*34th EPS Conference on Plasma Phys. Warsaw, 2 - 6 July 2007 ECA Vol.**31F**, P-4.064 (2007)*

Cross-field plasma particle transport in strong three dimensional magnetic turbulence with significant collisionality

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Introduction. There is strong interest in plasma particle transport perpendicular to turbulent magnetic fields in a wide range of plasmas. Applications span the terrestrial magnetosphere [1], the solar wind [2], cosmic ray acceleration and propagation [3-5], and magnetically confined fusion plasmas [6]. Cross-field transport is due to the complex interplay between collisional scattering, resonant scattering, field line wandering and finite Larmor radius effects. Of particular interest are regimes where the characteristic Larmor radius r_L of the particle population is comparable to the characteristic wavelength k_B^{-1} of the the magnetic field variation, such that $r_L k_B \approx 1$; or where the amplitude of the magnetic field variation δB is comparable to, or greater than, the background field B_0, so that $\delta B/B_0 \geq 1$. Both these regimes are unreachable by analytic treatment alone. We present a study of cross-field electron transport using a new finite difference Vlasov-Fokker-Planck code based on the spherical harmonic formulation [7] that enables accurate computation of collisional effects. Our approach involves quantifying the evolution of distributions of particles in space and pitch angle, (Fig.3) rather than tracking individual particles, thus it does not suffer from statistical noise.

Model. The turbulent magnetic field is represented by a superposition of harmonic modes: $\mathbf{B}(\mathbf{x}) = B_0\hat{\mathbf{z}} + \sum_i \mathbf{B}_i \cos(\mathbf{k}_i \cdot \mathbf{x} + \phi_i)$ where the dependence of B_i on k_i reflects the chosen power law scaling of the turbulence and the phases ϕ_i are random, furthermore the turbulence is isotropic; see Figs.1 and 2. The magnetic field is incorporated in the KALOS implementation

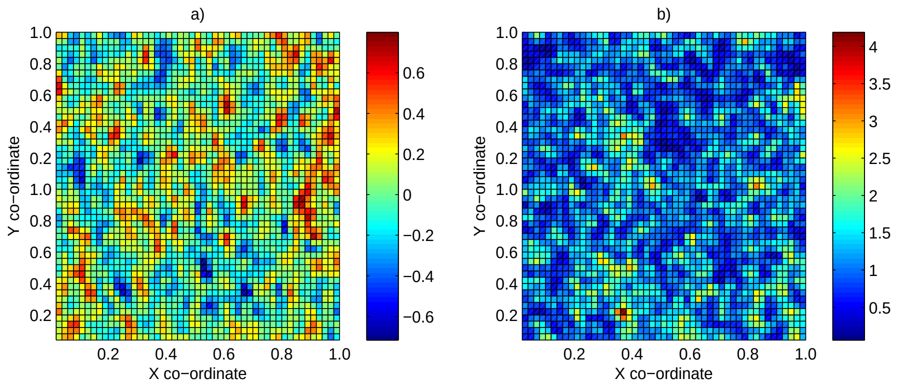

Fig. 1: Slice through computational box in the X-Y plane of the $a)$ X component and $b)$ magnitude of the magnetic field, here modelled as a superposition of many hundred modes, whose squared magnitudes follow Kolmogorov scaling $\propto k^{-5/3}$.

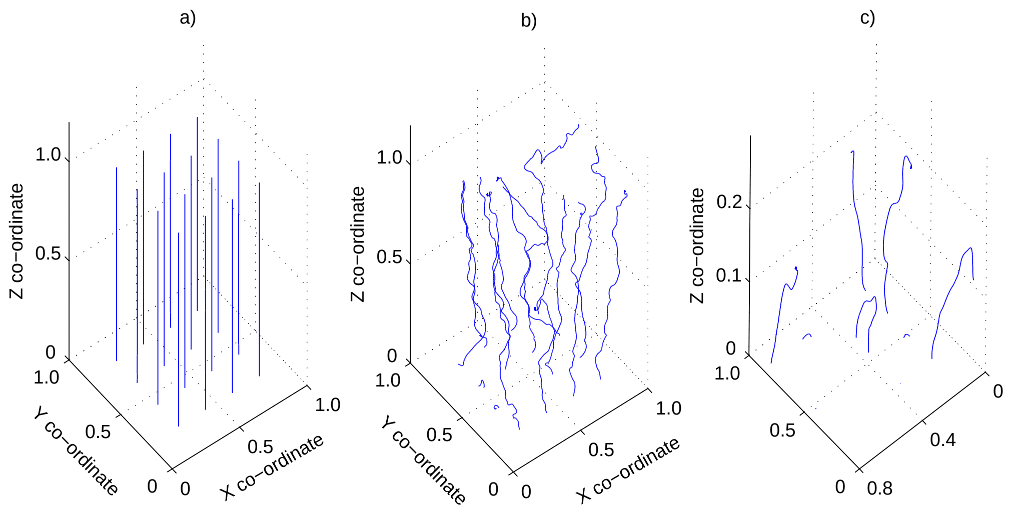

Fig. 2: Characteristic magnetic field lines from simulations when $\delta B/B_0 \approx$ (a) 0; (b) 1; (c) 10.

