Critical transition model with radial structure

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Recently, the experimental results for the emergence of the plasma shear flow layer in TJ-II have been explained as a second-order phase transition like process by using a simple model of envelope equations for the fluctuation level, the averaged poloidal velocity shear, and the pressure gradient [1]. The model employed is an extension of a local phase transition model for the L-H bifurcation [2] and contains the dependence of the coefficients on the pressure gradient.

Here, we extend this model by incorporating radial coupling. In this way, we obtain a transport model similar to the one proposed in Ref. [3], except for the equation of the averaged poloidal velocity shear. In this equation, the expression for the Reynolds stress (derived from a quasi-linear calculation) is similar to the one proposed in Ref. [4] and has two terms. The one responsible for the generation of flow is a negative viscosity term and the other is a hyper-viscosity term.

The model is formulated in terms of three fields: the normalized turbulence fluctuation level E; the poloidal flow shear σ and the flux-averaged pressure p.

$$\frac{\partial E}{\partial t} = N^{2/3}E - N^{-1/2}E^2 - N^{-1/3}\sigma^2 E + \frac{\partial}{\partial x}\left[(D_1 E + D_0)\frac{\partial E}{\partial x}\right], \tag{1}$$

$$\frac{\partial \sigma}{\partial t} = \sigma - \alpha_3 \frac{\partial^2}{\partial x^2}\left(N^{-4/3}E^2\sigma\right) - \frac{\partial^2}{\partial x^2}\left[\left(D_2 N^{-5/3}E^2 + D_3\right)\frac{\partial^2 \sigma}{\partial x^2}\right], \tag{2}$$

$$\frac{\partial p}{\partial t} = S(x) + \frac{\partial}{\partial x}\left[(D_1 E + D_0)\frac{\partial p}{\partial x}\right], \tag{3}$$

where $N = -\partial p/\partial x$. We assume that $S(x)$ is zero, except for a constant flux Γ_0 from the core, which determines the boundary condition at $x = 0$ according to

$$\Gamma_0 = -(D_1 E + D_0)\left.\frac{\partial p}{\partial x}\right|_0, \quad p(1) = 0 \tag{4}$$

For the other equations we use zero derivative boundary conditions,

$$\left.\frac{\partial E}{\partial x}\right|_0 = \left.\frac{\partial E}{\partial x}\right|_1 = \left.\frac{\partial \sigma}{\partial x}\right|_0 = \left.\frac{\partial \sigma}{\partial x}\right|_1 = \left.\frac{\partial^3 \sigma}{\partial x^3}\right|_0 = \left.\frac{\partial^3 \sigma}{\partial x^3}\right|_1 = 0 \tag{5}$$

Apart from the trivial solution, $E = \sigma = 0$, Equations (1) and (2) have one fixed point solution with $\sigma = 0$ and $E = E_0$ (constant). We obtain p by integrating Eq. (3). For stationary solutions,

$$D_1 E + D_0 = \Gamma_0 / N \tag{6}$$

By substitution of expression (6) in Eq. (1), we obtain for the fixed point solution,

$$D_1 N_0^{13/6} + D_0 N_0 - \Gamma_0 = 0, \quad E_0 = N_0^{\ 7/6} \tag{7}$$

For solutions close to the fixed point is possible to derive a simplified description of the system using a multiple scale perturbation analysis. We introduce a small parameter δ representing the size of the perturbation, and we consider the following expansion

$$\sigma = \delta\sigma_1, \quad E = E_0 + \delta^2 E_2, \quad N{=}N_0 + \delta^2 N_2 \tag{8}$$

By substituting the expansion in Eq. (2), we get at first order

$$\frac{\partial\sigma_1}{\partial t} = -\sigma_1 - \alpha_3 N_0 \frac{\partial^2\sigma_1}{\partial x^2} - (D_2 N_0^{2/3} + D_3)\frac{\partial^4\sigma_1}{\partial x^4} \tag{9}$$

