

## Shear Alfvén waves generated by a high power pulse at the plasma frequency: Experiments and Theory

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### Experimental Setup

The experiment is performed in the upgraded **L**Arge **P**lasma **D**evice (LAPD) [1]. The LAPD is a linear device, which produces a highly magnetized quiescent plasma, capable of supporting Alfvén waves. A schematic of the machine is shown in the figure below. The plasma is formed by a pulsed discharge ( $I_d \simeq 3.5$  kA) between cathode and anode. The electron beam thus formed collisionally ionizes the fill gas (Helium). The discharge typically lasts for 8 - 10 ms, and is pulsed at 1 Hz. The plasma is highly reproducible. It is 18 m long, and has a diameter of 50 cm, which is more than 100 ion larmor radii across at a background field of 1000 G.

In the experiment [2, 3], a high power pulse of microwaves is launched through a side window in the machine. The microwave source is a magnetron with fixed frequency at 9 GHz, and pulse length  $.5 \mu\text{s}$  or  $2.5 \mu\text{s}$  (not continuously variable). The output power of the source can be varied continuously to a maximum output of 100 kW. The microwaves are launched with a pyramidal horn antenna. The waves are launched in O-mode polarization ( $\mathbf{E} = E_z \mathbf{z} \parallel \mathbf{B}_0$ ), and propagate into the radial density gradient of the plasma, across the background field  $\mathbf{B}_0$ . The experiment is performed after the discharge shut-off ( $T_e \simeq 0$ ).

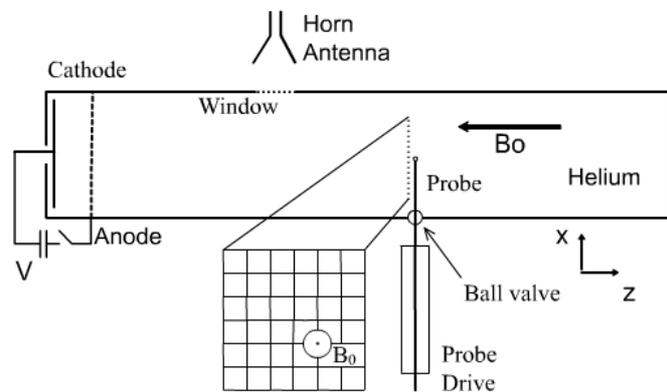


Figure 1: Schematic of the experiment

### Experimental Results

Measurements of the microwave electric field were done with a 3-axis dipole probe. The signals from the probe were rectified with calibrated crystal diodes, so that only amplitude levels are measured. The left panel of Figure 2 shows the electric field intensity of the microwaves in a transverse plane. This data is taken in the initial stage of the microwave pulse, right after

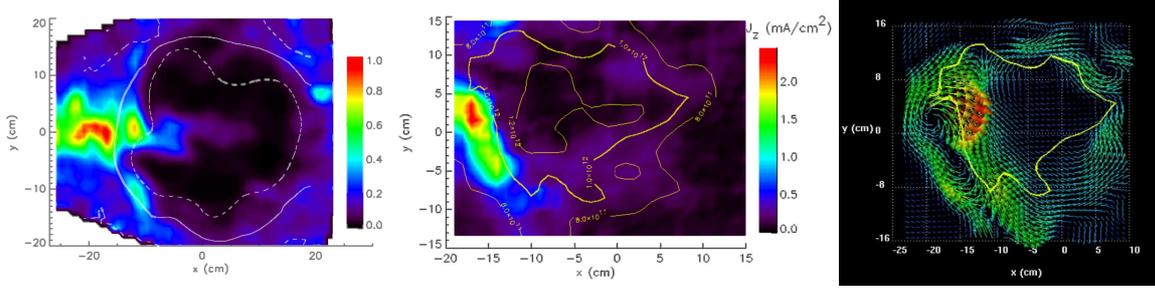


Figure 2: Planes transverse to  $\mathbf{B}_0$  of microwave intensity (left), fast electron signature (middle) and magnetic perturbation due to Alfvén waves (right).

the turn-on of the source. The contours on the plot correspond to the location of the L cutoff for X-mode (inner contour), the plasma frequency cutoff (center contour) and the upper hybrid resonance (outer contour). The waves propagate up to the  $\omega_{pe}$  cutoff for O-mode. The amplitude peaks at densities just below the O-mode cutoff.

Signatures of suprathermal electrons can be found in the Mach probe data. A snapshot of the current density, measured with the Mach probe, is plotted in the middle panel of Figure 2. The strongest currents are observed on the underdense side of the O-mode cutoff (thick contour), nearly in line with the horn. This is consistent with the microwave electric field measurements. Note that the data from the left and middle panel of Figure 2 were obtained in different data runs, which explains why the density contours are different. Nonetheless, these pictures show that both the microwaves and the suprathermal electrons have their peak at the same  $x - y$  locations with respect to the cutoff location, and the suprathermal electrons are therefore accelerated in the vicinity of these strong electric fields.

