Confinement Regime Transition Connected to Spontaneous Rotation Reversal and Collision Rates at the Plasma Edge*

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I. Introduction

The accretion theory of the spontaneous rotation phenomenon observed in axisymmetric plasmas links this to the transport processes prevailing at the edge as well as in the main body of the plasma column. In order to have a relatively simple balance of angular momentum, the theory considers modes excited at the edge that eject particles carrying angular momentum to the material wall surrounding the plasma and inducing recoil rotation in the opposite direction. Another set of modes is involved in carrying the (recoil) angular momentum from the periphery toward the center of the plasma column. In particular, a collisional mode is considered to be responsible for the ejection of angular momentum to the wall, and the reversal of the spontaneous rotation direction in the transition from the L-confinement regime to the H-regime is related to the reversal of the mode's phase velocity in the toroidal direction. A discussion of the modes that can produce the inward transport of angular momentum in the main body of the plasma column has been given in Ref. [3].

II. Collisional Modes at the Plasma Edge

We deal with the main features of the modes that can be excited at the edge: i) ballooning modes that are standing along the magnetic field; ii) traveling modes that propagate along the field. For simplicity we simulate a toroidal confinement configuration by a plane geometry in which the magnetic field is in the z-direction and the relevant gradients are in the x-direction. Thus ballooning modes are represented by the following form of the perturbed density:

$$\hat{n} = \tilde{n}(x)\ \exp\left(-i\ \omega t + i\ k_y\ y\right)\ \cos\left(k_{\parallel}\ z\right) \qquad (1)$$

where, given the complexity of the ballooning mode geometry, the effective longitudinal mode number $k_{\parallel}$ is difficult to assess. The local effect of the magnetic field curvature is simulated by an effective gravity. Thus, by using the guiding center approximation, we can derive the complete dispersion relation

$$\left(\omega - \omega_{*e}^{T}\right)\left[\omega_A^2 - \gamma_I^2 - \left(\omega + i\ \nu_\mu\right)\left(\omega + \omega_{di}\right)\right] = i\ \nu_m\left[\gamma_I^2 + \left(\omega + i\ \nu_\mu\right)\left(\omega - \omega_{di}\right)\right] \qquad (2)$$

where $\nu_\mu = D_\mu^{\perp\perp} k^2 + \nu_{in}$, $D_\mu^{\perp\perp}$ is the effective "viscous" diffusion coefficient for transverse (poloidal) flows, ν_{in} is the ion-neutral collision frequency, $\nu_m = D_m k^2$, $D_m = \nu_{ei}^{\parallel} c^2 / \omega_{pe}^2$ is the magnetic diffusion coefficient, $\omega_A = k_\parallel \mathrm{v}_A$, $\mathrm{v}_A \equiv B / \sqrt{4\pi n m_i}$, $\gamma_I^2 = -2(dp/dx)/R_c\rho$, R_c is the magnetic field curvature radius in the simulated torus, $\omega_{di} = k_y \mathrm{v}_{di}$, $\mathrm{v}_{di} = [c/(eBn)](dp_i/dx)$, $\omega_{*e}^T = -[c/(eBn)](dp_e/dx + \alpha_T \; n \, dT_e/dx)$, and $\alpha_T \simeq 0.7$.

The limit in which Eq. (2) can be easily solved is that in which the relevant mode can be treated as electrostatic, corresponding to $\left| D_m k^2 / (\omega - \omega_{de}) \right| > 1$. Then we find, for $\omega = \omega_R + i\gamma$,

$$\omega_R \left(\frac{\omega_A^2}{\nu_m} + \nu_\mu + 2\gamma \right) \approx \omega_{*e}^T \left[\frac{\omega_A^2}{\nu_m} - \left| \frac{\omega_{di}}{\omega_{*e}^T} \right| (\nu_\mu + \gamma) \right], \tag{3}$$

$$\gamma \approx \frac{\gamma_I^2 + \omega_R (\omega_R - \omega_{di})}{\dfrac{\omega_A^2}{\nu_m} + \nu_\mu + \gamma}. \tag{4}$$

These equations show that the growth rate has two components, one due to $\gamma_I^2 \propto |dp/dr| / R_c$ and the other related to the finiteness of $\omega_R > 0$ that is the excitation factor of the collisional drift modes. As shown by Eq. (2), ω_R / ω_{de} changes sign when

$$\nu_m (\nu_\mu + \gamma) = \omega_A^2 \left| \omega_{*e}^T / \omega_{di} \right|. \tag{5}$$

In the case of cylindrical plasma columns [4] like those produced by Q-machines, $\gamma_I = 0$, and γ vanishes when ω_R vanishes. In fact this feature was found [4] in order to experimentally identify drift modes for the first time. We observe that if $D_\mu^{\perp\perp}$ is given by the transverse classical viscosity, as in the case of the Q-machine experiments, then the parameter ω_A^2 / ν_μ is proportional to B^4. Therefore $D_\mu^{\perp\perp} k^2 / \omega_A^2$ at the edge of a well confined toroidal plasma is not significant. Therefore the contribution of ion-neutral collisions becomes significant. We consider the fact that viscosity for poloidal flows is larger than that for longitudinal flows and likely to be anomalous.

