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Anomalous dispersion observed in numerical simulation of electron gas dynamics in the external corona of laser generated plasma

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Introduction

The study of the phase space evolution of electron gas in the presence of stimulated Raman scattering in a non-relativistic and homogeneous laser plasma was undertaken by a solution of the Vlasov equation (including a weak collision term) simultaneously with the Maxwell equations in a 1D periodic slab model by a transform method. The parameters of the computation were chosen to be compatible with the plasma in the laser corona typically generated by the nanosecond PALS system at the high flux end attainable by a tight focusing $\sim 10^{16}$ W/cm^2. The computation was tuned to a regime with a predominant Raman back-scatter of the heating laser wave and a strong tendency for particle trapping in the daughter electrostatic plasma wave generated in the scattering process. The outcome of the numerical simulation strongly depends on the density of permissible discrete Fourier spectrum of the participating electrostatic waves. With a sparse spectrum just a secondary scattering process of once back-scattered electromagnetic wave is observed causing an electron acceleration in the reversed direction. However, with a higher density of the Fourier modes, which allows for a growth of sidebands of the electrostatic plasma wave, a development of trapped particle instability and the of energy flow from the principle mode to the lower sideband is observed, leading to an intermittency as first described in [1]. The group velocity of wave packets generated by this process has for our particular simulation parameters a tendency to be in the opposite direction with respect to the direction of the phase velocity. We interpret this phenomenon of anomalous dispersion as a regime predicted in [2], where a separate analytic treatment of the trapped particle dynamics predicted a separate non-linear dispersion branch tied to the former linear electrostatic wave dispersion curve by a transitional part with the anomalous dispersion.

Trapped particles

The electron trapping is the forward propagating electrostatic plasma wave, which is generated as the daughter wave in the Raman back-scattering process is the most characteristic feature of the particle-wave interaction. In the advanced stage of trapping it turns out that the trapped particles with the dynamics of their own behave as a “new" particle species giving rise to a new unstable mode leading to a growth of the sidebands. Here we shall, however, be concerned with the influence of the trapped population on the form of the plasma wave dispersion relation. In the following we shall follow the line of reasoning as presented in [3]. In a strong monochromatic electrostatic wave characterized by the longitudinal electrostatic field $E_x = E_0 \cos(kx - \omega t)$ the trapped electron wobbling frequency near the bottom of potential troughs is given as $\omega_B = \sqrt{ekE_0/m}$ (e is the elementary charge and m the electron mass, E_0 is the wave amplitude, ω its angular frequency, with $\vec{k} = (k,0,0)$ as the wave vector) the electron distribution function splits in the phase space in two parts distinguished by the parameter κ meaning the ratio of the depth of wave potential trough to the overall energy of the wobbling electron in the wave frame, which discriminates between the passing (κ<1) and trapped (κ>1) electrons

$$\kappa = \frac{2eE_0/k}{\mathscr{E} + eE_0/k} \tag{1}$$

and E is the wave frame energy of electron in the electrostatic wave

$$\mathscr{E} = m(v_x - \omega/k)^2/2 - (eE_0/k)\sin(kx - \omega t) \tag{2}$$

through which the electron distribution function composed of the passing electron distribution $f_{pass}(x, v_x, t)$ and the trapped electron distribution $f_{tr}(x, v_x, t)$ depends on v_x and x

$$f_{pass} = \frac{n_0}{\sqrt{\pi} v_{T_e}} \exp\big[-\big(\frac{\omega}{k v_{T_e}} \mp \frac{2\sqrt{2}}{\pi v_{T_e}} \sqrt{(\mathscr{E} + eE_0/k)/m}\, E(\kappa)\big)^2\big], \;\kappa > 1 \tag{3}$$

$$f_{tr} = \frac{n_0}{\sqrt{\pi} v_{T_e}} \exp\big\{-\big[\big(\frac{\omega}{k v_{T_e}}\big)^2 + \frac{1}{m}\big(\frac{2\sqrt{2}}{\pi v_{T_e}}\big)^2 (\mathscr{E} + eE_0/k)\big(\sqrt{\kappa} E(1/\kappa) - \frac{\kappa - 1}{\sqrt{\kappa}} K(1/\kappa)\big)^2\big]\big\}$$
$$\times \cosh\big[\frac{4\sqrt{2}\omega}{\pi v_{T_e}^2 k} \sqrt{(\mathscr{E} + eE_0/k)/m}\, \big(\sqrt{\kappa} E(1/\kappa) - \frac{\kappa - 1}{\sqrt{\kappa}} K(1/\kappa)\big)\big], \;\kappa < 1 \tag{4}$$

n_0 is the mean electron number density, $v_{T_e} = \sqrt{k_B T_e/m}$ is the electron thermal speed (T_e is the electron temperature, k_B Boltzmann constant), K and E are the complete elliptic integrals of the first and second kind. Notice that (3) reduces to a Maxwell distribution for $E_0 \to 0$, whereas (4) goes to zero. Substituting (3, 4) directly in the Poisson equation and taking just the first Fourier component a non-linear dispersion relation is found ($\varphi = kx - \omega t$)

$$1 + (4en_0/kE_0)\big[\int_0^{2\pi} d\varphi \sin\varphi \big(\int_{(\kappa<1)} dv_x f_{pass} + \int_{(\kappa>1)} dv_x f_{tr}\big)\big] = 0 \tag{5}$$

