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Resistive Wall Modes Stabilization in the Presence of 3D Wall Structures

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Introduction

The "advanced tokamak" concept is economically attractive if the ideal external kink beta limit is raised substantially. This is possible only if the resistive wall mode (RWM) is stabilized by either passive (close fitting conducting wall) and active (feedback coils) stabilization, or/and rapid plasma rotation. In contrast to the usual ideal MHD treatment with the extended energy principle, the present analysis is generally non-self-adjoint and includes the necessary perturbed magnetic pressure for satisfying pressure balance. One solving method could be to found a new basis of orthogonal eigenvectors to ensure the self-adjointness property of the energy operator - the normal mode approach [1-2], or to replace a perfect conducting wall with one of finite conductivity and to deduce a modified energy principle [3]. Numerical MHD stability studies in the presence of toroidal rotation, viscosity, resistive walls and current holes, by using the CASTOR FLOW code are presented in Ref. [4]. Modelling of RWMs has been made by using the VALEN code [5] with an equivalent surface current model for the plasma. Extensive study and theoretical development has also been presented by Bondeson and his co-workers [6,7] by using the MARS code. In a first stage, we have adopted the second approach [3], where the assumption that the plasma displacement trial function, in the situation where a conducting wall is located at finite distance from the plasma is considered to be identical to that used in the evaluation of the potential energy when the conducting wall is moved infinitely far away has been made. We considered such assumption too limiting and have developed an other approach.

Writing the expression for the potential energy in terms of the perturbation of the flux function, and performing an Euler minimization, we have obtained a system of ordinary differential equations in that perturbation [8]. This system of equations describes a tearing mode or an external kink mode, the latter if the resonance surface is situated at the plasma boundary. Usually, vanishing boundary conditions for the perturbed flux function at the magnetic axis and at infinity are considered. From single layer potential theory, we have developed an approach to fix "natural" boundary conditions for the perturbed flux function just at the plasma boundary, replacing thus the vanishing boundary conditions at infinity [8]. Now, in the presence of a resistive wall, the boundary conditions of the external kink mode at the plasma boundary are determined by the reciprocal interaction between the external kink perturbation of the plasma and the toroidal wall (in its thin wall approximation). A general toroidal geometry has been considered. By using the concept of a surface current [9], the description and calculation of the influence of the modes outside the plasma were greatly simplified. The normal component of the magnetic field perturbation, at the plasma boundary, has been considered as excited by toroidally coupled external kink modes and it is that component that gives the normal to the wall component of the exciting field causing the wall response via the induced eddy currents.

General expression of the potential energy

By writing, in a coordinate system with straight field lines (a, θ, ζ), the general expression of the potential energy W in terms of two test functions $u(a, \theta, \zeta)$ and $\lambda(a, \theta, \zeta)$ instead of the displacement, one obtains

$$\begin{aligned} W &= \frac{1}{2\mu_0}\int da d\theta d\zeta \left\{ \frac{g^r_{11}}{\sqrt{g^r}}\left(\frac{\partial\psi}{\partial\theta}\right)^2 - 2\frac{g^r_{12}}{\sqrt{g^r}}\frac{\partial\psi}{\partial\theta}\frac{\partial\psi}{\partial a} + \frac{g^r_{22}}{\sqrt{g^r}}\left(\frac{\partial\psi}{\partial a}\right)^2 \right. \\ &+ \psi\frac{\partial u}{\partial\theta}\left[\left(\frac{F'}{\Psi'}\right)' + \left(\frac{4\pi^2 p'}{\Psi'}\right)'\frac{\sqrt{g^r}}{\Phi'}\right] - \frac{\partial}{\partial a}\left(\psi\frac{J' + \partial\nu/\partial\theta}{\Phi'}\frac{\partial u}{\partial\theta}\right) \\ &- \frac{4\pi^2 p'}{\Psi'}\frac{\partial u}{\partial\zeta}\left[\frac{\partial u}{\partial a}\frac{\partial}{\partial\theta}\frac{g^r_{33}}{F} - \frac{\partial u}{\partial\theta}\frac{\partial}{\partial a}\frac{g^r_{33}}{F}\right] + \frac{g^r_{33}}{\sqrt{g^r}}\left(\frac{\partial\lambda}{\partial\theta}\right)^2 + 2\frac{g^r_{12}}{\sqrt{g^r}}\frac{\partial\psi}{\partial\theta}\frac{\partial\lambda}{\partial\zeta} \\ &- \left. 2\frac{g^r_{22}}{\sqrt{g^r}}\frac{\partial\psi}{\partial a}\frac{\partial\lambda}{\partial\zeta} + \frac{g^r_{22}}{\sqrt{g^r}}\left(\frac{\partial\lambda}{\partial\zeta}\right)^2 - 2\frac{J' + \partial\nu/\partial\theta}{\Phi'}\frac{\partial u}{\partial\theta}\frac{\partial\lambda}{\partial\zeta} + 2\frac{F'}{\Phi'}\frac{\partial u}{\partial\theta}\frac{\partial\lambda}{\partial\theta} \right\}, \end{aligned} \tag{1}$$

