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The general fishbone-like dispersion relation: a unified description for shear Alfvén Mode excitations*

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Introduction.

Perturbations of the shear Alfvén wave (SAW) spectrum generally consist of singular (inertial) and regular (ideal MHD) structures. For this reason, via asymptotic analyses it is always possible to derive a general *fishbone-like* dispersion relation in the form [1, 2]

$$i\Lambda(\omega) = \delta\hat{W}_f + \delta\hat{W}_k \ . \qquad (1)$$

Here, $i\Lambda(\omega)$ is the inertial layer contribution due to thermal ions, while the right hand side comes from background MHD and Energetic Particle (E.-P.) contributions in the regular ideal regions. In particular, Alfvén Eigenmode (AE) collective excitations by E.-P. are possible due to the coupling of the E.-P. pressure perturbation with the SAW vorticity equation via magnetic curvature drifts [2]. On the basis of this dispersion relation, Eq. (1), two types of modes exist: a discrete Alfvén Eigenmode (AE), for $\mathbb{R}\text{e}\Lambda^2 < 0$; and an Energetic Particle continuum Mode (EPM) [1] for $\mathbb{R}\text{e}\Lambda^2 > 0$. The combined effect of $\delta\hat{W}_f$ and $\mathbb{R}\text{e}\delta\hat{W}_k$, which determines the existence conditions of AE by removing the degeneracy with the SAW accumulation point, depends on the plasma equilibrium profiles. Thus, various effects in $\delta\hat{W}_f + \mathbb{R}\text{e}\delta\hat{W}_k$ can lead to AE localization in various gaps, i.e. to different species of AE [3]. In the case of EPM, meanwhile, ω is set by the relevant energetic ion characteristic frequency and mode excitation requires the drive exceeding a threshold due to continuum damping [5, 6, 7, 8]; i.e., $\mathbb{I}\text{m}\delta\hat{W}_k > \mathbb{R}\text{e}\Lambda$ [4] in Eq. (1). This AE Zoology [9] is consistently described by the single and general dispersion relation, discussed here. In this work, we discuss different applications of practical interest of the general *fishbone-like* dispersion relation and show how it can be the starting point for systematic extensions of our analyses to the nonlinear regime.

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The general fishbone-like dispersion relation: applications of practical interest and extensions to the nonlinear regime.

Equation (1) can be derived by asymptotic matching the regular (ideal MHD) mode structure with the general (known) form of the SAW wave field in the singular (inertial) region, as the spatial location of the shear Alfvén resonance, $\omega^2 = k_\parallel^2 v_A^2$, is approached. Here, v_A is the Alfvén speed. In the simple high-frequency ideal MHD case, one has $\Lambda(\omega) = |s| q R_0 \omega / v_A$ [2], with $s = rq'/q$ the magnetic shear, q the safety factor and R_0 the torus major radius.

A celebrated example of EPM is the *fishbone* instability [4, 10], where $i|s| \left[\left(R_0^2/v_A^2 \right) \omega \left(\omega - \omega_{*pi} \right) (1+\Delta) \right]^{1/2} = \delta \hat{W}_f + \delta \hat{W}_k$, ω_{*pi} is the core ion diamagnetic frequency and $\Delta \propto q^2$ is the enhancement of plasma inertia due to geodesic curvature [11, 12].

