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1-D model for the emergence of the plasma edge shear flow layer with momentum conserving Reynolds stress

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Introduction

In the last years, a great deal of attention has been directed to the formation of shear flow layers and the corresponding region of radial electric field gradient to understand the improved confinement regimes. Many of the recent theoretical developments in this direction have been focused on barrier formation [1] or zonal flows [2, 3]. On the experimental side, much progress has been done in the visualization of edge turbulence and flows ([4], [5]), including specific applications to the emergence of the shear flow layer in TJ-II ([6]), in which we are especially interested.

The emergence of the plasma edge shear flow layer as the density increases in the TJ-II stellarator [7] is shown [8, 9] to have the characteristic properties of a second order phase transition. It is consistent [10] with a simple transition model that couples shear flow amplification by turbulence [11, 12] with turbulence suppression by sheared flows [13]. The model used in interpreting the TJ-II results is based on a transition model [14] initially introduced to explain the transition from the low confinement mode (L mode) to the high confinement mode (H mode) [15] in magnetically confined plasmas. This model consists of two envelope equations for the fluctuation level and mean poloidal flow. A later extension of the model [16] included a third equation to account for the pressure gradient contribution to the radial electric field. This second model shows the existence of two critical points, the second one causing the first order transition that has been associated with the L to H transition. The first critical point leads to a second order transition, which has been identified with the emergence of the plasma edge shear flow layer.

In the present work, we focus on this second order transition and discuss an extension to 1-D of the original model used in comparison with the experimental data [10]. In contrast with previous 1-D extensions of the transition model [17, 18], here we formulate the Reynolds stress term as a momentum conserving term.

The model

The relevant plasma edge region to which this model is applied corresponds to $r \in [r_0, a]$, $(a - r_0)/a \approx 0.1$, where a is the minor radius of the plasma. We take the slab geometry approximation in representing this region and use as coordinate $x := (r - r_0)/(a - r_0)$, so that $x \in [0, 1]$.

The fields of our model will be the fluctuation level envelope $E := \langle (\tilde{n}_k/n_0)^2 \rangle^{1/2}$, the averaged poloidal shear flow $U := \partial \langle V_\theta \rangle / \partial r$ and (minus) the averaged pressure gradient $N := -\partial \langle p \rangle / \partial r$, where $\langle \cdot \rangle$ denotes ensemble average. A suitable one-dimensional generalization of the model discussed in [10] requires a form of the Reynolds stress which conserves momentum. Using a pressure-gradient-driven turbulence model and assuming densely packed turbulence, a quasi-linear calculation yields the following form for the Reynolds stress:

$$\langle \tilde{V}_x \tilde{V}_\theta \rangle = \alpha_3 E^2 \partial_x \langle V_\theta \rangle + D_2 E^2 \partial_x^3 \langle V_\theta \rangle, \tag{1}$$

which is very similar to the expression previously derived in Ref. [19] in the context of zonal flow dynamics. Notice that α_3 measures the strength of the Reynolds stress and non-zero D_2 is needed for the spectrum of the instability to be bounded.

The equations defining our model are:

$$\partial_t E = NE - E^2 - U^2 E + \partial_x [(D_0 + D_1 E)\partial_x E] \tag{2a}$$

$$\partial_t U = -\mu_1 U + \mu_2 \partial_x^2 U - \alpha_3 \partial_x^2 (E^2 U) - D_2 \partial_x^2 (E^2 \partial_x^2 U) \tag{2b}$$

$$\partial_t N = \partial_x^2 [(D_3 E + D_4) N] \tag{2c}$$

where D_0 and D_4 are neoclassical diffusivity coefficients, D_1 and D_3 are the coefficients of anomalous diffusivity multiplying the fluctuation level, and μ_1, μ_2 are the coefficients of the collisional flow-damping terms. Finally, the boundary conditions are: $\partial_x U(0,t) = \partial_x^3 U(0,t) = \partial_x U(1,t) = \partial_x^3 U(1,t) = 0,\ \partial_x E(0,t) = \partial_x E(1,t) = 0,\ (D_3 E + D_4)N\big|_{(0,t)} = \Gamma,\ \partial_x N\big|_{(1,t)} = 0,\ \forall t$. Here, Γ is the particle flux and is the natural control parameter of the model.

Linear stability analysis of the critical point

The only non-trivial fixed point of the model is

$$U_0 = 0,\ E_0(\Gamma) = N_0(\Gamma) = \frac{1}{2D_3}\left(\sqrt{D_4^2 + 4D_3\Gamma} - D_4\right). \tag{3}$$

A linear stability analysis reveals that there exist unstable modes if and only if

$$\alpha_3 E_0^2 - \mu_2 - 2E_0\sqrt{\mu_1 D_2} \geq 0. \tag{4}$$

Now observe that the boundary conditions imply the quantization of k. Namely,

$$k = n\pi,\ 0 \leq n \in \mathbb{Z}. \tag{5}$$

Therefore (4) is only a necessary condition for the existence of instabilities. The critical point is defined by the minimum value of the flux, Γ_c, for which there exists an unstable mode $k_c = n_c\pi$. Define $E_c := E_0(\Gamma_c)$. Then, one can show that

$$E_c^2 = \frac{\mu_1 + \mu_2 k_c^2}{k_c^2(\alpha_3 - D_2 k_c^2)}\ . \tag{6}$$

Thus, in particular, if $\alpha_3/D_2 \leq \pi^2$ there are no unstable modes, no matter how much we increase Γ. If $\alpha_3/D_2 > \pi^2$ at least $k = \pi$ can become unstable. Actually, if $\sqrt{\alpha_3/D_2} \in (\pi, n\pi)$, there exist $n-1$ potentially unstable modes $k = \pi, 2\pi, \ldots, (n-1)\pi$.