where ψ is the perturbation of the flux function Ψ. After developing the perturbed values in Fourier series, performing an Euler minimisation of the energy functional and then integrating with respect to the angles θ and ζ the result of that minimisation, one obtains a system of coupled ordinary differential equation of the form

$$\mathbf{Y}'' = \mathbf{f^{-1}} \cdot (\mathbf{G} \cdot \mathbf{Y} + \mathbf{V} \cdot \mathbf{Y}') \tag{2}$$

where $\mathbf{f}$, $\mathbf{V}$ and $\mathbf{G}$ are matrices, and $\mathbf{Y}$ is the flux function perturbation vector, with the non-diagonal terms representing both toroidicity and shape coupling effects.

Close to the magnetic axis ($a \to 0$), we have found the following behaviour of the amplitude of the flux function perturbation

$$Y^m(a) \sim a^m - \frac{2a^{m+2}}{(m+1)a^2_{m/n}} - \frac{(m-1)a^{m+4}}{(m+1)(m+2)a^4_{m/n}}, \tag{3}$$

where $a_{m/n}$ is the resonance radius corresponding to the wave numbers m and n.

We have to consider a "natural" boundary condition just at the plasma boundary [8]. From potential theory we know that a continuous surface distribution of simple sources extending over a not necessarily closed Liapunov surface ∂D and of density $\sigma(\mathbf{q})$, generates a simple-layer potential at $\mathbf{p}$, in ∂D. After some tedious calculations, the boundary condition at the plasma boundary becomes

$$\mathbf{Y}_{k+1} = \left(\mathbf{I} - h\mathbf{F}\cdot\mathbf{D}^{-1} - \vec{\alpha}_k\right)^{-1}\cdot\vec{\beta}_k, \tag{4}$$

with $\mathbf{I}$ the unit matrix and h the "radial" integration mesh, $\mathbf{D}$ and $\mathbf{F}$ $[M\times M]$ complex matrices, with the elements given by the metric coefficients, normal and tangential magnetic field components. $\vec{\alpha}_k$ is a known $[M\times M]$ coefficient matrix and $\vec{\beta}_k$ a known $[M]$ coefficient vector, both resulting from a forth-order Runge-Kutta integration scheme. For unit perturbations $Y_{2/1}$ ($m=2$, $n=1$) and $Y_{3/2}$ ($m=3$, $n=2$), the corresponding surface charge distributions are given in Fig.1.

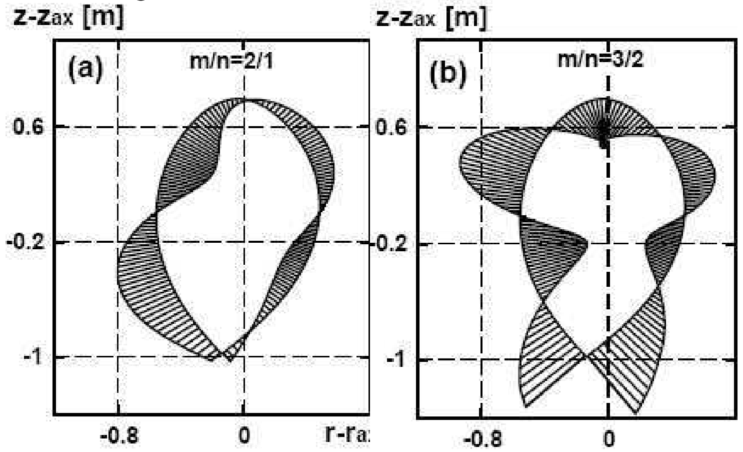

Fig. 1 The surface charge distribution along the plasma boundary for unit flux perturbations $Y_{2/1}$ and $Y_{3/2}$, respectively. The plasma configuration of the ASDEX Upgrade tokamak corresponding to the shot no. 13476 at 5.2 s has been considered.