Taking just the first Fourier component of the perturbed electron density means that we not only ignored all the higher harmonics, but also excluded all the possibility for a sideband growth, which means that the trapped particle instability in this model is not admitted. If solved for the $\omega = \omega(k)$ dependence (5) has two roots, of which one corresponds to the electron plasma wave, the other, which is smaller exists without a substantial damping just for a final E_0 and represents a low lying non-linear branch of the dispersion with $\omega \sim \omega_B \ll \omega_{pe}$ (electron plasma frequency) for $k \to 0$. These two branches of the dispersion curve are interconnected with a narrow transitional part belonging to the upper plasma wave dispersion with a negative slope - anomalous dispersion as is demonstrated by fig. 1, which was taken over from [3]. In the outer corona of the plasma generated by a nanosecond laser with a typical intensity in the focus $\sim 10^{16}$ W/cm^2 a strong electrostatic wave develops quite naturally as the forward propagating daughter wave of the back-scattering Raman instability. The intensity of such a wave in the developed stage of the instability is very large, $E_0 \sim 10^8$ V/cm and the normalized field $E_N = eE_0/(m\omega_{pe}v_{Te}) \sim 1$ (parameter in fig. 1). In addition, due to the generation of the side bands the wave is no longer monochromatic, which may deplete its potential minima of the trapped electrons. Hence, to get a deeper insight a more complete, self-consistent solution is necessary.

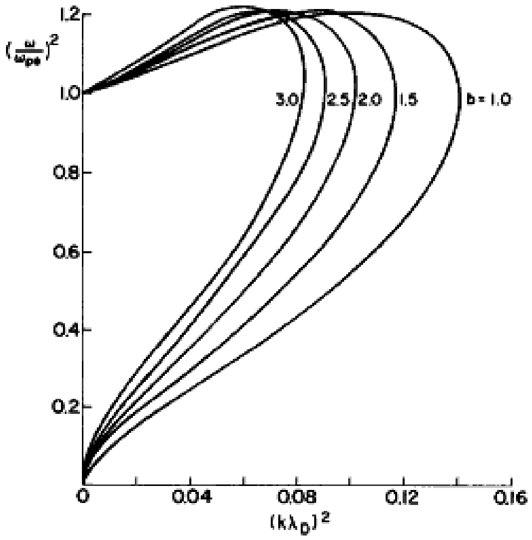

Figure 1: Dispersion curve of the electron plasma wave deformed by the presence of trapped particles. Parameter E_N^2. The anomalous dispersion part joins the upper and lower branch of the dispersion. Taken from [3].

Self-consistent model

A self-consistent model in a simple 1D geometry must account for a propagation and generation of electromagnetic waves to accommodate the Raman back-scatter of the heating laser wave and the generated forward propagating electrostatic plasma wave as well as the development in the phase-space by a Vlasov equation with a weak collision term added

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + \frac{e}{m}\left(\frac{\partial \varphi}{\partial x} - \frac{e}{m} A \frac{\partial A}{\partial x}\right)\frac{\partial f}{\partial v} = \tag{6}$$

$$= v_c \left(\frac{\partial (vf)}{\partial v} + < v^2 > \frac{\partial^2 f}{\partial v^2}\right),$$

$$\left[\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\omega_{pe}^2}{c^2}\frac{n_e}{n_0}\right] A = 0, \tag{7}$$

$$\frac{\partial^2 \varphi}{\partial x^2} = \frac{e}{m}(n_e - n_0), \tag{8}$$

$$\frac{n_e}{n_0} = \int_{-\infty}^{\infty} f\, dv, \tag{9}$$

where A is the only non vanishing transverse component of vector potential $\vec{A} = (0, A, 0)$ under a Coulomb calibration, φ is the electrostatic potential, n_e is the electron number density, c is the speed of light, x the spatial coordinate (propagation direction), t is the time, v_x is the velocity in the parallel direction and n_0 is the initial number density of electrons, $\nu_c \ll \omega_{pe}$ is the electron-ion collision frequency. This is a closed system of equations, which was solved by a transform method [4, 5].

Results and discussion

The solution of (6)-(9) renders the evolution of electron distribution function in the phase space. In the advanced stage a new unstable mode appears leading to a growth of the sidebands. This new mode is a well known trapped particle instability recently discussed in [1]. As the sidebands grow, the trapping electrostatic field becomes more complex, the potential troughs of the trapping wave open and the trapped resonant electrons are freed. This, however, precludes the growth of the trapped particle instability and the sidebands disappear. The primary electrostatic wave starts to trap the electrons anew and the whole process is repeated. This means that in the computation a gros periodicity or an intermittency appears. All these perturbations of the fundamental spatial periodicity of the electron distribution function move in the negative direction, against the laser beam, which we interpret as a negative group velocity in conformity with [3]. Moreover, the energy flow between the main electrostatic mode and the sidebands is asymmetric and invariably favours the lower sideband. We surmise that this should be interpreted as a non-linear frequency lowering in the case of predominating quadratic non-linearity due to the finite amplitude effect of the primary wave, as also noticed in [6]. Support by the grant No. 202/05/2745 of the Grant Agency of the Czech Republic is gratefully acknowledged.

References

[1] S. Brunner, E.J. Valeo, Phys.Rev.Lett. **93** (2004) 145003.

[2] A. Salceda, R.J. Focia, A.K. Ram and A. Bers, Nucl.Fusion **43** (2003) 1759-1770.

[3] V. Krapchev, A.K. Ram, Phys. Rev. A **22** (1980) 1229.

[4] T.P. Armstrong, R.C. Harding, G. Knorr, D. Montgomery, “Methods in Computational Physics" vol. 9 (Academic Press 1970) 29.

[5] M. Mašek, K. Rohlena, Czech. J. Phys. **55** (2005) 973.

[6] H.A. Rose, D.A. Russel, Phys.Plasmas **8** 4784 (2001).