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Generation of Relativistic Even-Harmonics in Underdense Plasmas

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Abstract. The generation of phase-matched even-harmonics in underdense plasmas is analyzed directly from one-dimensional (1D) and two-dimensional (2D) particle-in-cell (PIC) simulations. The phase-matching conditions are derived and the previously predicted growth rates for the second-harmonic are observed to be in good agreement with the simulation results.

1. INTRODUCTION

Harmonic generation is an active field of research as it is a promising approach for generating extremely bright coherent X-ray pulses in the femtosecond and attosecond regime, thus opening the way for measurements with unprecedented temporal resolution [1].

At high intensities, laser-plasma interactions can lead to relativistic harmonic generation, due to the nonlinear orbit of the plasma free electrons in the presence of the intense laser field, leading to coherent harmonic generation in the forward direction (propagation direction of the laser pulse). Although this process has been studied before, generating harmonics efficiently is still a challenge for a wide range of applications, mainly due to the difficulties in guaranteeing phase-matching during the conversion process over long interaction lengths. A few phase-matching schemes were proposed for relativistic third-harmonic generation [2], where the transversal currents driven by the propagation of an intense laser pulse (driver) in an underdense unmagnetized plasma have an odd-harmonic content, thus acting as a source of odd-harmonics. In the presence of a modulated transverse magnetic field, phase-matched second-harmonic generation was predicted by Rax *et al*. [3].

In this paper, we derive the phase-matching conditions for the generation of even-harmonics of arbitrary order in the presence of a transverse modulated magnetic field, and we analyze the evolution of the generated harmonics directly from PIC simulations, for short pulse regimes, where ultra-high intensities are already available in the near-infrared region.

2. PRINCIPLES OF EVEN-HARMONIC GENERATION IN MAGNETIZED PLASMAS

In the presence of a linearly polarized high intensity laser pulse, the relativistic oscillatory motion of the plasma free electrons resembles an 8 (figure-eight motion) and consists of a transverse oscillation at odd-harmonics of the laser frequency and a longitudinal oscillation at even-harmonics of the laser frequency. When a transverse modulated magnetic field is considered in the laser-plasma interaction region, phase-matched even-harmonics can be generated. On one hand, the role of the magnetic field is to convert the longitudinal oscillations at even-harmonics of the laser frequency into transverse oscillations, due to the $\vec{v} \times \vec{B}$ term in the Lorentz force. On the other hand, the modulation of the magnetic field allows for a control of the phase-velocity of the driven currents, making it possible to have regimes with phase-matching between the driven currents and the generated harmonics.

We consider a linearly polarized driver pulse with vector potential $\vec{a}_0 = a_0 \cos[\omega_0 t - k_0 z + \phi(z)]\, \vec{e}_1$, where ω_0 is the initial frequency of the driver pulse, k_0 its initial wavenumber, $\omega(z)$ its phase, z its propagation direction and $\vec{e}_1$ the unit vector in its polarization direction, and a transverse magnetic field of the form $\vec{b} = b\cos[\times_n z]\vec{e}_b$, where ω_n is the wavenumber of the magnetic field and $\vec{e}_b$ is the unit vector in its polarization direction. In this configuration, the driven currents are given by $\vec{j}_n = j_n \cos[n\omega_0 t - nk_0 z + n\phi(z)]\cos[\kappa_n z]\, \vec{e}_n$, where n is an even integer indexing the order of the harmonic currents and $\vec{e}_n = \vec{e}_z\, \omega \vec{e}_b$ is the unit vector in the currents direction. The generated even-harmonics are of the form $\vec{a}_n = a_n \cos[n\omega_0 t - nk_0 z + \phi_n(z)]\, \vec{e}_n$, with $\times_n$ being its phase. The phase-matching conditions are obtained by requiring the phase velocity of the driven currents to be the same as the phase velocity of the generated harmonics, yielding $\kappa_n = n\, \partial\phi / \partial z - \partial\varphi_n / \partial z$, with $\partial\phi / \partial z = \omega_p{}^2 / (2\omega_0 c)$ and $\partial\varphi_n / \partial z = \omega_p{}^2 / (2n\omega_0 c)$, where $\omega_p = (4\pi\, n_e / m_e)^{1/2}$ is the electron plasma frequency and c is the velocity of light. Therefore, the magnetic-field wavenumber required to phase-match the n^{th} order even-harmonic is

$$\kappa_n = \frac{\omega_p{}^2}{2\omega_0 c} \frac{n^2 - 1}{n}, \qquad (1)$$

which for $n = 2$ leads to the result of Ref. [3].

