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Anisotropic instability in a laser heated plasma revisited

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In the problem of interaction of the electromagnetic radiation with plasmas, the magnetic fields play an important role by affecting the energy absorption and transport processes. The magnetic fields are self-generated either by the rotational part of the pressure (the crossed gradients of the density and the temperature or the laser intensity) or as an instability due to the anisotropy of the electron distribution function. The former process manifests itself in inhomogeneous plasma and therefore it involves the ion motion in the time scale of hundred of ps. The later mechanism or the Weibel instability is a faster process developing in a homogeneous plasma in the ps time scale. It has been investigated by a number of authors, who have considered the combined effect of the electron collisions and the laser-induced anisotropy on the growth rate of the instability and the domain of unstable modes. The Weibel instability in the context of laser-plasma interaction can be generated either due to the temperature gradient, or by the inverse Bremsstrahlung absorption of the laser light [1, 2]. It is shown in the present paper that the instability growth rate was strongly overestimated in Refs. [1, 2], because the direct coupling between the quasistatic and laser fields has been ignored [3].

The main point of our revised theory is that the effect of laser field does not reduce to the anisotropy of the electron distribution function, but it needs to be accounted for directly in the equation for the perturbations. That is, the calculation of the electron distribution function anisotropy and the stability analysis have to be fulfilled within the same formalism.

Let us consider a homogeneous plasma of a density n_e in linearly polarized high frequency electric and magnetic fields, $\boldsymbol{E}_l(t) = \boldsymbol{E}_0 \cos\omega_0 t, \boldsymbol{B}_l(t) = \boldsymbol{B}_0 \sin\omega_0 t$. The plasma is transparent for the laser field, that is, $\omega_0 > \omega_{pe}$, where ω_{pe} is the electron plasma frequency. We are interested in the stability of a low frequency, $\omega \ll \omega_{pe}$, electromagnetic perturbation of the wave vector $\boldsymbol{k}$ and having the electric and magnetic fields $\boldsymbol{E}_s$ and $\boldsymbol{B}_s$. Under the effects of quasistatic and oscillating fields, $\boldsymbol{E} = \boldsymbol{E}_s + \boldsymbol{E}_l$, $\boldsymbol{B} = \boldsymbol{B}_s + \boldsymbol{B}_l$, and of the electron-electron, and electron-ion collisions, the electron distribution function f_e satisfies the following kinetic equation:

$$\partial_t f_e + \boldsymbol{v}\cdot\boldsymbol{\nabla} f_e - \frac{e}{m_e}(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})\,\partial_{\boldsymbol{v}} f_e = C_{ei}(\boldsymbol{v}, f_e) + C_{ee}(f_e, f_e). \quad (1)$$

Assuming that the electron quiver energy in the laser field, $W_e = e^2E_0^2/2m_e\omega_0^2$, is smaller than the electron temperature: $W_e \ll T_e$, we derived from (1) an equation for the electron distribution function averaged over the laser period:

$$\partial_t \tilde{f}_e + \boldsymbol{v}\cdot\boldsymbol{\nabla}\tilde{f}_e - \frac{e}{m_e}(\boldsymbol{E}_s + \boldsymbol{v}\times\boldsymbol{B}_s)\,\partial_{\boldsymbol{v}}\tilde{f}_e - C_{ei}(\boldsymbol{v};\tilde{f}_e) - C_{ee}(\tilde{f}_e) = I_P + I_{IB} + I_B. \tag{2}$$

The ponderomotive term, I_P, and the term I_{IB}, that accounts for the inverse Bremsstrahlung heating, are well-known [4]. The term I_B was derived recently in [3], it describes a direct coupling between the quasistatic and laser fields. It plays a stabilizing role on the perturbation growth, and it can be written as follows:

$$I_B = -\frac{e}{m_e^2} W_{lm}\partial_{v_m}\partial_{v_i}(\boldsymbol{E}_s + \boldsymbol{v}\times\boldsymbol{B}_s)_i\,\partial_{v_l}\tilde{f}_e, \tag{3}$$

where $W_{lm} = W_e e_l e_m$ is the tensor of the ponderomotive potential and $\boldsymbol{e} = \boldsymbol{E}_0/E_0$ is the unit vector in the direction of the laser field. The electron-ion collision operator is taken in the Lorentz approximation

$$C_{ei}(\boldsymbol{v};f_e) = A\,v^{-3}\partial_{v_i}\left[(v^2\delta_{ij} - v_i v_j)\partial_{v_j} f_e\right] \tag{4}$$

where $A = Zn_e e^4 \ln\Lambda/8\pi\varepsilon_0^2 m_e^2$, Z is the ion charge, and $\ln\Lambda$ is the Coulomb logarithm.