of the Vlasov-Fokker-Planck (VFP) equation, Eq.(29) of Ref.[7]. In contrast to [7] we solve the VFP equation in all three spatial dimensions. By evolving this equation, diffusion coefficients are calculated from the study of a population of electrons that is initially monoenergetic, but uniformly distributed in pitch angle, in a stationary ion background. Figure 3 shows how an initial electron density profile relaxes within our stationary magnetic field model. The diffusion coefficients are calculated from the relaxation rate of this perturbation.

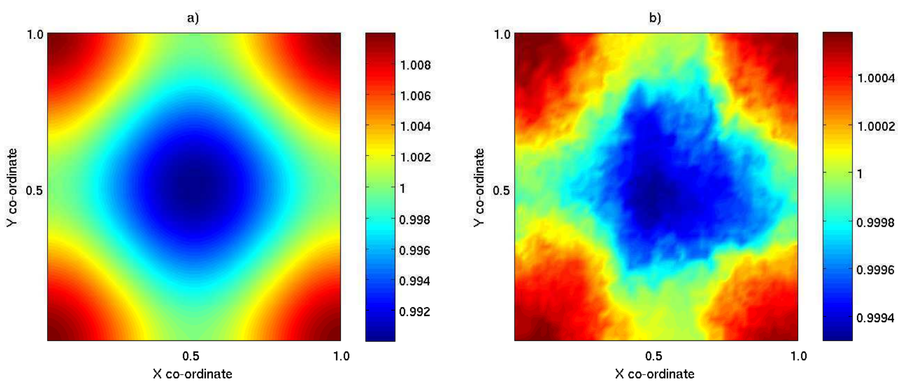

Fig. 3: $a)$ Initial electron density profile, showing a 1% harmonic perturbation. $b)$ Electron density after $\sim$ 600 Larmor periods. All simulation data are calculated on a Cartesian grid $n_x = n_y = n_z = 60$. Boundaries are periodic.

Results. Figure 5 shows key results from our code, which is benchmarked (Fig.4) against a reduced Braginskii treatment, where the mean free path is significantly less than the length scale of thermal and density gradients and the analytical diffusion coefficient is $D_\perp = {v_{th}}^2\tau/(3(1 + (\omega_0\tau)^2)$; here v_{th}, τ, and ω_0 denote repectively the electron thermal velocity, collision timescale, and cyclotron frequency.

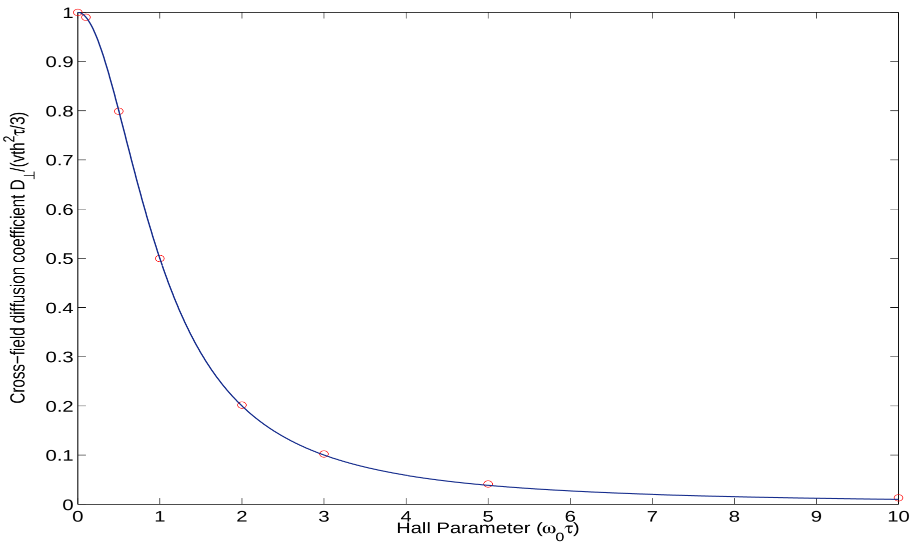

Fig. 4: Calculated cross-field diffusion coefficient versus Hall parameter $\omega_0\tau$. Circles: coefficients inferred from individual code runs. Line: theoretical curve.

