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Characterisation of noise in gyrokinetic full-f particle simulation

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Introduction: Small scale, low frequency instabilities drive turbulence, which is believed to be one of the main processes leading to the enhanced transport observed in tokamaks. In order to reproduce interesting processes where the particle distribution function may deviate from Maxwellian, kinetic methods are preferred. Gyrokinetic plasma simulation, either with continuum or particle codes, has become a standard tool for such transport analysis. While continuum codes often are more productive inside the plasma, particle-in-cell (PIC) simulations are easier to implement at the plasma edge. However, particle noise is often an issue, which must be considered. While standard techniques include the use of smoothing, finite size particles and field-aligned coordinates, modern turbulence codes further reduce noise significantly by using a so-called delta-f technique where only perturbed quantities are simulated. Still, there are reasons not to use such technique (e.g. near the edge) and full-f simulation is needed to simulate strongly perturbed plasmas including wide orbit effects, steep gradients and rapid dynamic changes.

The gyrokinetic full-f code Elmfire [1] includes both turbulence and neoclassical physics but there are serious restrictions in terms of CPU and memory resources. Recently, it was shown that noise can have significant effect on results even in delta-f simulations [2], but the diagnostics developed there are not directly applicable for full-f simulations using constant particle weights. In the present paper, the effect of noise on fluctuation levels and macroscopic quantities in gyrokinetic full-f particle simulation code Elmfire [1] is analyzed.

Density and potential fluctuations: The present code Elmfire solves the gyrokinetic full-f equations for quasi-neutrality with a stochastic PIC algorithm based on method developed in Ref. [3]. The code is electrostatic and parallel motion of kinetic electrons is solved implicitly. The numerical methods in the code are described in Ref. [1].

In the code, the value of density in each cell is sampled from ensemble of test particles, which each are carrying information of both of their location and velocity components affecting the Larmor radius. Even the coefficient matrix, from which the potential is solved, is sampled from test particles without any assumption of velocity distribution.

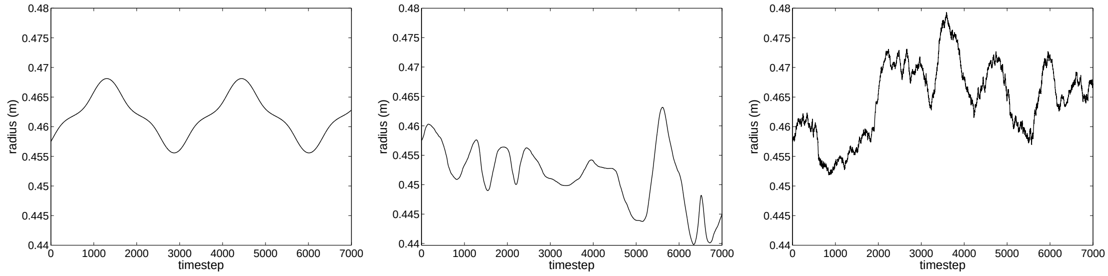

Figure 1: *Radial coordinate of a) unperturbed orbit, b) orbit with Coulomb collisions, and c) orbit with Coulomb collisions and numerical noise.*

Variations of density from averaged value in each flux surface consist of both numerical effects such as noise and initialisation as well as physics including neoclassical effects and turbulence:

$$\delta n/n = \tilde{X}_{turb} + \tilde{X}_{NC} + \tilde{X}_{ini} + \tilde{X}_{noise} \quad (1)$$

Here, the noise level in the particle simulations reduces as $\tilde{X}_{noise} \propto 1/\sqrt{N}$, where N is the average number of simulation particles in each cell. Fluctuations of fully developed turbulence can be quantified using the mixing-length estimate $\tilde{X}_{turb} = 1/k_{\perp}L_n$ for the nonlinear saturation level of physical fluctuations. Neoclassical effects have dependence $\tilde{X}_{NC} \propto (r/R)\cos\theta$, as given in Ref. [4] and, at the beginning of simulation, there is also an effect from initialisation, $\tilde{X}_{ini}$. Here, r and R are the minor and major radius, respectively. In figure 1, using typical ASDEX Upgrade edge parameters, it is shown how a radial motion of unperturbed particle trajectory changes when we first add Coulomb collisions and then also particle noise corresponding to simulation with 500 particles per cell.

As an example of noise-dominated simulation, in figure 2, the rms level of the density and potential fluctuations in the plasma mid-radius for a varying number of simulation particles is shown. Here, parameters similar as in Cyclone Base case are used. The noise problem is introduced by the choice of relatively low temperature ($T = 100$ eV) and weak density gradient ($L_n = 0.5$ m) predicting $\tilde{X}_{turb} \sim 0.01$ (for $k_{\perp}\rho_i \approx 0.2-0.3$ and $L_n/\rho_i = 500$). With these parameters, the $1/\sqrt{N}$ noise is higher than the level expected for physical fluctuations and fluctuations are observed to follow the $1/\sqrt{N}$ law fairly well. Also, after some 20 timesteps, there is no evolution of the fluctuation level in time supporting the fact that the noise dominates the fluctuations in the present case. For steeper density gradient and higher temperature, the physical density fluctuations would be much better resolved at saturation according to the mixing-length prediction.