For the parameters in the experiment  $|\mathbf{E}| \simeq 70$  kV/m, the jitter velocity is on the order of  $2 \cdot 10^7$  cm/s ( $v_{osc} = \frac{eE}{m\omega}$ ). At this speed an electron can move about  $10^{-3}$  cm before the electric field turns around. Therefore, direct acceleration out of the interaction region ( $> 10$  cm wide) is not possible. The current thinking is that the incident microwaves first dig a density cavity through the ponderomotive force. The density profile becomes inhomogeneous along  $z$ , with a density hole at  $z = 0$  and sharp density increases on both sides of it. The pump frequency now matches the plasma frequency in a much more narrow region around this density hole, narrow enough to allow for direct acceleration of electrons out of the microwave interaction region.

The suprathermal electrons are followed by a series of oscillations, as measured with a 3-axis magnetic loop probe. These oscillations were identified as shear Alfvén waves. The right panel of Figure 2 shows a snapshot in time of the transverse magnetic field vectors. Overplotted is the density contour that corresponds to  $\omega = \omega_{pe}$ . The strongest waves are excited in the plasma on

the side nearest the horn. Note that the location of the strongest waves exactly corresponds to the location of the fast electrons in the middle panel of Figure 2. The fact that the waves are observed right after the suprathermal electrons pass by, and the fact that the waves are centered on the same field lines where the suprathermal electrons are observed, leads to the theory that the waves are generated by these electrons.

### Theoretical Model

A theoretical model [4] is proposed based on the observation that field aligned suprathermal electrons are observed in conjunction with Alfvén waves. These suprathermal electrons will be modeled by an external current in Maxwell's equation,  $\mathbf{J}_{ext} = -\frac{evN_b}{\pi d_{\perp}^2} \delta(z-vt) \exp\left(-\frac{x^2+y^2}{d_{\perp}^2}\right) \hat{z}$ .  $\mathbf{J}_{ext}$  represents the current from a moving charged beam with  $N_b$  electrons in the beam, with velocity  $v$  along the  $z$ -axis, with a transverse gaussian width  $d_{\perp}$ , and no spatial extent in  $z$ . In Fourier space, the wave equation is obtained as  $\mathbf{k} \times \mathbf{k} \times \mathbf{E} + \frac{\omega^2}{c^2} \tilde{\epsilon} \cdot \mathbf{E} = -\frac{4\pi i \omega}{c^2} \tilde{\mathbf{J}}_{ext}$  where  $\tilde{\mathbf{J}}_{ext} = -2\pi veN_b \delta(\omega - k_z v - i\nu) \exp\left(-\frac{k_{\perp}^2 d_{\perp}^2}{4}\right) \hat{z}$ .  $\nu$  is a small positive number which will make sure causality is preserved. The factor  $\delta(\omega - k_{\parallel} v)$  reveals that this is a Cherenkov problem. The general form of the solution will be proportional to  $\int d^3k \int d\omega \frac{S(k, \omega)}{D(k, \omega)} e^{i\mathbf{k} \cdot \mathbf{x} - i\omega t}$ , with  $S$  a factor dependent on the shape of the source term  $\mathbf{J}_{ext}$  and  $D$  a factor containing the dispersion relation.

For the inertial regime ( $\bar{v}_e \ll v_A$ ) and for low frequencies ( $\omega < \Omega_{ci} \ll \omega_{pi} \ll \Omega_{ce}, \omega_{pe}$ ):  $\epsilon_{\parallel} \Rightarrow -\frac{\omega_{pe}^2}{\omega^2}$  and  $\epsilon_{\perp} \Rightarrow \frac{c^2}{v_A^2} \frac{1}{1 - \frac{\omega^2}{\Omega_{ci}^2}}$ . This leads to the solution of the perturbed magnetic field

$\tilde{B}_x = \frac{4\pi i}{c} \frac{\Omega_{ci}^2}{v_A^2} \frac{k_y \lambda_e^2}{k_{\parallel}^2 - \frac{\Omega_{ci}^2}{v_A^2} \left( \frac{k_{\parallel}^2 v_A^2}{\omega^2} - 1 - k_{\perp}^2 \lambda_e^2 \right)} \tilde{J}_{ext,z}$  with  $\lambda_e = c/\omega_{pe}$ . The far field is determined by the poles of the integrand (zeros of the dispersion relation) and the Cherenkov relation  $\omega = k_{\parallel} v$ . There are no propagating waves for  $v > v_A$  and also not for  $k_{\perp} > \frac{1}{\lambda_e} \sqrt{v_A^2/v^2 - 1}$  if  $v < v_A$ .