Rax *et al.* [3] studied the second-harmonic generation in the ordinary mode (B-field in the laser polarization direction) and in the extraordinary mode (B-field in the other transverse direction), assuming long laser pulses, such that no wakefield is generated. In the next section, we resort to 1D and 2D PIC simulations, performed with the OSIRIS 2.0 framework [4], to check the validity of the derived phase-matching conditions and to study the second-harmonic growth in short-pulse regimes. Details of the simulation parameters will be presented elsewhere.

3. SIMULATION RESULTS

In Fig. 1, results of the second-harmonic generation are presented for the ordinary mode, since in the extraordinary mode the driver pulse and the generated harmonics have the same polarization, thus making it harder to analyze the evolution of the generated harmonics. The slow modulations observed are due to the wakefield generated by the driver pulse. In Fig. 2, we plot the comparison between the simulation results and the growth of the second-harmonic predicted in Ref. [3] for both phase-matched (PM) and non-phase-matched (n-PM)

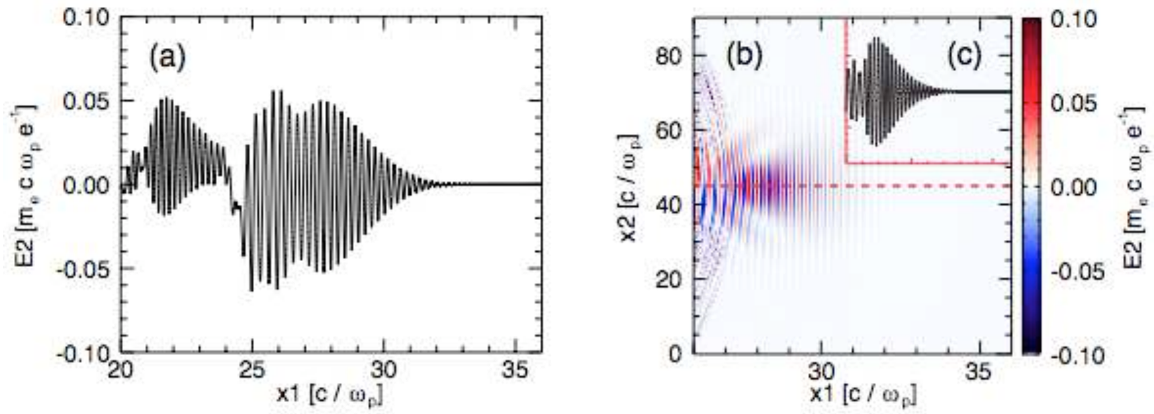

FIG. 1: Electric field of the generated second-harmonic, in the ordinary mode, for a B-field modulation matched to second-harmonic growth: (a) 1D simulation, (b) 2D simulation, and (c) lineout on axis of the 2D simulation. The amplitude of the B-field is $b \sim 58$ T . The plots correspond to a propagation distance of 2 mm.

regimes. The 1D results are in good agreement with the 1D theory. The differences observed in the 2D simulation are due to the enhancement of the driver vector potential as self-focusing occurs ($P/P_{cr} >> 1$).

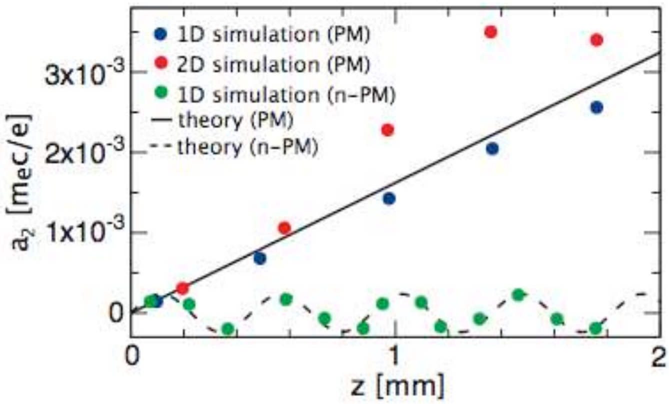

FIG. 2: Comparison between the second-harmonic growth from PIC simulations and from the 1D theory [3], for both phase-matched (PM) and non-phase-matched (n-PM) cases.

