Review on kinetic theory and applications of electromagnetic radiation scattering by dust induced fluctuations in plasmas

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Abstract

The present review deals with the modern understanding of the physics of incoherent electromagnetic wave scattering on plasma fluctuations as the sum of scattering on individual plasma particles with amplitudes being the sum of amplitudes of Thomson and transition scattering. In usual plasmas this description is a correct physical interpretation of well known and widely used results but is at present the basis for generalizations of scattering processes in non-equilibrium plasma states and for the understanding of the role played by the dust induced fluctuations. The review describes the first theoretical and observational investigations of scattering of electromagnetic radiation on dust grains in lower ionosphere. It is demonstrated that the effects due to presence of dust appear not only for the electromagnetic wave transition scattering by dust grains (as previously thought) but also as result of influence of dust on scattering by electrons and ions through the dependence of the screening factors on the effective collisions of electrons and ions with dust. The new dust dependent effects are due to rather effective scattering and absorption of plasma particles by dust grains, which is substantially broadening the scattering resonances. It is demonstrated that instead of an exponential type of broadening as in the case of Doppler effects for scattering on ions in absence of dust, the dust induced broadening has a power law tails. For backscattering and forward scattering with small changes in frequencies there appear new resonances due to dust attraction which can enhance the scattering. These effects can be used for dust diagnostic. The effects introduced by dust are most pronounced for the difference of the frequencies of the incident and scattered waves on the order of the frequency of electrom/ion scattering and absorption by dust grains. The most important for this effect to be measurable is the large cross-sections of absorption and ion scattering which can exceed the dust geometric cross-sections by many orders of magnitude.

1. Physics of scattering

The first consideration of scattering of electromagnetic radiation on grains in plasmas was made in [1] and applied for interpretation of enlarged radar scattering in lower atmosphere (by noctilucient clouds) in [2]. The theoretical model in [1,2] is based on effect of transitional scattering [3,4] the physics of which is related with the electron oscillation in the screening cloud of grains. Since such screening cloud has a charge proportional to the dust charge $Z_d e$ and $Z_d$ could be large the scattering on dust can be $Z_d P; P = n_d Z_d n_e$ larger than the scattering on electrons if $P$ is of the order of 1 and $Z_d$ is large. The further investigations of radar scattering and measured grain sized indicates that the observed enhancement of scattering can have a more complex nature.
At that point we should explain the general physics of scattering of electromagnetic radiation in plasmas including the dusty plasmas. This is often called as Thomson scattering on fluctuations (see the review [5]) and is widely used in plasma diagnostics. The general theory was developed in early 1960’s by many authors (see for example [6,7]) and is based on the theory of plasma fluctuations. Even before that there was formulated a common statement that the incoherent scattering on fluctuations is not the sum of scattering on individual particles even in very rarified plasmas. This statement is wrong. In 1964-1967 the development of plasma physics discovers that the plasma wave scattering is mainly produced by ions and than this process was used as the standard accepted point of the basics of plasma physics. At the first time this scattering was called as non-linear scattering [8] and later (in more broad ranges) as transitional scattering [3,4]. The usual theory of scattering on fluctuations also contains terms known as electron fluctuation induced by ion motions. There are proportional to $\delta(\omega - \omega' - (k - k') \cdot v)$ where $\{\omega, k\}$ and $\{\omega', k'\}$ are frequency and wave number of the incident and scattered wave respectively. This expression from simple arguments describes the conservation of energy and momentum for scattering of radiation by individual ions. In 1985 [9] it was noticed that if one simply adds to the amplitude of Thomson scattering the amplitude of transition scattering and calculates the probability of scattering by squaring these amplitudes one finds that the sum of scattering on electrons and ions exactly coincides with that obtained by the fluctuation theory. In this sense the fluctuation theory only takes properly into account the transition scattering and total scattering is than interpreted as sum of scattering on individual particles. The question arises whether by this physical interpretation we only pronounce different words for the same effect? It was proved that the latter is not correct [10] since the equations for scattering can be generalized for any non-equilibrium distributions of electrons and ions and for non-equilibrium radiation and than it is possible to find how the ions are affected by the scattering. The result is that the changes of the ion distributions is regulated by the same probability of scattering as obtained from expressions for transition scattering or in fluctuation theory. The correct physical interpretation of scattering shows obviously that any heavy particle including the dust particle can take part in scattering of electromagnetic radiation. The progress was also made in derivation of scattering by using the non-linear plasma responses up to the third power in the field strength. These responses were used in plasma physics to calculate the stimulated scattering [11] which due to Einstein relation between stimulated and spontaneous emissions give also the probabilities of scattering. The Einstein relations was proved to be valid for non-equilibrium distributions both of particles and radiation and in presence of external sources of radiation. The spontaneous scattering is also simple to calculate from non-linear responses by taking into account the so called natural plasma fluctuations. These achievements simplifies the investigations of influence of dust on electromagnetic scattering in dusty plasmas