On the basis of the magnitude of the growth rate of the ballooning mode, whose phase velocity is in the direction of the electron diamagnetic velocity in regimes where the edge of the plasma column is relatively weakly collisional, we find that the confinement should be improved relative to the opposite case. At the same time as confinement is improved, the thermal energy content of the plasma column increases and with this the rate of inward transport of angular momentum.

We envision that, in the full nonlinear evolution of the ballooning modes, blobs are produced and ejected toward the wall, carrying angular momentum in the same direction as that of the modes [3]. From this point of view the process of angular momentum ejection may not differ in the H- and the L-regimes. As pointed out earlier, the core of the plasma column rotates in the direction of the ion diamagnetic velocity in the H-regime and in the opposite direction in the L-regime. As further evidence that spontaneous angular momentum and transport are connected we note that the power threshold needed to access the H-regime from the L-regime has been observed experimentally to be related to the variation of rotation velocity [5].

The simplest representation of an important class of toroidal ballooning modes can be given by adopting the "disconnected" mode approximation [6] for an equilibrium with circular magnetic surfaces, as

$$\hat{n} = \tilde{n}(r,\theta)\exp\left\{-i\ \omega\, t + i\ n^0\left[\varphi - q(r)\theta\right]\right\} \tag{6}$$

where θ and φ indicate the poloidal and toroidal angles, $-\pi+\varepsilon<\theta<\pi-\varepsilon$, $\varepsilon<1$, n^0 is an integer, $q(r)\approx r\ B_\varphi/(R_0\ B_\theta)$ is the unwinding parameter, and $\tilde{n}(r,\theta)$ is a localized function of θ around $\theta\approx 0$. The toroidal phase velocity is $\omega_R\ n^0/R_0$, and the mode angular momentum can be evaluated by extending the well known derivation given for instance in Ref. [7] for the limit where $\gamma \ll |\omega_R|$. The maximum of $\tilde{n}(r,\theta)$ is at $\theta = 0$, and the value of $k_\parallel$ considered for the mode represented by Eq. (1) is an indication of the "peakedness" of $\tilde{n}(r,\theta)$ in an appropriate range of θ (e.g., $-\pi/2 \le \theta \le \pi/2$) around $\theta = 0$.

We note, in this context, that radially convective modes can carry angular momentum toward the wall.

III. Velocity-Gradient-Driven Resistive Mode

In order to assess the transport of angular momentum we must also consider the role of collisional traveling modes which can coexist with the ballooning modes that have been discussed earlier. In particular, finite electrical resistivity is the relevant factor, and, when the local value of $\eta_i = (d\ln T_i/dx)/(d\ln n/dx)$ is well above unity, the mode phase velocity can be found both in the electron diamagnetic velocity direction (vanishing collisional frequency) and in the opposite direction.

The mode is represented by

$$\hat{\Phi} = \tilde{\Phi}(x)\ \exp(-i\ \omega\, t + i\ k_\perp\ y + i\ k_z\ z) \tag{7}$$

In the limit where $k_\parallel^2 \lambda_e^2 \ll 1$, λ_e being the mean free path, the perturbed electron density is given by

$$\frac{\hat{n}_e}{n} \approx \frac{e\hat{\Phi}}{T_e}\left[\frac{\omega_*}{\omega} + i\,\frac{k_\parallel^2 T_e}{m_e\, \nu_{ei}^\parallel\, \omega^2}(\omega - \omega_{de})\right]. \tag{8}$$

For $\omega^2 > k_\parallel^2\ \mathrm{v}_{th,i}^2$, the perturbed ion density is

$$\frac{\hat{n}_i}{n} \approx \frac{e\hat{\Phi}}{T_e}\left[\frac{\omega_*}{\omega} - \frac{k_\parallel\, k_y}{\omega^2} D_B\left(\frac{dV_\parallel}{dx} + \frac{k_\parallel}{m_i\ \omega}\frac{dT_i}{dx}\right)\right] \tag{9}$$

where $V_\parallel(x)$ is the inhomogeneous flow velocity that is present in the equilibrium state. Then the dispersion relation is

$$\omega \approx i\frac{k_y}{k_\parallel}\frac{\nu_{ei}^\parallel}{\Omega_{ce}}\left(\frac{dV_\parallel}{dx} + \frac{k_\parallel}{m_i\ \omega}\frac{dT_i}{dx}\right) + \omega_{*e}^T. \tag{10}$$

This shows that the velocity gradient can drive the mode growth if $k_y\ k_\parallel(dV_\parallel / dx) > 0$ and, as expected, that the ion temperature gradient contributes to the mode growth if $\mathrm{sgn}(\omega_R / k_y) = \mathrm{sgn}(dT_i/dx)$. Given the small value of $\nu_{ei}^\parallel / \Omega_{ce}$ we may argue that this mode can play a role in heavily collisional regimes and for relatively short transverse wavelengths.

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