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Statistical analysis of two point correlation functions for flow velocity measurements

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Introduction

In a previous work [1] the statistics of the autocorrelation function (ACF) has been investigated in connection to the autocorrelation-width-method (ACFWM) used for the one point detection of poloidal flow modulations especially zonal flows. Another possibility for studying this phenomenon is the time delay estimation method which has been used e.g. in the DIII-D tokamak for beam emission spectroscopy [2] and elsewhere. This method is based on the determination of the time shift ($\tau_{\max}$) of the cross-correlation function (CCF) calculated from short time intervals of two fluctuating measurement signals (called evaluation time windows $\Delta T_{\rm etw}$). Due to the finite length of the analyzed time record, inevitable uncertainties arise when we would like to determine the location of the peak correlation value. If $\Delta T_{\rm etw}$ is too short, the uncertainty of the velocity determination can be higher than the velocity modulations so the modulations are not detectable. So the $\Delta T_{\rm etw}$ must be adjusted correctly, but it is only possible when the scatter of the time delay is known. This contribution presents the determination of the scatter of the time delay if the noise is dominated by statistics of the events.

Mathematical model of the fluctuating signal

In order to determine the scatter of the time shift, a mathematical model of the fluctuating signals must be made. According to this model the signal $S(t)$ is represented as a sum of a finite number of statistically independent random turbulent structures (eddies, blobs, events etc.), every event $(s(t,\vec{\mu}_i))$ can be characterized by its random variables ($\vec{\mu}_i$) like its spatial scale, its lifetime, its amplitude, etc.:

$$S(t,\mu) = \sum_{i=1}^{N} s\left(t,\overrightarrow{\mu_i}\right). \tag{1}$$

A simple model of a turbulent event could be given by taking Gaussian-shaped structures with uniform amplitudes evolving in time as a Gauss-function and moving in one dimension, locally detected at *x* (see Fig. 1):

$$s(t,\vec{\mu}_i) = e^{-\frac{\left(x-x_i^*-v\left(t-t_i^*\right)\right)^2}{2w_x^2}-\frac{\left(t-t_i^*\right)^2}{2w_t^2}}, \tag{2}$$

where x_i^* and t_i^* are the uniformly but otherwise randomly distributed spatial and temporal centers of the ith event respectively, w_x is the poloidal width of structures and w_t is the lifetime.

Cross-correlation measurements for flow detections

From practical point of view the most important moment of a fluctuating signal is the time average:

$$\overline{S} = \frac{1}{\Delta T} \int_{-\frac{\Delta T}{2}}^{\frac{\Delta T}{2}} S(t)\, dt. \qquad (3)$$

In our one dimensional model we assume that the structures move poloidally. This is a valid approximation as the toroidal size of the structures is typically 2-3 orders of magnitude longer than their poloidal size. In this case we can define the cross-correlation function (CCF) of the two time records measured at x_a and x_b as:

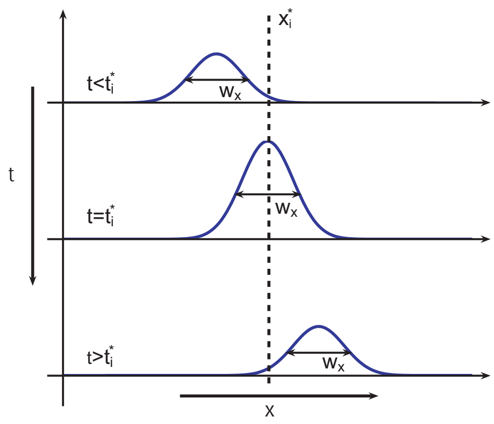

Figure 1: Illustration of propagating Gauss-shaped structures

$$C_{a,b}(\tau) = \overline{\left(S_a(t) - \overline{S_a}\right)\left(S_b(t+\tau) - \overline{S_b}\right)} = \overline{S_a(t)\, S_b(t+\tau)} - \overline{S}_a \overline{S}_b. \qquad (4)$$

