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Rotational stabilization and destabilization of the resistive wall modes: predictions of the competing models

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The rotational stabilization of the resistive wall modes (RWM) is analyzed within the single mode cylindrical Fitzpatrick–Aydemir model [1, 2] and the Boozer model [3, 4]. These models are based on different sets of equations and use different physical variables. To make possible a comparison of the predictions either expressed in different languages or implicitly contained in the models we accept here the approach described in [5], introduce common notation and derive the necessary consequences in terms of the measurable values, the mode growth rate γ_0 and the mode rotation frequency Ω_0.

In both cases the dispersion relation is derived by matching the inner and outer solutions for the magnetic perturbation $\mathbf{b}$. In the outer region the solution can be found, with a desired accuracy, using standard electromagnetic methods. The reliability of the final result will then depend on the boundary conditions at the plasma surface, which are different in [1, 2] and [3, 4]. The main issue addressed here is the stability of rotating modes versus the locked modes.

In the outer region the amplitude of the (m,n) harmonic b_m of $\mathbf{b}\cdot\mathbf{e}_r$ is described by

$$\tau_w \frac{\partial B_m}{\partial t} = \Gamma_m B_m - \Gamma_m^0 B_m^{ext}, \qquad (1)$$

where $B_m = b_m(r_w)$, r_w is the wall radius, τ_w is the 'wall time', B_m^{ext} is the contribution to B_m from the currents behind the wall, $\Gamma_m^0 \approx -2|m|$ for the modes with small m and n. In the plasma-wall vacuum gap

$$b_m / b_m^{out} = 1 + (\Gamma_m^0 / \Gamma_m - 1) x^{-2|m|}, \qquad (2)$$

which gives us the Γ_m through b_m / b_m^{out} at the plasma boundary $r = r_{pl}$. Here $b_m^{out} = b_m - b_m^{pl}$, b_m^{pl} is the part of b_m due to the currents in the plasma, and $x = r / r_w$. In more detail the model is described in [5, 6].

From (1) it follows that, when $B_m^{ext} = \text{const}$,

$$\Gamma_m = \tau_w \left(\gamma_0 + in\Omega_0\right), \qquad (3)$$

where γ_0 is the growth/decay rate of the mode, and Ω_0 is the angular frequency of its toroidal rotation. Equating this to Γ_m from (2) yields the dispersion relation in terms of b_m^{out} / b_m.

The Boozer theory assumes that, at the plasma boundary,

$$-b_m^{out}/b_m(r_{pl}) = s - i\alpha\,, \tag{4}$$

which is equivalent to Eq. (15) in [4] with real s and α. Here $s=0$ represents the marginal stability of the locked (nonrotating) modes, while the mode locking corresponds to $\alpha = 0$.

In the Fitzpatrick–Aydemir model [1, 2]

$$-b_m^{out}/b_m(r_{pl}) = s_c(\kappa - \delta_\alpha)\,, \tag{5}$$

where $s_c = c/(1-c)$ with $c = (r_{pl}/r_w)^{2|m|}$, the real quantity κ is the measure of the instability drive defined by Eq. (11) in [2],

$$\delta_a = (g - i\omega_\phi)^2 + \nu(g - i\omega_\phi) \tag{6}$$

with $g = p/n\Omega_1$ and $\omega_\phi = \Omega_\phi/\Omega_1$, the complex growth rate $p = \Gamma_m/\tau_w$ comes from $\exp(pt)$ dependence of the perturbation amplitude, Ω_ϕ is the plasma toroidal angular velocity within the inertial layer (one of the important elements in the model), Ω_1 is a fitting parameter, and ν describes the dissipation. Expression (6) corresponds to Eq. (73) in [1] or Eq. (5) in [2].

The different conditions for $\mathbf{b}$ at the plasma boundary in the Fitzpatrick–Aydemir and the Boozer theories must give different dependencies $\gamma_0(\Omega_0)$ under constraints of each model. We compare them and discuss some properties of the results.

With Boozer boundary condition (4), equations (2) and (3) give us $\gamma_0(s,\alpha)$ and $\Omega_0(s,\alpha)$ and, finally the dispersion relation [5]

$$(\gamma_0 - \gamma_{\min})^2 = (n\Omega_{\max})^2 - (n\Omega_0)^2\,, \tag{7}$$

which relates the mode growth rate γ_0 to its rotation frequency Ω_0 with s a parameter. Here

$$\gamma_{\min} = 0.5(\gamma_l - \bar{\gamma}_w)\,, \qquad n\Omega_{\max} = 0.5(\gamma_l + \bar{\gamma}_w)\,, \tag{8}$$

and

$$\gamma_l \equiv \bar{\gamma}_w \frac{s}{s_c - s}\,, \qquad \bar{\gamma}_w \equiv \frac{2|m|}{\tau_w(1-c)}\,. \tag{9}$$

From (7) it follows that $\gamma_l(s)$ is the growth rate of the locked mode ($\Omega_0 = 0$).