We try as solutions modes like $\sigma_1 = \sigma_{10}\cos(k\pi x)$ which satisfy the boundary conditions. To study their stability properties, we consider a temporal and spatial dependence like

$$\sigma(x,t) = \delta\sigma_{10} e^{\gamma t} \cos(k\pi x) \tag{10}$$

Then from Eq. (9) we get

$$\gamma = -1 + \alpha_3 N_0 (k\pi)^2 - (D_2 N_0^{2/3} + D_3)(k\pi)^4 \tag{11}$$

This means that the range of possible unstable modes is given by the relation

$$k_-^2 < k < k_+^2 \tag{12}$$

where

$$(k_\pm \pi)^2 = \frac{\alpha_3 N_0 \pm \sqrt{(\alpha_3 N_0)^2 - 4(D_2 N_0^{2/3} + D_3)}}{2(D_2 N_0^{2/3} + D_3)} \tag{13}$$

From this expression, we obtain a necessary condition to have instability, and that is the existence of real solutions,

$$(\alpha_3 N_0)^2 \geq 4(D_2 N_0^{2/3} + D_3) \tag{14}$$

This expression gives us a threshold for N_0, and, consequently, a threshold for the flux through Eq. (7),

$$\Gamma_c = D_1 N_0^{13/6} + D_0 N_0 \tag{15}$$

This condition may not be sufficient. To have a sufficient condition for instability, there should be an integer k between k_- and k_+. This means that the critical value for

N_0 corresponds to a solution of Eq. (13) for the closest integer to the k-value obtained when condition (14) is verified.

By applying Eq. (13) to different k-values, we can obtain the flux threshold (Fig. 1). We have done a scan in D_2 with k going from 1 to 7, and the rest of parameters being

$$D_0 = 10^{-3}, D_1 = 10^{-2}, D_3 = 10^{-6}, \alpha_3 = 0.0175$$

By substituting now expansion (8) in Eqs. (1) to (3) we can obtain analytical expressions for E_2, N_2, and σ_{10}. In Fig. 2 we compare the analytical and numerical results for $D_2 = 1.5 \times 10^{-3}$ and $k = 1$.

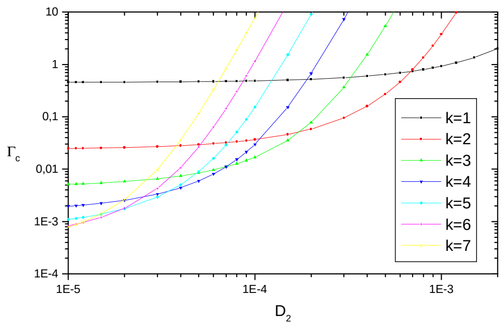


Figure 1.

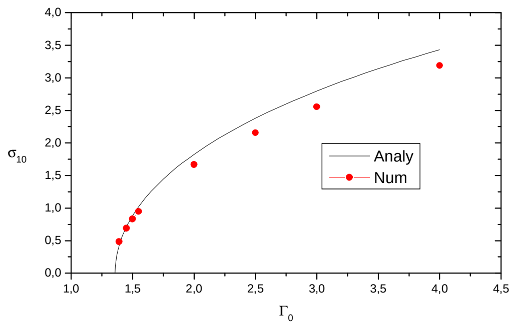


Figure 2.

External Torque

To study the effect of an external torque, we add a term, $\tau = \tau_0 \cos(k\pi x)$, to the r.h.s. of Eq. (2). For low values of τ_0, we can use the same expansion as before assuming σ of order δ. However, for moderate values of τ_0, the third term of the r.h.s. of Eq. (1) cannot be assumed of higher order than the first two terms. So, in general, we will write

$$E = E_0 + \widetilde{E}, N = N_0 + \widetilde{N}, \sigma = \sigma_1 \cos(k\pi x) + \widetilde{\sigma}$$

and we will assume that $\widetilde{E} = E_2 \cos(2k\pi x) + ..., \widetilde{N} = N_2 \cos(2k\pi x) + ..., \widetilde{\sigma} = \sigma_3 \cos(3k\pi x) + ...$

At lowest order, it is easy to find a system of equations for E_s, N_s, and σ_1. The solution of this system of equations gives similar results to the ones obtained by numerically solving the evolution equations.