Determination of the vacuum field due to a helical perturbation

The perturbation field $\tilde{\mathbf{B}} = \nabla\Phi$ outside of the plasma can be assumed to be produced by the surface current equations

$$[\tilde{\mathbf{B}}\cdot\nabla a]\ \ \nabla a\times[\tilde{\mathbf{B}}] = \mu_0\mathbf{K} = -\mu_0\nabla s\times(\nabla\kappa + \nabla\times\mathbf{g}) = -\mu_0\nabla a\times\kappa, \tag{5}$$

where [ ] means the jump across the plasma surface. $\kappa(\theta,\zeta,t)$ being the time-dependent stream function of the surface current, and $\mathbf{g}$ an arbitrary vector. In terms of the external and internal magnetic scalar potentials (with respect to the plasma boundary) one has

$$\mu_0\kappa = -\Phi_e + \Phi_i = -\int_{\partial D} g(\mathbf{p},\mathbf{q})\sigma_e(\mathbf{q})dq + \int_{\partial D} g(\mathbf{p},\mathbf{q})\sigma_i(\mathbf{q})dq \tag{6}$$

The normal component of the perturbed magnetic field has been considered as excited by a flux function perturbation ψ of unit amplitude $Y(1)$ on the plasma boundary and resulting from a external kink mode. $\tilde{B}_n$ can be calculated with the relation

$$\tilde{B}_n = \frac{\nabla a}{\mid\nabla a\mid}\cdot\tilde{\mathbf{B}} = \frac{1}{\sqrt{g_{22}g_{33}}}\frac{\partial\psi}{\partial\theta},\quad \psi = \sum_m Y_m(a)\exp[i(m\theta - n\zeta)]. \tag{7}$$

Determination of the diffusion equation in the wall

In an orthogonal curvilinear coordinate system (u, v), with h_u and h_v the Lamé coefficients, the diffusion equation for the eddy current stream function $U(u, v, t)$ in a thin wall looks like

$$\frac{1}{h_u h_v}\left[\frac{\partial}{\partial u}\left(\frac{h_v}{h_u}\frac{\partial U}{\partial u}\right)+\frac{\partial}{\partial v}\left(\frac{h_u}{h_v}\frac{\partial U}{\partial v}\right)\right]-\frac{1}{d}\mu\sigma_s\frac{\partial U}{\partial t}=\sigma_s\frac{\partial B_n^{ext}}{\partial t}, \quad (8)$$

with the initial and the boundary conditions

$$U(u, v, 0) = 0, \quad F(u, v, U_t, U_u, U_v, t) = 0. \quad (9)$$

Considering the following input data: $d = 10^{-3}$ m, $\mu = 4\pi^{-7}$ H/m, $\sigma_v = 10^7$ $1/\Omega$/m, $\sigma_s = 10^4$ $1/\Omega$, the function $U(x, y, t)$ at different time, excited by a m/n =3/2 external kink mode in a thin wall with holes is presented in Fig. 2.

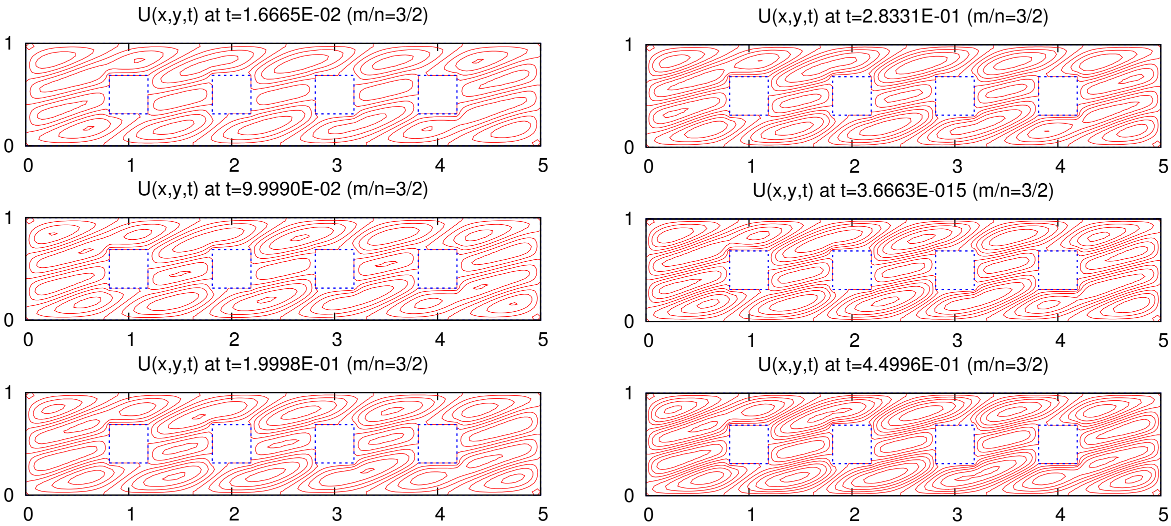

Fig. 2 Eddy current stream function $U(x, y, t)$ at different time, excited by a m/n =3/2 external kink mode in a thin wall with holes.

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