Figure 5 a) shows the cross-field coefficient as a function of $\delta B/B_0$ for electrons in a turbulent field with a Kolmogorov spectrum in the case where collisional scattering is significant. The diffusion coefficient is normalised to the gyro-Bohm coefficient $D_B = {v_{th}}^2/3\omega_0 \sim r_L^2\omega_0$ assuming $\tau \sim \omega_0^{-1}$. The left hand asymptote corresponds to the collisional diffusion coefficient $D_\perp$. Collisions are the dominant transport process until the magnetic turbulent transport processes, including field line wandering and resonant scattering, become significant at $\delta B/B_0 \approx 0.01$. There is a smooth transition from the collisonal regime to quasilinear scaling $D_\perp \sim (\delta B/B_0)^2$, which continues until a resonant peak at $\delta B/B_0 \approx 2$. In the case where collisions are negligible ($\omega_0\tau \gg 1$), the dominant transport processes are field line wandering and resonant scattering. Both processes are important here, since we choose the characteristic Larmor radius to be comparable to the length scale of the magnetic turbulence. When r_L becomes smaller than the characteristic turbulence length, we observe a suppressed peak, consistent with a reduction of resonant scattering. Figure 5 b) shows the cross-field diffusion coefficient as a function of $\delta B/B_0$ for three different turbulent spectral indices γ, where $\mid \delta B_k \mid^2 \sim k^{-\gamma}$, corresponding to standard models. The height and position of the resonant peak is insensitive to the spectral index. In the low turbulence limit ($\delta B/B_0 \ll 1$) we again see a quasilinear scaling $(\delta B/B_0)^2$ in agreement with analytic work. Furthermore we see that the coefficient never reaches our estimate of the gyro-Bohm limit.

Conclusions. We have written and comprehensively benchmarked a Vlasov-Fokker-Planck finite difference code for the study of electron transport in magnetic turbulence. Uniquely it allows tuning of all key parameters, including: the ratios of Larmor radius to turbulence length-

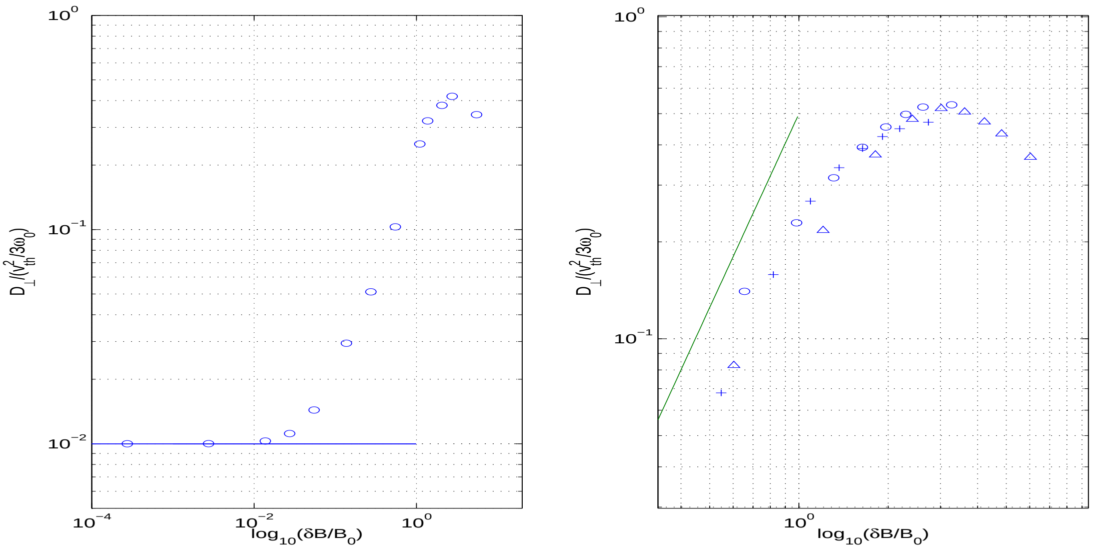

Fig. 5: The cross field diffusion coeffient normalised to the gyro-Bohm coefficient as a function of $\delta B/B_0$ in isotropic magnetic turbulence with a) significant electron-ion collisionality ($\omega_0\tau = 10^2$) and b) low collisionality ($\omega_0\tau = 10^6$). The horizontal line in a) marks the collisional limit. Graph b) shows data around the peak at $\delta B/B_0 \sim 1$ for three different spectral indices: Kolmogorov ($\gamma = 5/3$ - Crosses); Iroshnikov-Kraichnan ($\gamma = 3/2$ - Circles); and inverse ($\gamma = 1$ - Triangles).

scale and collisional mean free path; and the magnitude and degree of complexity of the magnetic turbulence, and its spectral index. It has been used to obtain new results on the effect of magnetic turbulence on cross-field electron transport, in regimes of widespread interest that are otherwise inaccessible.

Acknowledgements. This work was supported in part by the EPSRC and Euratom.

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