The inverse Fourier transform is performed as  $\frac{1}{(2\pi)^4} \int k_{\perp} dk_{\perp} dk_{\parallel} d\alpha d\omega \tilde{B}_{x,y}(\mathbf{k}, \omega) e^{i\mathbf{k} \cdot \mathbf{r} - i\omega t}$ .

After integrating over  $\omega$  and  $\alpha$  one obtains

$B_{\phi} = -\frac{veN_b}{\pi c} \frac{\Omega_{ci}^2}{v_A^2} \int k_{\perp} dk_{\perp} dk_{\parallel} \frac{k_{\perp} \lambda_e^2 J_1(k_{\perp} \rho) \exp\left(-\frac{k_{\perp}^2 d_{\perp}^2}{4}\right)}{k_{\parallel}^2 - \frac{\Omega_{ci}^2}{v_A^2} \left( \frac{k_{\parallel}^2 v_A^2}{(k_{\parallel} v + iv)^2} - 1 - k_{\perp}^2 \lambda_e^2 \right)} e^{ik_{\parallel}(z-vt)}$  and  $B_z = B_{\rho} = 0$ . Contour integration can be used for the  $k_{\parallel}$  integration. Both propagating poles have negative imaginary part.

The final solution then becomes

$$\mathbf{B}(\mathbf{x}, t) = 0 \quad \text{for } t < z/v, \quad \text{otherwise one finds}$$

$$\mathbf{B}(\mathbf{x}, t) = B_{\phi} \hat{\phi} = -2 \frac{veN_b}{c} \frac{\Omega_{ci}^2}{v_A^2} \int_0^{k_{\perp, M}} dk_{\perp} k_{\perp}^2 \lambda_e^2 \exp\left(-\frac{k_{\perp}^2 d_{\perp}^2}{4}\right) J_1(k_{\perp} \rho) \frac{\sin(k_s(z-vt))}{k_s} \hat{\phi}$$

with  $k_s = \frac{\Omega_{ci}}{v_A} \sqrt{v_A^2/v^2 - 1 - k_{\perp}^2 \lambda_e^2}$  and  $k_{\perp, M} = \frac{1}{\lambda_e} \sqrt{v_A^2/v^2 - 1}$  This is then numerically integrated, for different  $t$ ,  $z$  and  $\rho$ .

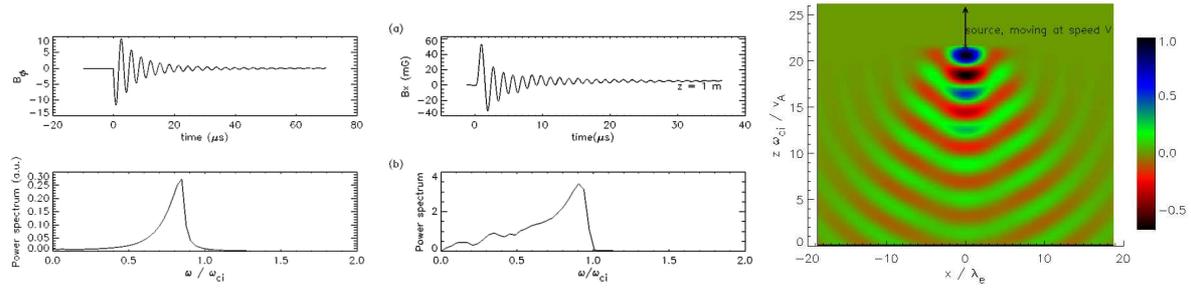


Figure 3: Comparison theory (left) and experiment (middle). Snapshot of wave pattern (right).

A direct comparison can now be made between theory and experiment. Typical experimental parameters are  $d_{\perp} = 1.5$  cm,  $n = 10^{12}$  cm $^{-3}$ ,  $B_0 = 1000$  G,  $v/v_A = .5$ . The left and middle panel of Figure 3 show that there is excellent agreement. The right panel of Figure 3 shows a snapshot of the propagating wave. The phase fronts are inverted because the inertial shear Alfvén wave is a backward wave in the perpendicular direction ( $v_{ph,\perp}$  is opposite to  $v_{g,\perp}$ ). Higher  $k_{\perp}$  or lower  $v/v_A$  would result in a more collimated pattern.

## Conclusions

A series of experiments performed at LAPD have shown that microwaves in the electron plasma frequency range launched across the magnetic field into the radial density gradient lead to the production of fast particles. Alfvén waves are observed in conjunction with these suprathermal electrons. A theoretical model is proposed, detailing the Cherenkov radiation of shear Alfvén waves, which captures the essential features of the experimental observations.

## References

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