The cross-correlation function calculated from signals having finite temporal length, contains an error originating from the finite number of overlapping events. This is called event statistical noise. The variance of the cross-correlation function originating from this noise can be written in standard form:

$$\sigma^2(\tau) = \left\langle C_{a,b}(\tau)^2 \right\rangle - \left\langle C_{a,b}(\tau) \right\rangle^2, \qquad (5)$$

where the mean value calculating procedure $\langle \cdots \rangle$ has to be done for all of $\overrightarrow{\mu_i}$ random variables except the time averaging which is denoted by an overbar. The variance of the CCF as well as the mean value of CCF can be analitically evaluated only in very special cases.

The velocity of the plasma will be the ratio of the distance of the probes $(x_b - x_a)$ and the mean value of the time shift $(\langle \tau_{\max} \rangle)$ if the events change slowly.

Relative scatter of the CCF

Using the Gauss-model described above, the mean value and the variance of the cross-correlation function can be derived analytically.

If some reasonable simplifications are made (such as: $\frac{w_x}{v} \ll w_t$: the structures change slowly, $\frac{w_x}{v} \ll \Delta T_{\text{etw}}$: during the evaluation time window significant part of an event can be detected, etc.) the relative scatter of the cross-correlation function can be calculated. Setting the parameters of

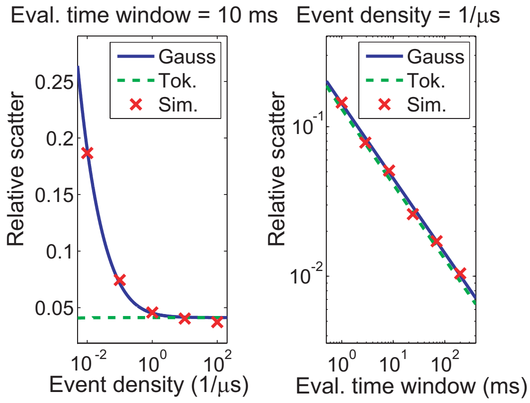

Figure 2: Parameter dependence of the relative scatter

the turbulence to typical magnitudes found in fusion measurements and having many events during one evaluation time window, the relative scatter becomes:

$$\sigma_{\mathrm{rel}}(\tau) = \frac{\sigma(\tau)}{\langle C_{a,b}\rangle(\tau)} = \sqrt[4]{2\pi}\sqrt{\frac{\frac{w_x}{v}}{\Delta T}}\sqrt{\frac{e^{-\frac{1}{2}\frac{v^2}{w_x^2}\left(\tau-\frac{x_b-x_a}{v}\right)^2} e^{-\frac{1}{2}\frac{(x_a-x_b)^2}{v^2 w_t^2}}+1}{e^{-\frac{1}{4}\frac{v^2}{w_x^2}\left(\tau+\frac{x_a-x_b}{v}\right)^2} e^{-\frac{1}{4}\frac{(x_a-x_b)^2}{v^2 w_t^2}}}}. \tag{6}$$

Fig. 2 shows the relative scatter of the CCF at $\langle\tau_{\mathrm{max}}\rangle$. The green line shows Eq. (6) while the blue line is an extended expression without assuming high number of events. A numerical simulation was done to check these analytical expressions, its result is shown by red crosses. Two main conclusions can be drawn from this figure: (i) The relative scatter will not depend on the event density if the number of events is high enough and (ii) the relative scatter depends inversely proportional to the square root of ΔT_{etw} (Fig. 2 right).