Dynamics near marginal stability

Our aim is to find approximate equations for the dynamics of Eqs. (2) near (and above) the critical point. To that end we perform an expansion with parameter $\delta = \sqrt{\Gamma/\Gamma_c - 1}$, for small δ. Explicitly, we take $E = E_c + \delta^2 E_2 + \ldots,\ U = \delta U_1 + \ldots,\ N = N_c + \delta^2 N_2 + \ldots,\ \Gamma = \Gamma_c(1+\delta^2)$ and perform a rescaling of the coordinates $\overline{\eta} = \delta x,\ \overline{\tau} = \delta^4 t$.

Expanding Eqs. (2a) and (2c) gives

$$N_2 = E_c\left(1 - \frac{D_3 E_2}{D_4 + D_3 E_c}\right), \quad E_2 = \frac{D_4 + D_3 E_c}{D_4 + 2D_3 E_c}\left(E_c - U_1^2\right). \tag{7}$$

Consequently, for weakly unstable states, the problem of studying the dynamics of our model consists in finding an approximate equation for the dynamics of U, E and N being determined at the end of the day from the slaving conditions (7). At low collisionality the magnetic pumping dominates and $\mu_2 = 0$ is a good approximation (*collisional drag*). If the collisionality at the plasma edge is high enough, the damping is essentially diffusive and the relevant limit is $\mu_1 = 0$ (*collisional diffusion*). Next, we study both cases separately.

– *Collisional drag:* Let us set $\mu_2 = 0$, $\mu_1 \neq 0$, so that $E_c := 1/\sqrt{\mu_1^{-1} k_c^2(\alpha_3 - D_2 k_c^2)}$. In order to find a reduced equation for the weakly non-linear dynamics of U we expand (2b), the final result being

$$\partial_t U = -\mu_1 U - E_0(\Gamma)^2\left(\alpha_3 \partial_x^2 U - D_2 \partial_x^4 U\right) + 2E_c \frac{D_4 + D_3 E_c}{D_4 + 2D_3 E_c}\left[\alpha_3 \partial_x^2 U^3 + D_2 \partial_x^2\left(U^2 \partial_x^2 U\right)\right].$$

– *Collisional diffusion:* Now, we take $\mu_1 = 0$ and $\mu_2 \neq 0$. In this situation the critical point is given by $E_c = 1/\sqrt{\mu_2^{-1}(\alpha_3 - k_c^2 D_2)}$ and, unlike the case of collisional drag, the critical mode is always $k = \pi$. For typical values of α_3 and D_2, $\alpha_3 \gg \pi^2 D_2$, and we can make the approximation $E_c = \sqrt{\mu_2/\alpha_3}$. Using this, expanding Eq. (2b) and performing the change of variables:

$$\sigma = \frac{1}{\sqrt{E_c}} U_1, \quad \tau = \frac{1}{D_2}\left(\frac{2\alpha_3 E_c (D_4 + D_3 E_c)}{D_4 + 2D_3 E_c}\right)^2 \overline{\tau}, \quad \eta = \sqrt{\frac{2\alpha_3 (D_4 + D_3 E_c)}{D_2 (D_4 + 2D_3 E_c)}}\ \overline{\eta} \tag{8}$$

we obtain the definitive form of the equation describing the weakly non-linear dynamics of U:

$$\partial_\tau \sigma = -\partial_\eta^2\left[\sigma - \sigma^3 + \partial_\eta^2 \sigma\right]. \tag{9}$$

Using that at the boundary, $\partial_x \sigma = \partial_x^3 \sigma = 0$, and taking an initial condition such that $\int_0^1 U(x) \mathrm{d}x = 0$, the stationary solutions of Eq. (9) are exactly the solutions of $\sigma - \sigma^3 + \partial_\eta^2 \sigma = 0$, which is the time-independent Ginzburg-Landau equation for second-order phase transitions. It is the same equation as in the case of non-momentum-conserving Reynolds stress with a collisional drag [20]. The stationary solutions can be computed analytically. In terms of the original variables:

$$U(x) = \delta \sqrt{E_c b_-}\ \mathrm{sn}\left(\sqrt{\frac{b_+ \alpha_3 (D_4 + D_3 E_c)}{D_2 (D_4 + 2D_3 E_c)}}\ \delta x + A \middle| m\right). \tag{10}$$

where $b_{\pm} = 1 \pm \sqrt{1-4C}$, $m = b_{-}/b_{+}$, A and C are integration constants determined by the boundary conditions and $\mathrm{sn}(y|m)$ stands for the Jacobi elliptic function, which is periodic in y with period $P = 4K(m)$,

$$K(m) = 4\int_0^{\pi/4} \frac{\mathrm{d}\theta}{\sqrt{1-m\sin^2\theta}}. \tag{11}$$

Conclusions and further work

We have introduced a one-dimensional version of the phase transition model considered in [10] including a Reynolds stress term with manifest momentum conservation. The analysis performed in this work shows some interesting differences with respect to previous one-dimensional versions of the model in which momentum conservation is not implemented (see Ref. [20]), the most relevant of them concerning the nature of the fixed points and the instabilities.

The next step would be to extend the model including the necessary ingredients to be compared with experimental data. This amounts to introduce the appropriate dependences on N of the coefficients which have been taken constant in the present work and to incorporate a dynamical equation for neutral particles. This issue will be addressed in future publications.

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