Equation (7) describes two branches. We are interested in the most unstable one:

$$\gamma_0 = \gamma_{\min} + \sqrt{(n\Omega_{\max})^2 - (n\Omega_0)^2}\,. \tag{10}$$

This shows that, for fixed s, the mode rotation can decrease the mode growth rate from γ_l to $\gamma_{\min} = 0.5(\gamma_l - \bar{\gamma}_w)$. Therefore, within the Boozer model, the complete rotational stabilization is possible for the modes with $\gamma_l < \bar{\gamma}_w$ only. The latter means [5] $C_\beta < 0.5$, with C_β a standard measure of the beta gain between the no wall and ideal wall stability limits. In the DIII-D experiments, however, the rotational stabilization of RWM was efficient up to $C_\beta \approx 1$ [7].

The Fitzpatrick–Aydemir model with (5) gives the dispersion relation

$$\kappa - \delta_a = \Gamma_m / (\bar{\gamma}_w \tau_w + \Gamma_m) . \qquad (11)$$

For the modes near the marginal stability (precisely, $|\gamma_l|$ and $|\Gamma_m| / \tau_w$ much smaller than $\bar{\gamma}_w$) this is reduced to

$$\Gamma_m \approx \bar{\gamma}_w \tau_w (\kappa - \delta_a) \approx \gamma_l \tau_w + \bar{\gamma}_w \tau_w (\omega_\phi^2 + i \nu \omega_\phi) , \qquad (12)$$

which predicts the increase of γ_0 with increasing rotation frequency, which is opposite to the dependence $\gamma_0(\Omega_0)$ in (10). According to (12), the mode with $\gamma_l < 0$, stable in the nonrotating plasma (and $\Omega_0 = 0$ in this case), must be destabilized by the plasma rotation with $\omega_\phi^2 > -\gamma_l / \bar{\gamma}_w$.

This destabilizing effect of the mode rotation in the Fitzpatrick–Aydemir model can be seen in the exact dispersion relation (11). At the marginal stability $\gamma_0 = 0$ we have $\Gamma_m = in\Omega_0 \tau_w$, $g = ig_I$ with real g_I, and (11) reduces to

$$\kappa + (\omega_\phi - g_I)^2 + i \nu (\omega_\phi - g_I) = in\Omega_0 / (\bar{\gamma}_w + in\Omega_0) , \qquad (13)$$

which gives us

$$\kappa_{marg}^F = (1 - y)(1 - y / \nu^2) \qquad (14)$$

with $y \equiv \bar{\gamma}_w^2 / (\bar{\gamma}_w^2 + n^2 \Omega_0^2)$. For the nonrotating modes ($\Omega_0 = 0$) we have $y = 1$ and $\kappa_{marg}^F = 0$. The mode rotation makes $y < 1$ and, for $\nu^2 < 1$, we obtain negative κ_{marg}^F when

$$\nu^2 < y < 1 . \qquad (15)$$

The smaller κ at the marginally stable state means deterioration of the plasma stability. Since this happens because of the mode rotation, it must be interpreted as the RWM rotational *destabilization*. In other words, the Fitzpatrick–Aydemir model implies that, in the plasma with weak dissipation (the original restriction of the model [1, 2]), the locked modes must be more stable than the rotating modes. This contradicts to the Boozer theory and to observations [8] demonstrating better stability of the rotating modes.

The Fitzpatrick–Aydemir model was developed assuming a weak dissipation, which was emphasized in [1, 2]. Therefore, $\nu^2 < 1$ would be more consistent (and even more, $\nu^2 << 1$) with the model initial constraints than the opposite choice. However, the only way to completely avoid the rotational destabilization in the Fitzpatrick–Aydemir model is to accept $\nu^2 \geq 1$. But even in this case this model will not agree with the Boozer model on the range of the rotational stabilization, as shown by the relation between the marginal κ's:

$$\kappa^F_{marg} = \kappa^B_{marg} - \kappa^B_{marg}(1-\kappa^B_{marg})/\nu^2 . \qquad (16)$$

With $\nu^2 < 1$ the Fitzpatrick–Aydemir model gives the rotational stabilization at small y only which means large Ω_0. The rotation frequency Ω_0 of RWM measured in the DIII-D tokamak is described as $O(1/\tau_w)$ [7]. With $n\Omega_0 = \bar{\gamma}_w$, which can be a compromise between the need of large $|n\Omega_0|$ in the model and small $|n\Omega_0|$ observed in experiments, we obtain $\kappa^F_{marg} = 0.5 - 0.25/\nu^2$. This does not allow the rotational stabilization of RWM up to β near the ideal-wall stability limit (or $\kappa \approx 1$) which was achieved in the DIII-D experiments [7].

Several experimental tests of the two theories can be proposed. For example, verification of the relations (7), (10) and (12). Also, equation (11) implies that, near the stability boundary,

$$\Omega_\phi / \Omega_0 = \text{const}/(\bar{\gamma}_w^2 + n^2\Omega_0^2) , \qquad (17)$$

if $|\Omega_0| << |\Omega_\phi|$, which is always satisfied in RWM experiments with fast plasma rotation [7]. This becomes especially simple when $n^2\Omega_0^2 << \bar{\gamma}_w^2$ and can be easily compared with data obtained in the tokamaks with varying plasma rotation.

The above analysis makes it clear that predictions of the two models on the rotational stabilization of RWM are incompatible at any choice of the fitting parameters. Besides, they do not allow the RWM stability at β close to the ideal wall limit while the stable operation in this region with plasma rotationally stabilized was repeatedly demonstrated in the DIII-D experiments [7].

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