The combined effect of $\delta \hat{W}_f$ and $\mathrm{Re}\,\delta \hat{W}_k$, which determines the existence conditions of AE by removing the degeneracy with the SAW accumulation point, depends on the plasma equilibrium profiles. Thus, various effects in $\delta \hat{W}_f + \mathrm{Re}\,\delta \hat{W}_k$ can lead to AE localization in various gaps, i.e. to different species of AE, as described in [3]. This AE Zoology [9] is consistently described by the single and general dispersion relation Eq. (1), as stated in the Introduction. A simple example is the Toroidal AE (TAE) near the lower accumulation point, ω_ℓ, of the toroidicity induced frequency gap in the SAW continuum [13]. In this case, $\Lambda^2 v_A^2/(q^2 R_0^2) = \omega_\ell^2 - \omega^2$ and the TAE mode existence condition reads $\delta \hat{W}_f + \mathrm{Re}\,\delta \hat{W}_k > 0$ [14]. Similar pictures could be easily extended to TAE localized near the upper SAW accumulation point, and to Alfvén Cascades (AC) [15] or Reversed Shear AE (RSAE) [16]. In this latter case, *e.g.* , $\Lambda^2 v_A^2/(q^2 R_0^2) = (\omega^2 - k_\parallel^2 v_A^2)/(k_\parallel q R_0)$ [2] and the gap mode existence condition, $\delta \hat{W}_f + \mathrm{Re}\,\delta \hat{W}_k > 0$, is favored by the Mercier stability condition at $s = 0$ [17]. The effect of plasma compressibility on ACs was also analyzed recently [17, 18], when the mode frequency becomes comparable with that of the low-frequency SAW accumulation point $\omega \simeq \beta_i^{1/2} (7/4 + T_e/T_i)^{1/2} v_A / R_0$ [19, 20], with β_i the thermal ion beta. Generally, causality constraints imply that the existence condition for gap modes with frequency just above a SAW continuum accumulation point are characterized by $\delta \hat{W}_f + \mathrm{Re}\,\delta \hat{W}_k > 0$. Meanwhile, $\delta \hat{W}_f + \mathrm{Re}\,\delta \hat{W}_k < 0$ is the necessary condition for gap modes with frequency just below an accumulation point of the SAW continuum.

The low frequency SAW continuum frequency gap deserves a special note since, in this case, the mode frequency can be comparable with thermal (core) ion diamagnetic and/or transit frequencies; i.e., $|\omega| \approx \omega_{*pi} \approx \omega_{ti}$. We could generally refer to this SAW frequency gap as the Kinetic Thermal Ion (KTI) gap. In fact, the ideal MHD accumulation point ($\omega = 0$ at $k_\parallel = 0$) is shifted by either the ion diamagnetic drift (as in the Kinetic Ballooning Mode (KBM) case [21]), or by parallel and perpendicular ion compressibility (as for Beta induced AE (BAE) [22]), or, more generally, by the combined effects of finite ∇T_i and wave-particle resonances with thermal (core) ions (as for Alfvén Ion Temperature Gradient driven mode (AITG) [23]) in the inertial layer. For the AITG, the SAW continuum accumulation point could be shifted to the complex ω plane and, thus, become unstable for modes with sufficiently short wavelength ($\lambda_\perp \gtrsim \rho_i$, the ion Larmor radius). The mode localization condition then leads to the excitation of an unstable discrete AITG even in the absence of the E.-P. drive. Obviously, similar physics considerations and results will be applicable to RSAE/AC.

The general fishbone like dispersion relation, Eq. (1), can be the starting point for systematic extensions of our analyses to the nonlinear regime. Due to the intrinsic EPM resonant character and their localization at the radial position where the drive is strongest, EPM rapidly redistribute E.P.s. Simulation results indicate that, above the linear stability threshold, strong EPM induced fast ion transport occurs in *avalanches* [24]. Such strong transport events occur on time scales of a few inverse linear growth rates (generally $100 \div 200$ Alfvén times, $\tau_A = R_0/v_A$) and have a ballistic character [25] that basically differentiates them from the diffusive and local nature of weak transport. In addition, the radial mode structure evolves on the same time scale of fast ion transport: thus, the regime is strongly non-perturbative. EPM induced avalanches consist of a radially propagating unstable front producing a convective E.-P. radial redistribution [24]. This peculiarity is due to the fact that EPM growth rate has a stronger n dependence, producing a narrow toroidal mode number unstable spectrum, in contrast with the AE case. As a consequence, single-n non-linear dynamics may be expected to dominate the initial rapid convective phase. The radial propagation of a single-n non-linear EPM localized mode structure takes place via couplings between poloidal harmonics and their interplay with non-linear distortion of the E.-P. source. An analytical description has been proposed for elucidating the EPM avalanche paradigm

$$D^{\ell}_{EPM}(-i\omega+\partial_t,\partial_r,r)A(r,t) = \delta\hat{W}^{n\ell}_k\left(\partial_t,\partial_r,r,|A|^2\right)A(r,t)\ , \qquad (2)$$

where $A(r,t)$ is the radial envelope of the single-n EPM poloidal harmonics, $D^{\ell}_{EPM}(-i\omega+\partial_t,$ $\partial_r,r)$ is the linear EPM dispersion function, including radial dispersiveness, and $\delta\hat{W}^{n\ell}_k(\partial_t,\partial_r,$ $r,|A|^2)$ is the E.-P. $\delta\hat{W}_k$ due to non-linear wave particle interactions.

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