The spectrum of the generated second-harmonic in both ordinary and extraordinary modes, for 1D simulations, can be observed in Fig. 3. The second-harmonic can be clearly identified and its amplitude in the extraordinary mode is about three times greater than in the ordinary mode, as predicted by the theory [3].

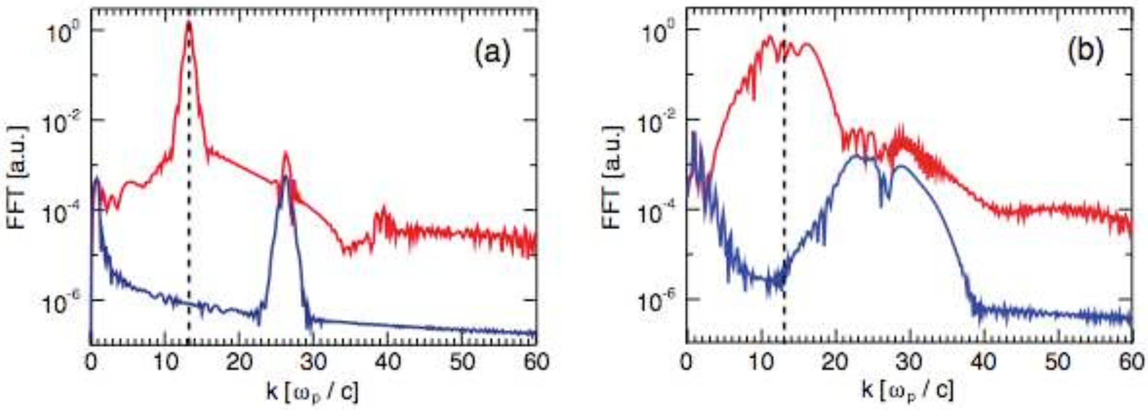

FIG. 3: Spectrum of the electric field along the polarization direction of the driver pulse in the extraordinary mode (red) and along the other transverse direction in the ordinary mode (blue). The B-field modulation is matched to second-harmonic growth and its amplitude is $b \sim 58$ T. The plots correspond to propagation distances of (a) 0.33 mm and (b) 0.66 mm. The dashed line corresponds to the initial frequency of the driver pulse.

Finally, in order to check the phase-matching condition for the fourth-harmonic, illustrative 1D simulations were performed for a modulated magnetic field, matched to fourth-harmonic growth, and for a constant magnetic field. From Fig. 4, we can observe that the matched modulation enhances the fourth-harmonic growth, diminishing the second-harmonic growth.

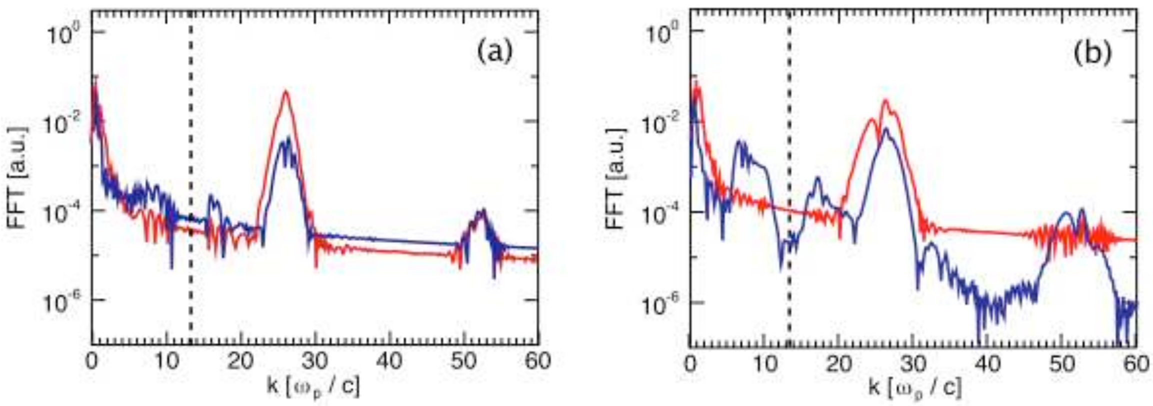

FIG. 4: Spectrum of the electric field of the generated harmonics, in the ordinary mode, for a B-field modulation matched to fourth-harmonic growth (blue) and for a constant B-field (red). The amplitude of the B-field is $b \sim 5800$ T in both cases. The plots correspond to propagation distances of (a) 0.14 mm and (b) 1.4 mm. The dashed line corresponds to the initial frequency of the driver pulse.

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