Scatter of the location of peak correlation value

In order to get information about the event statistical noise of flow measurement in the case of usage of cross-correlation functions, it is essential to link the relative scatter of the CCF calculated above to the standard deviation of the time delay of the maximum of the CCF. In order to get this let us suppose that the deviation of the τ_{max} from its average value is due to the superposition of the mean value of CCF and a single Gaussian 'difference'-function ($G_{a,b}(\tau)$) with the amplitude determined by the scatter of the correlation function at the maximum to the average CCF and with FWHM like the FWHM of the mean CCF:

$$C_{a,b}(\tau) = \langle C_{a,b}(\tau)\rangle + G_{a,b}(\tau) = Ke^{-\frac{\tau^2}{2s^2}} + \sigma(\tau)e^{\frac{(\tau-dm)^2}{2s^2}}, \tag{7}$$

where s and K are parameters depending on w_x, w_t, ΔT_{etw}, v, x_a, x_b, but do not depends on τ and dm is the random variable of the location of the 'difference'-function maximum. The location of CCF maximum is determined by its derivative with respect to τ.

According to Fig. 3 the location of maximum peak value:

$$-\frac{d\left\langle C_{a,b}\right\rangle(\tau)}{d\tau}\bigg|_{\tau=\tau_{\max}} \approx \frac{d\,G_{a,b}(\tau)}{d\tau}\bigg|_{\tau=\langle\tau_{\max}\rangle}. \quad (8)$$

It can be considered that in a CCF the ’difference’-functions are in pairs symmetrically to $\langle\tau_{\max}\rangle$ so the effect of the ’difference’-function pair which causes the highest displacement must be investigated. Supposing uniform distribution to dm the scatter of $\tau_{\max}$ can be calculated by numerical integration:

$$\sigma_{\max} = 0.62\,\sigma_{\mathrm{rel}}(\tau_{max})\frac{w_x}{w_t}\frac{x_b - x_a}{v^2} \sim \frac{1}{\sqrt{\Delta T_{\mathrm{etw}}}}. \quad (9)$$

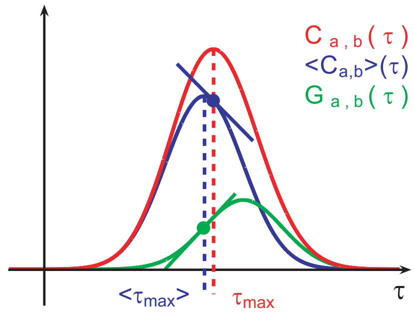

Figure 3: The effect of ’difference’-function on the displacement of the CCF maximum.

The parametric dependencies in this result were tested in simulations and excellent agreement was found. Some initial data analysis has also been done on Langmuir probe fluctuation data from the CASTOR tokamak, where the square root dependence of the scatter of the CCF maximum clearly showed up. The above analytical results will be used in the future to help designing proper two-point flow velocity measurements in fusion plasmas, regardless of the measurement method itself.

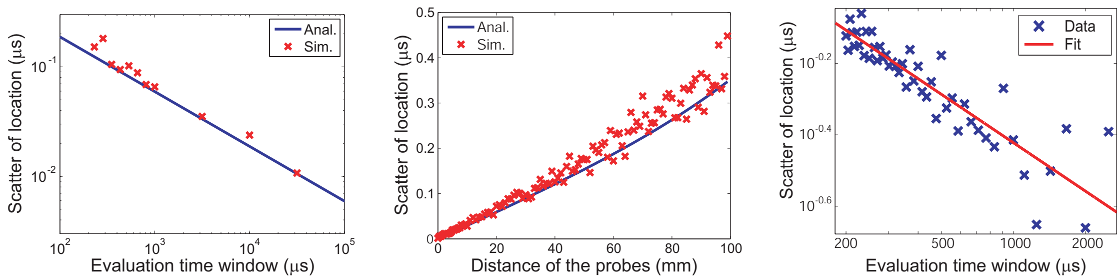

Figure 4: Analitic (blue line) and simulated (red cross) dependence of the uncertainty of the time shift on the length of the evaluation time window and probe distance: left and middle. Dependence of the uncertainty of the time shift on the length of the evaluation time window in measured data (blue cross) and the fitted function (red line): right.

References

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[2] D. K. Gupta, R. J. Fonck et al., Physical Review Letters **97**, 125002 (2006)