Finite beta effects on an ITG turbulence-zonal mode system in tokamak plasmas

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Introduction

Finite beta effects on a coupled system of ion temperature gradient (ITG) driven turbulence and zonal modes are investigated by global fluid simulations of electromagnetic ITG turbulence. Turbulent ion heat transport and zonal flows decrease with increasing beta. Although the Reynolds stress drives the zonal flows and the geodesic transfer effect acts as a sink in an almost whole plasma, it is found that these contributions change sign at low order rational surfaces. The Maxwell stress contribution to zonal flow generation is not dominant even at the beta at which the most linearly unstable mode is changing from the ITG mode to kinetic ballooning mode (KBM). Zonal magnetic fields generated from the turbulence are strong at the low order rational surfaces. A coarse choice of toroidal modes fails to capture nonlinear interactions which affect generation of the zonal magnetic fields.

Numerical model

To investigate finite beta effects on the interaction between the ITG turbulence and the zonal modes, we use a five-field electromagnetic Landau-fluid model\cite{1}. Five fields are density fluctuation $\tilde{n}$, electrostatic potential $\tilde{\phi}$, parallel ion velocity $\tilde{v}$, parallel magnetic potential $\tilde{A}$ and ion temperature fluctuation $\tilde{T}$. Parameters used in the calculations are $R/a=4$, $\rho_i/a=0.01$, $T_e=T_i$, $n_{eq}=0.8 + 0.2 e^{-2(r/a)^2}$, $T_{eq}=0.35 + 0.65(1-(r/a)^2)^2$, $q=1.05 + 2(r/a)^2$. Numerical calculations are done by Fourier mode expansion in the poloidal and toroidal directions and finite difference in the radial direction. All toroidal modes up to $n=50$ are included in the calculations, where $n$ is toroidal mode number. The number of Fourier modes taken is 1525 and the radial grid number is 512. Finite beta has stabilizing effect on the linear ITG mode, while the KBM is destabilized by beta and becomes dominant at high beta\cite{2}. In the parameters used, the most unstable linear mode is the ITG mode for $\beta < 0.5\%$ and the KBM for $\beta > 0.6\%$, where $\beta = (n_c T_c)/(B_0^2/\mu_0)$ is a half of the beta value evaluated on the plasma center\cite{1}. Nonlinear simulation is done for $\beta =0.1\%$, $0.3\%$, $0.5\%$ and $0.6\%$, and besides, the calculation in the electrostatic ($\beta=0$) limit with adiabatic electrons is performed for comparison. Pressure profile evolution is allowed in the nonlinear simulation.
Finite beta effect on turbulent transport and zonal mode energy

Figure 1 shows temporal evolutions of an electrostatic component of turbulent ion heat flux. An electromagnetic component is negligibly small compared to the electrostatic one. The heat flux decreases with increasing beta, although the heat flux in the electrostatic case is almost the same level as the one for $\beta = 0.1\%$ for $t > 300$. Temporal evolutions of zonal flow energy and zonal magnetic field energy are shown in Fig. 2. The zonal flow energy in the electrostatic case is about twice larger than that in the $\beta = 0.1\%$ case. Therefore the turbulent transport in the electrostatic case seems to be suppressed by the zonal flows well. The zonal flow energy decreases with increasing beta up to $\beta = 0.5\%$. The zonal flow energy for $\beta = 0.6\%$ is almost the same as the one for $\beta = 0.5\%$. On the other hand, the zonal magnetic field energy increases with beta.

Figure 1: Temporal evolution of turbulent heat flux.

Figure 2: Temporal evolutions of zonal flow energy (left) and zonal magnetic field energy (right).

Zonal flow generation

The time evolution equation for the zonal flow energy is

$$
\frac{\partial}{\partial t} \frac{1}{2} \langle v_E \rangle^2 = -\langle v_E \rangle \langle v_E \hat{\Omega} \rangle + \frac{\beta}{n_{eq}} \langle v_E \rangle \langle \hat{B}_r j \rangle - 2 \frac{a}{n_{eq} R} \langle v_E \rangle \langle p \sin \theta \rangle,
$$

(1)
Figure 3: Time averaged zonal flow energy drives as a function of radius in the electrostatic (left) and the $\beta = 0.1\%$ (right) cases.

where $\langle \cdot \rangle$ denotes the flux surface average, $\langle v_E \rangle = \partial \phi_0 / \partial r$ is the zonal flow, $v_{Er} = -1/r (\partial \tilde{\phi} / \partial \theta)$ is the radial E×B drift velocity, $\tilde{\Omega} = \nabla^2_\perp \tilde{\phi}$ is the vorticity, $\langle p \sin \theta \rangle$ is the pressure sideband.

As shown in the above equation, the zonal flow energy in toroidal plasmas is dominated by Reynolds stress, Maxwell stress and geodesic transfer. Figure 3 shows time averaged zonal flow drives as a function of radius in the electrostatic and the $\beta = 0.1\%$ cases. In both the cases, the Reynolds stress drives the zonal flows and the geodesic transfer acts as a sink in an almost whole plasma. It is seen that the Reynolds stress contribution is negative and the geodesic transfer contribution is positive locally at the $q = 2$ surface. The reversal of the Reynolds stress and the geodesic transfer contributions at the $q = 2$ surface is observed in the other $\beta$ cases.

**Zonal magnetic field generation**

In the finite beta case, the turbulence can generate nonlinearly zonal magnetic fields as well as the zonal flows. The zonal magnetic fields are generated at low order rational surfaces[3]. Figure 4 shows temporal evolution of zonal magnetic field for $\beta = 0.3\%$. It is seen that the zonal magnetic field is the most strongly excited at the $q = 2$ surface. The zonal magnetic field tends to flatten the $q$ profile around $q = 2$ locally. The flattening is stronger for higher beta due to larger zonal magnetic field at the $q = 2$ surface. In electrostatic tur-
Figure 5: Temporal evolutions of zonal magnetic field energy for $\Delta n = 1$ (solid) and $\Delta n = 2$ (dash).

In turbulence simulations, a coarse choice of toroidal modes like $\Delta n = 2, 4, \cdots$ is often used to reduce computational time, where $\Delta n$ is the interval of the toroidal mode number ($n$) taken in numerical simulations. We have compared the $\Delta n = 2$ case with $\Delta n = 1$ for $\beta = 0.3\%$. The zonal magnetic field in the $\Delta n = 2$ case is larger than that in the $\Delta n = 1$ case as shown in Fig. 5. In the $\Delta n = 2$ case, the zonal magnetic field is the largest at the $q = 1.5$ surface instead of the $q = 2$ surface. We have investigated nonlinear zonal magnetic field drive at the $q = 1.5$ surface for each $n$. In the $\Delta n = 2$ case, the zonal magnetic field at the $q = 1.5$ surface is mainly driven by the $n=12, 14, 16, 18$ modes. On the other hand, the $n=13, 15, 17$ modes which are not included in the $\Delta n = 2$ case act as a sink for the zonal magnetic field at the $q = 1.5$ surface in the $\Delta n = 1$ case, although the $n=14, 16, 18$ modes drive the zonal magnetic field. Thus the coarse choice of toroidal modes fails to capture some of nonlinear interactions which affect the zonal magnetic field generation. Therefore all toroidal modes should be included in numerical turbulence simulations in the regime where the zonal magnetic fields play an important role.

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References