For the stability analysis we consider the electron distribution function, $\tilde{f}_e = f_0 + f_a + \delta f_e$, as a sum of a stationary homogeneous nonperturbed part, $f_0 + f_a$, and a perturbation, $\delta f_e \sim \exp(\gamma t + i\boldsymbol{k}\cdot\boldsymbol{r})$. The stationary distribution function, is split in the the isotropic part, f_0, and the anisotropic part, f_a. The equation for f_0 has been analyzed by Langdon [5]. In the present analysis f_0 is taken to be a Maxwellian function.

The anisotropic part of the stationary electron distribution function, f_a, in the limit of a high ion charge, $Z \gg 1$, can be simply expressed as balance between the inverse Bremsstrahlung heating term and electron-ion collisions:

$$f_a = \frac{W_{ij}}{3m_e} v^2 \left(v_i v_j - \frac{1}{3}\delta_{ij} v^2\right)\frac{\partial}{\partial v}\left(\frac{1}{v^4}\frac{\partial f_0}{\partial v}\right). \tag{5}$$

It is of the order of W_e/T_e compared to f_0 and it corresponds to a higher effective electron temperature in the direction $\boldsymbol{E}_0$. Therefore it should be unstable with respect to magnetic field and current perturbations.

The linearized equation (2) for the perturbation δf_e in the limit $\gamma \ll \nu_{ei}, kv_{Te}$ takes the following form:

$$i\boldsymbol{k}\cdot\boldsymbol{v}\,\delta f_e - C_{ei}(\boldsymbol{v};\delta f_e) = \frac{e}{m_e}\boldsymbol{E}_s\cdot\partial_{\boldsymbol{v}} f_0 + \frac{e}{m_e}\boldsymbol{v}\times\boldsymbol{B}_s\cdot\partial_{\boldsymbol{v}} f_a + I_B(f_0). \tag{6}$$

The last term in this equation was ignored in previous studies [1, 2], although it is of the same order as the second term of the left-hand side of Eq. (6). Moreover, these two terms partially cancel out each other and their combinaison leads to a completely different functional velocity dependence.

To calculate δf_e, we choose the coordinate system with the polar axis z directed along the vector $\boldsymbol{k}$ and the axis y directed along the vector of magnetic field, $\boldsymbol{B}_s$. We assume also that the electric vector of the laser field $\boldsymbol{e}$ lies in the x,z-plane at the angle θ_E with respect to the z-axis. The expansion of δf_e in the spherical functions enables us to carry on the calculations for the arbitrary value of the collisionality parameter, $k\lambda_{ei}$.

The dispersion equation for the instability growth rate follows from the combination of the Faraday and Ampère's laws with the current found from Eq. (6):

$$\gamma = \frac{\nu_{ei}}{H(k)} \left(-\frac{k^2 c^2}{\omega_{pe}^2} - \frac{1}{3}\frac{W_e}{T_e} G(k) \cos 2\theta_E \right), \tag{7}$$

where the wavelength dependence of the growth rate is defined by two dimensionless functions, H and G. The first one is proportional to the plasma resistivity:

$$H(k) = -\frac{4\sqrt{2\pi}}{9 n_e v_{Te}^3} \int_0^\infty \frac{dv\, v^6}{h_{1,1}(kv^4/2A)} \frac{\partial f_0}{\partial v}, \tag{8}$$

where the function $h_{1,1}(x)$ is defined through the chain rule:

$$h_{l,m}(x) = \frac{1}{2} l(l+1) + \frac{(l+1)^2 - m^2}{4(l+1)^2 - 1} \frac{x^2}{h_{l+1,m}}.$$

This function accounts for the strong anisotropy of δf_e in the limit of short wavelengths, $k\lambda_{ei} \gtrsim 1$. The second function accounts for the effect of the laser-induced anisotropy:

$$G(k) = -\frac{4\pi v_{Te}^2}{3n_e} \int_0^\infty \frac{dv}{v^3} \left(1 - \frac{1}{h_{1,1}(kv^4/2A)} \right) \frac{\partial}{\partial v} \left(v^5 \frac{\partial f_0}{\partial v} \right). \tag{9}$$

