phi = 0$ and $B_r, B_\theta \ll B_\phi$. The evolution of the field and the fluxes are given by the following equation: $\partial_t X = MX + Z$. Where:

$$X = \begin{pmatrix} E_r \\ \Gamma_r \\ \Gamma_\theta \end{pmatrix} \quad M = \begin{pmatrix} 0 & -\frac{e}{\epsilon_0} & 0 \\ -\frac{en}{m} & -v_r & \omega_c \\ 0 & -\omega_c & -v_\theta \end{pmatrix} \quad Z = \begin{pmatrix} 0 \\ -p'/m \\ 0 \end{pmatrix}$$

The equilibrium solution is $E_r = -p'/en$, $\Gamma_r = \Gamma_\theta = 0$, and the time evolution is given by $X(t) = e^{Mt} X(0) + \int_0^t e^{M(t-t')} Z(t') dt'$. The variations of $Z(t)$ will correspond to transport time scales, much slower than those of interest in transient regimes of ECH driven phenomena. Accounting for such slow variations cannot be done without (at least) one transport equation with E_r -dependent transport coefficients. We leave this for a future work and concentrate on the dynamics of E_r itself. The characteristic times of our problem are given by the eigenvalues

of M , whose characteristic equation is: $P(\lambda) = \lambda^3 + (v_r + v_\theta)\lambda^2 + (v_r v_\theta + \omega_{UHR}^2)\lambda + v_\theta \omega_p^2 = 0$. Where ω_{UHR} is the Upper Hybrid Resonance frequency. Firstly, let us check that the former systems of equations is stable, i.e., the solutions of the set of homogeneous linear equations, with $Z=0$, vanishes when $t \rightarrow \infty$. Equivalently, we must show that if λ is a root of $P(\lambda)$, then $\text{Re}(\lambda) < 0$. According to Strelitz theorem this is satisfied if all the coefficients of the polynomial are positive as well as the product of first and second power coefficients. So that $\text{Re}(\lambda_i) < 0 \Rightarrow v_\theta \neq 0$, which means that the poloidal viscosity is the key ingredient that governs the electric field dynamics. $P(\lambda)$ is a real cubic polynomial, hence it has at least a real root, which we denote by λ_l . It is also hold that $v_r, v_\theta \ll \omega_{UHR}$, then $\lambda_l \approx -v_\theta (\omega_p^2 / \omega_{UHR}^2)$. The two reminder roots satisfy: $\lambda_2 = \lambda_3^* \approx 1/2 [v_r + v_\theta (\omega_c^2 / \omega_{UHR}^2)] + i\omega_{UHR}$. It turns out that they are complex, giving rise to fast oscillations. We get rid of these extremely fast, unobservable oscillations by averaging the solution $X(t)$ over a few periods:

$$\begin{pmatrix} E_r \\ \Gamma_r \\ \Gamma_\theta \end{pmatrix} (t) = e^{\lambda_l t} \left\langle \begin{pmatrix} E_r \\ \Gamma_r \\ \Gamma_\theta \end{pmatrix} \right\rangle (0) + \int_0^t e^{\lambda_l (t-t')} \begin{pmatrix} -\frac{p'}{en} \\ 0 \\ 0 \end{pmatrix} dt'$$

We therefore expect the observable time-scales of E_r and Γ_θ to be the same and essentially governed by the poloidal viscosity. Within our hypotheses, any initial condition $\Gamma_r \neq 0$ must decay to the equilibrium solution following the exponent given by λ_l . Therefore, if we start from an equilibrium field that satisfies $\Gamma_e = \Gamma_r$, the final field will be the equilibrium one plus the extra field due to the increase in electron pressure gradient due to the reduction of heat transport, provided that the perturbation of ion flux is negligible. Obviously, our variable radial flux must be interpreted in this context as $\Gamma_r = \Gamma^{ECH}$. The time scale of the modification of electrostatic potential in TJ-II is about 50 μs , which gives a value for the poloidal time decay $v_\theta \approx 80 \times 10^3 \text{ s}^{-1}$. The equilibrium value of the electrostatic potential after a gyrotron has been switched on will be given by the value of the electron temperature and the enhanced flux is zero once the steady state has been reached.

-
- [1] M. A. Ochando and F. medina. Plasma Physics and Controlled Fusion 45 (2003) 221
 - [2] F. Castejón et al. Nuclear Fusion. 42 (2002) 271
 - [3] M. Yokoyama et al. Fusion Science and Technology 50 (2006) 327
 - [4] S. Murakami. Nucl. Fusion 40 (2000) 693
 - [5] F. Castejón and S. Eguilior. Plasma Phys. and Control. Fusion 45 (2003) 159
 - [6] F. Castejón et al. Nucl. Fusion 44 (2004) 593