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A new theory of spontaneous toroidal rotation and some thoughts page numberon neoclassical poloidal rotation in tokamaks

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For an axisymmetric, magnetically confined toroidal plasma, the rate of change of angular momentum density is given by

$$\frac{\partial}{\partial t}\left(\rho R v_\phi + R\varepsilon_0\left(\underline{E}\times\underline{B}\right)_\phi\right) + \frac{1}{R}\frac{\partial}{\partial R}\left(\rho R^2 v_\phi v_R + R^2 P_{\phi R} - R^2 T_{\phi R}\right)$$

$$+\frac{\partial}{\partial Z}\left(\rho R v_\phi v_Z + R P_{\phi Z} - R T_{\phi Z}\right) = 0 \quad (1)$$

Here $P_{\alpha\beta}$ and $T_{\alpha\beta}$ are the plasma and Maxwell stresses, and $\varepsilon_0\left(\underline{E}\,\beta\,\underline{B}\right)$ is the electromagnetic momentum density. This equation clearly shows that toroidal angular momentum in an isolated, axisymmetric tokamak plasma is conserved.

A new theory of spontaneous toroidal rotation has been developed. A two fluid MHD model is required, in particular including the Hall term and the $\nabla p_e / n_e e$ term in Ohm's law. Then magnetic fields tend to be frozen to the electron fluid. Thus, magnetic field perturbations, ∇B, associated with both waves and instabilities will tend on average to move in the $\alpha\beta$ direction, i.e. the direction of electron current flow. A wall of finite resistivity η_w is required for this mechanism. Here the travelling magnetic perturbation will diffuse in, leading to a drag force on the wall and an equal and opposite force on the plasma. The force on the resistive wall can be calculated as follows. We consider a stationary perturbing magnetic field $\underline{B} = \left(B_x(x,z,t), 0, B_z(x,z,t)\right)$ and a planar conductor at $x \geq 0$ moving with a velocity v_z. Employing Maxwell's equations and Ohm's law gives

$$\left(\frac{\partial}{\partial t} + v_z\frac{\partial}{\partial z}\right)\left(B_x, B_z\right) = \frac{\eta_w}{\mu_0}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\left(B_x, B_z\right) \qquad (2)$$

Then, assuming $\partial/\partial z = -ik, \partial/\partial t = +i\omega$ and an x dependence of the form exp(-αx) where $\eta = \eta_R + i\eta_I$ is complex we obtain the average force per unit area of the conductor surface in the z-direction to be, in the limits of the magnetic Reynolds' number $R_m = \mu_o v_z / \nabla_w k$ being >>1 and $\nabla << k v_z$,

$$\overline{F_A} = -(\delta B_{x0})^2\left(\frac{v_z}{8\mu_0\eta_w k}\right)^{1/2} \quad (3)$$

where ηB_{x0} is the amplitude of the perturbing x component of magnetic field. We note the $v_z^{1/2}$ dependence of this force, because the penetration depth of the perturbation varies as $v_z^{-1/2}$. In the laboratory frame, the equal and opposite force on the plasma leads to toroidal acceleration. If ∇B_{xo} is constant in time the toroidal velocity is determined from

$$\pi a^2 \rho\, dv_\phi \big/ dt = 2\pi a \overline{F_A} \quad (4)$$

where now $v_z = v_{d\perp\phi} - v_\phi$ and $v_{d\alpha\beta}$ is the component of the perpendicular relative drift velocity that is in the ∇ (or z) direction. The reason for this is that in a two fluid description of MHD modes it is only the component of electron drift velocity that is perpendicular to the magnetic field that enters the electron momentum equation or Ohm's law. In the laboratory frame the magnetic perturbations in absence of resistivity in the plasma will have a ∇ component of velocity (actually negative for dp/dr <0) equal

$$v_{d\perp\phi} = \frac{B_\nabla}{neB^2}\frac{dp}{dr} \quad (5)$$

This velocity is typically ~0.1 times the total drift velocity of electrons in the - η direction, the latter having a typical magnitude for JET or ALCATOR of ~ 2 x 10^5 m/s (i.e. J_∇ / ne). Eq.(4) thus becomes, noting that usually $v_{d\perp\phi} < 0$,

$$\frac{dv_\phi}{dt} = \frac{2(\delta B_{x0})^2}{\rho a}\frac{(-v_{d\perp\phi} - v_\phi)^{1/2}}{(8\mu_0\eta_w k)^{1/2}} \quad (6)$$

the solution of which is

$$(\phi\, v_{d\delta\rho}\ \phi\ v_\rho)^{1/2} = (\phi\, v_{d\delta\rho})^{1/2}\ \phi\ \frac{(-B_{x0})^2 t}{2\perp a(2\mu_0\eta_w k)^{1/2}} \quad (7)$$

valid for $0\ \alpha\, t\ \alpha \beta_a$, where

$$\partial_a = \frac{2\rho a(2\mu_0\phi_w k|v_{d\varepsilon\times}|)^{1/2}}{(-B_{x0})^2} \quad (8)$$