The function G is significantly different from the results of previous publication [1]. Without the term I_B, accounting for the coupling between the quasistatic and high-frequency fields, it would have the following form:

$$\tilde{G}(k) = \frac{8\pi v_{Te}^2}{3n_e} \int_0^\infty dv\, v^6 \left(1 - \frac{1}{h_{1,1}(kv^4/2A)} \right) \frac{\partial}{\partial v} \left(\frac{1}{v^4} \frac{\partial f_0}{\partial v} \right). \tag{10}$$

Fig. 1a shows the $k\lambda_{ei}$-dependence of these three functions, H, G, and $\tilde{G}$, in the case where f_0 is a Maxwellian distribution, with $\lambda_{ei} = v_{Te}/\nu_{ei}$ being the electron mean free path.

By setting $\gamma = 0$ in Eq. (7), we find the instability threshold. Two domains of instability exist, according to the change of sign of the function G. In the long wavelengths limit the instability develops along the anisotropy main axis $\boldsymbol{k} \parallel \boldsymbol{E}_0$, where $\theta_E = 0$. The wavelength dependence of the growth rate is similar to what has been found in the long wavelengths limit in Ref. [1], but the direction of the wave vector of unstable mode is inverted due to the negative value of the function G. However, the domain of instability in the long wavelengths limit is very narrow region, $k\lambda_{ei} \lesssim 0.35$, as shown in Fig. 1. While the threshold of this instability is very low, typically below 10^{13} W/cm^2, the growth rate is also rather small, of the order of $1-2$ ns^{-1}, and it is not of interest for practical applications.

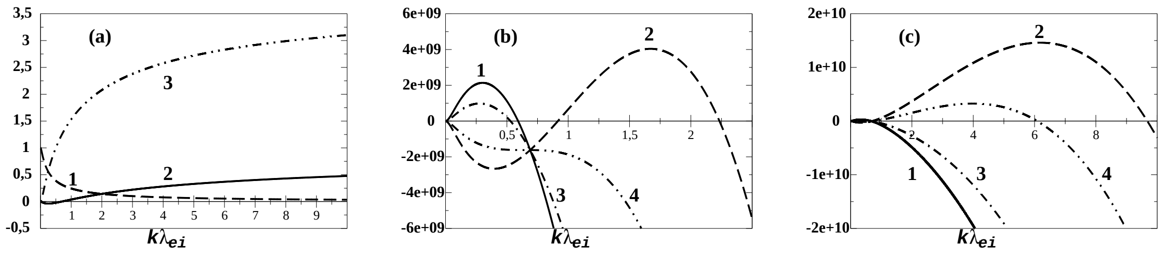

Figure 1: a) Dependence of functions $H(k)/H(0)$ (1), $G(k)$ (2) and $\tilde{G}(k)$ (3) on $k\lambda_{ei}$. b) Instability growth rate (in s^{-1}) as a function of $k\lambda_{ei}$ for $T_e = 2$ keV, $W_e/T_e = 0.045$, $Z = 4$, $n_e = 9\times10^{21}$ cm^{-3}. Lines 1 and 2 correspond to the linear polarization and 3 and 4 – to the circular polarization. Lines 1 and 3 correspond to the case $\theta_E = 0°$, and the lines 2 and 4 – to $\theta_E = 90°$. c) Same as b for the electron density of 10^{21} cm^{-3}.

In the perpendicular direction, $\boldsymbol{k} \perp \boldsymbol{E}_0$, that is, $\theta_E = \pi/2$, the instability is developing in the domain of shorter wavelengths, $k\lambda_{ei} \gtrsim 1$. Its growth rate is attaining the level of $10-20$ ns^{-1}, as it is shown in Fig. 1. In the collisionless limit, $k\lambda_{ei} \gg 1$, the expression for the growth rate is similar to the one obtained in Ref. [2]. However, the critical wave number is reduced by a factor more than two, and the maximum growth rate is reduced by a factor more than eight.

In conclusion, our analysis shows that the anisotropic instability is difficult to observe for realistic parameters, because expected growth time is of the same order as the laser pulse and plasma variation times and the unstable wavelengths are larger than the characteristic scale of the laser intensity inhomogeneites.

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