In the rest of the calculations of this paper, we apply the torque during a time $t = 10$, and then we remove the torque to analyze the decay of the shear flow. The evolution of the integral of σ^2 for different values of τ_0 when $\Gamma_0 = 1$ is shown in Fig. 3. In most of the cases, the shear flow has two decay scales and the change between them is more pronounced as τ_0 increases.

The square root of the integral of σ^2 decays like $\exp(\gamma_1 t)$ just after removing the external torque (first decay region), and like $\exp(\gamma_2 t)$ at larger times (second decay region). The first decay rate is easily understood from Eq. (2). As we remove the torque, the instantaneous exponential decay rate will be

$$\gamma_1 = -1 + \alpha_3 (k\pi)^2 N_s^{-4/3} E_s^2 - (k\pi)^4 (D_2 N_s^{-5/3} E_s^2 + D_3)$$

The second exponential decay rate is very similar for all the cases, with γ_2-values between -0.16 and -0.18.

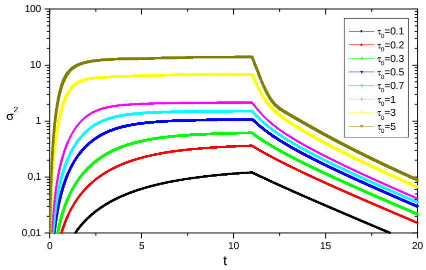
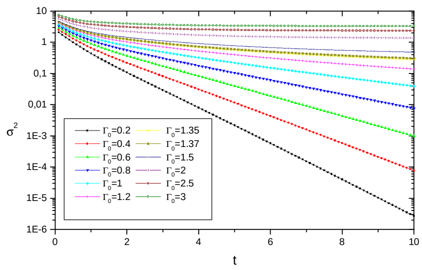


Figure 3.

Finally, we have done a scan in Γ_0 when the external torque has the value τ_0 = 2. The evolution of the integral of σ^2 when we suppress the external torque is shown in Fig. 4. This evolution is fitted to $y_0 + A_1 \exp(-t/T)$, as it is done in biasing experiments in TJ-II [5]. We show the values of the decay time T in Fig. 5. The slower decay corresponds to the flux values closer to Γ_c. The damping is close to the viscous damping only when the flux is far above the threshold. The results are similar when the torque is changed.



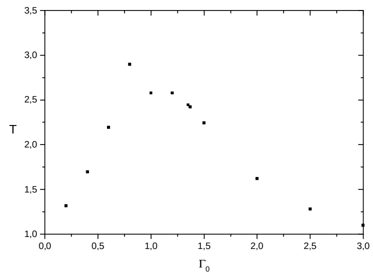


Figure 4. $D_2 = 1.5\times10^{-3}\,(k=1)$ **Figure 5.** Γ_c=1.35794

[1] B.A. Carreras, L. Garcia, M.A. Pedrosa, C. Hidalgo, Phys. Plasmas **13**, 122509 (2006)

[2] P.H. Diamond, Y.-M. Liang, B.A. Carreras, P.W. Terry, Phys. Rev. Lett. **72**, 2565 (1994)

[3] P.H. Diamond, V.B. Lebedev, D.E. Newman, B.A. Carreras, Phys. Plasmas **2**, 3685 (1995)

[4] P.H. Diamond, S.-I. Itoh, K. Itoh, T.S. Hahm, Plasma Phys. Controlled Fusion **47**, R35 (2005)

[5] M.A. Pedrosa, et al., at this conference.