∇_a is the characteristic acceleration time of the toroidal velocity leading to a final velocity v_ϕ at $t = \nabla_a$ of $|v_{d\alpha\beta}|$. From ∇.B=0 the relationship between ∇B_{xo} and ∇B_{zo} can be found to be

$$\delta B_{zo} = \pm \delta B_{x0}\sqrt{\alpha_R^2 + \alpha_I^2}\,/\,k \Rightarrow \delta B_{x0} R_m^{1/2} \qquad (9)$$

where now R_m is $\mu_0 v_{d\perp\phi}/(\eta_w k)$. As an example; for $v_{d\perp\phi} = -2\times10^4\ \mathrm{ms}^{-1}$, $\eta_w^{-1} = 5.8\times10^7\ \Omega^{-1}\mathrm{m}^{-1}$, $k = 2\pi\ \mathrm{m}^{-1}$ we find $R_m = 2.3\times10^5 >> 1$. The simplified solution of the theoretical model assumes that v_ϕ is slowly varying in time compared to the diffusion of the magnetic field into the conducting wall. This implies

$$\tau_a \geq (k v_{d\perp\phi})^{-1} \qquad (10)$$

while for this example $(k v_{d\perp\phi})^{-1} = 8\ \mu$sec. For $n_i = 10^{20} m^{\nabla 3}, m_i = 2m_p, a = 1m$, this gives a maximum perturbing magnetic field ∇B_{x0} of 2.5 x 10^{-3} tesla or ∇B_{zo} of 1.2 tesla. Weaker perturbing fields will lead to slower toroidal spin up. The torque on the plasma will be located at the relevant singular surface for each MHD mode being excited (Taylor, 2003). The equal and opposite torque on the conducting wall can be found through measuring the components of the magnetic stress tensor at the wall (Etherington and Haines, 1965; Hutchinson, 2001). The same mechanism will lead to a poloidal torque, and, indeed, because the component of the electron drift velocity in the poloidal or ∇ direction $v_{d\alpha\beta}$ is much larger and given by

$$v_{d\perp\theta} = -\frac{B_\phi}{neB^2}\frac{dp}{dr} \qquad (11)$$

But because of trapped particles there will be neoclassical poloidal rotation damping as a result of Fokker-Planck collisions between particles which are just trapped with those that are just passing. Five different exponential damping rates have been found (Stix 1972, Hirshman 1978, Shaing and Hirshman 1989, Taguchi, 1991 and Hsu et al 1994). Morris et al, 1996, employing an initial value approach obtained an algebraic decay, dependent on the initial distribution, and consistent with the up-down asymmetry which is necessary to obtain a change of poloidal angular momentum in an isolated toroidal plasma. The characteristic time of damping of poloidal angular momentum is essentially the ion-ion collision time.

Poloidal angular momentum is measured about the curved magnetic axis. Nevertheless for an isolated plasma torus it is conserved unless there is an up-down asymmetry in the pressure or stress tensor (Haines 1970). In an inverse aspect ratio expansion, giving

$$\frac{\partial}{\partial t}\left[\rho r u_\theta + \varepsilon_0 r\left(\underline{E}\times\underline{B}\right)_\theta\right] + \sum_\gamma \nabla_\gamma\left[r\rho u_\theta u_\gamma + rP_{\theta\gamma} - rT_{\theta\gamma}\right]$$
$$= -\frac{r\sin\theta}{R_0 + r.\cos\theta}\left[\rho u_\phi u_\phi + P_{\phi\phi} - T_{\phi\phi}\right] \qquad (12)$$

a source term in sin η appears. Thus if the stress tensor or pressure has a component that varies also as sin ∇, implying up-down asymmetry, there will be a net source of poloidal angular momentum when averaged over a magnetic surface. The spin-up of poloidal rotation was first proposed by Stringer, 1969 as an instability requiring an initial kick, to which Rosenbluth and Taylor, 1969 added parallel viscosity. In Haines, 1970 the spin occurs naturally evolving from rest in a 2-fluid theory with Pfirsch-Schlüter diffusion and flows; The outward flow associated with P-S diffusion leads to a higher density in the outboard mid-plane; guiding-centre motion advects this in the $\alpha\beta$ *direction* to give a pressure component which varies as sin ∇. Anomalous transport should enhance this effect. This represents a second source of poloidal rotation. A third source of poloidal rotation occurs in the presence of a toroidal electric field and leads to the Ware (1970) pinch effect of trapped particles. Haines and Martin (1996) have shown that this is associated with up-down asymmetry.

References

Etherington, R.J. and Haines, M.G., 1965 Phys.Rev.Lett. **14,** 1019

Haines, M.G. 1970 Phys.Rev.Lett. **25**, 1480

Haines, M.G. & Martin, P. 1996 Phys.Plasmas **3**, 4536

Hirshman, S.P. 1978 Nucl.Fusion **18**, 917

Hsu, C.T., Shaing, K.C. and Gormley 1994 Plasma Phys. **1**, 132

Hutchinson, I.H. 2001 Plasma Phys. Controlled Fusion **43**, 145

Morris, R.C., Haines, M.G. and Hastie, R.J. 1996 Phys. Plasmas **3**, 4513

Shaing, K.C. and Hirshman, S.P. 1989 Phys.Fluids B1, 705

Stix, T.H. 1972 Phys.Fluids **16**, 1260

Taguchi, M. 1991 Plasma Phys. Controlled Fusion **33**, 859

Taylor, J.B. 2003 Phys.Rev.Lett. **91**, 115002

Ware, A.A. 1970 Phys.Rev.Lett. **25**, 15