When looking at figure 2b, one observes that potential fluctuations are higher than predicted from the adiabatic relation $e\delta\phi/T = \delta n/n$. To further investigate the effect of kinetic electrons, series of runs varying different parameters was done measuring the noise level at the beginning of the run. For $\delta n/n$ only clear dependence was found on number of test particles. For density separation (before the neutralizing polarization step), we found $\delta(n_i - n_e) \propto (\delta\phi/B\Delta r)\Delta t$ which indicates strong role of $E \times B$ drift. For potential, noise had roughly the dependence $\delta\phi \propto \sqrt{T}B$ which is in agreement with $e\delta\phi/T = 1/\sqrt{N}k\rho_s$ given in [5].

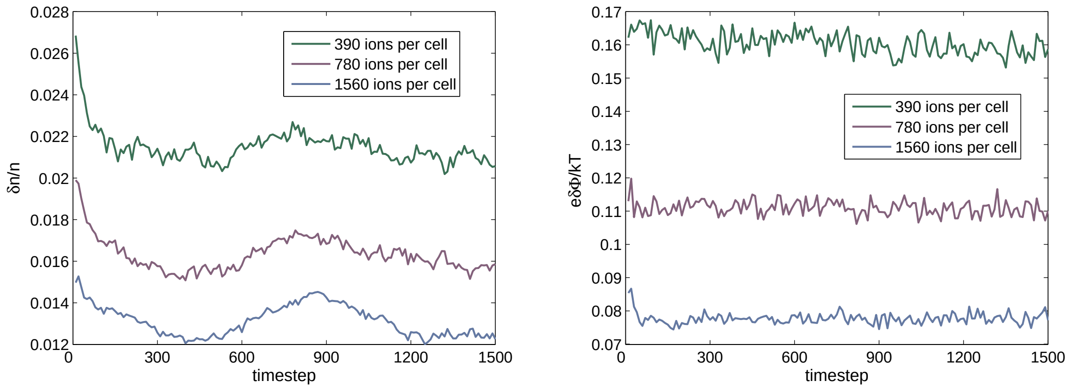

Figure 2: *Density and potential fluctuations as functions of time varying the number of test particles per cell.*

Contribution of noise to transport coefficients: The decorrelation time τ was measured from autocorrelation function in a series of simulations. In most of the cases, fluctuations are decorrelated in 3-5 time steps. As the time step in the simulations is limited to fraction of $\tau_e = \Delta z/v_{Te}$ due to stability reasons, this is consistent with the assumption that the dominant noise is created by electron free motion along the magnetic field, and is decorrelated by the time electrons pass one cell element along the field line. Here, Δz is the grid cell size along the magnetic field line.

To give a prediction for the contribution of the noise to ion heat conductivity, the radial diffusion coefficient is estimated from $D = \langle dr^2 \rangle/\tau$, where dr is the particle shift from the surface by the radial $E \times B$ velocity during the decorrelation time. Taking $dr = \langle\Phi\rangle_{\mathrm{rms}}\tau/B\Delta y$, we find an estimate $D = \langle\Phi\rangle_{\mathrm{rms}}^2\tau/B^2\Delta y^2$, where Δy denotes the grid cell size in the poloidal direction. In figure 3b, we compare our estimate for D calculated from measured τ and $\langle\Phi\rangle_{\mathrm{rms}}$ from series of simulations to the value of χ_{noise} measured at the beginning of each of these simulations. Here, a good agreement is found.

Contribution of noise to the radial electric field has been studied previously in [6]. There it was concluded that as the contribution of noise to the particle fluxes introduces non-ambipolar

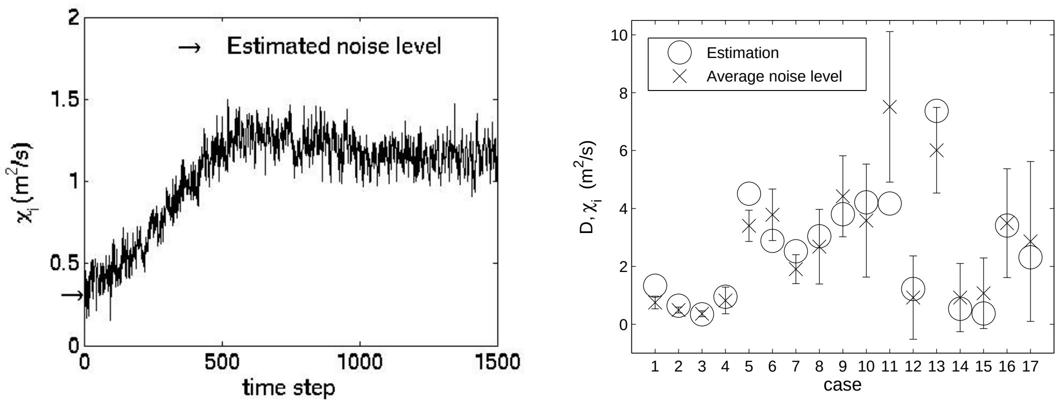

Figure 3: *a) Noise level is measured from the beginning of simulation, b)* χ_{noise} *follows* $D = \langle\Phi\rangle^2_{\rm rms}\tau/B^2\Delta y^2$ *fairly well.*

current components, it thus also affects the flux-surface averaged radial electric field such as given in Ref. [7] for fluctuations in general. This contribution may be small or even change the sign of radial electric field.

Summary: Parametric dependence of noise in density and potential fluctuations in gyrokinetic full-f simulations was studied. It is pointed out that noise not only makes it difficult to isolate physical fluctuations of plasma density and electrostatic potential from those ensuing from the finite number of simulation particles but also creates unphysical particle and heat flux and demolishes the neoclassical equilibrium.

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