2. Collisions of electrons and ions with grains

The influence of binary plasma collisions on the scattering of electromagnetic radiation is well investigated (see review [5]). In presence of dust one should add the dust-electron/ion collisions which have rather large cross-sections. The collisions can be due to absorption of electrons and ions by grains and described by frequencies $\nu_{abs}^{e/i,d} =$
\[ v \sigma_{\text{abs}}^{e/d}(v)n_d \] and due to the scattering by grains and described by the frequencies \( \nu_{\text{scat}}^{e/d} = v \sigma_{\text{scat}}^{e/d}(v)n_d \) where \( n_d \) is the dust density and \( \sigma's \) are the cross-sections. The latter depend on the parameter \( \beta = c^2 Z_d T_i / \lambda_{Di} \) where \( Z_d \) is the grain charge in units of electron charge and \( \lambda_{Di} \) is the ion Debye radius. For \( \beta \ll 1 \) the cross-section of absorption is about \( Z_d e^2 P/a T_i \) (where \( a \) is the grain size and \( P \) is the Havnes parameter \( P = Z_d n_d / n_i \)) larger than the geometric cross-section and the cross-section of scattering is even larger by the additional factor \( Z_d e^2 / a T_i \). For \( \beta \gg 1 \) the cross-section of absorption is not changed substantially but the cross-section for scattering is then determined by large angle scattering due to importance of nonlinearity of the grains fields in process of scattering. In this case the cross-section can be estimated as \( \pi R_{nl}^2 \) while the nonlinear screening radius \( R_{nl} \) can be estimated as \( 7 - 9 \) times larger than the Debye screening radius (still the cross-section of scattering exceeds the cross-section of absorption). Thus the large cross-sections of collisions of electrons and ions with grains indicate that these collisions can be important in scattering of electromagnetic radiation even for rather small dust densities. The condition that the rate of collisions of electrons and ions with dust is larger than the rate the binary collisions of plasma particles can be written as \( P Z_d \gg 1 \) and is fulfilled in most existing experiments in dusty plasmas.

3. Effects introduced by dust in scattering of electromagnetic waves

The intensity of scattering in dusty plasma can be found by the described procedure and is proportional to

\[
\Im \left\{ e^e - 1 \right\} \left( 1 - \frac{e^e - 1}{e^k} \right) = \left| e^i + e^d - 1 \right|^2 \Im e^e + \left| e^i - e^d \right|^2 \Im e^i + \left| e^e - 1 \right|^2 \Im e^d \tag{1}
\]

Where \( e^i,i,d \) are the dielectric functions separately of electrons, ions and dust (for the case where other component are not taken into account) at the difference of the frequencies of the incident and the scattered waves, \( \omega - \omega' \) and at the difference of the wave numbers of the incident and the scattered waves, \( k - k' \). The dust influences the scattering not only through the imaginary part of \( e^d \) (with Doppler shift related to dust motion as considered in [1,2]) but also in all screening factors through the dust-electron/ion collisions. These contributions became important if the difference of the frequencies is comparable with the dust-electron/ion collision frequency. Here we pay attention only to the three following new effects introduced in scattering by presence of dust:

1) Center part of the \textbf{ion scattering line where the frequency difference of the initial and scattered waves is much less than the ion Doppler shift} (of the order of \( k v T_i \) for not very small angles of scattering) the dust changes the factor \( f \) which in absence of dust is of the order of 1 to the value different from 1 in presence of dust:

\[
f = \frac{\nu_{\text{abs}} + \nu_{\text{scat}}}{\sqrt{\nu_{\text{abs}}(\nu_{\text{abs}} + 2 \nu_{\text{scat}})} \int_0^\infty \exp\left(-\frac{y^2}{y_0^2}\right) \left(1 - \frac{\arctan(y) - \frac{1}{1 + y^2}}{y_0^2}\right) dy}
\tag{2}
\]

where \( y_0^2 = 2 v T_i (k - k')^2 / \nu_{\text{abs}}(\nu_{\text{abs}} + 2 \nu_{\text{scat}}) \). For \( \nu_{\text{scat}} \gg \nu_{\text{abs}} \) the peak at the center of the scattering curve (for \( \omega = \omega' \)) is enhanced by a factor \( \sqrt{\nu_{\text{scat}} / \nu_{\text{abs}}} \).

2) \textbf{For frequency difference much larger than the ion Doppler shift} where in absence
of dust the Doppler curve shows that the scattering is exponentially small by some factor

\[ F = \frac{1}{\sqrt{2\pi v_{T_i}|k' - k|}} \exp\left( -\frac{(\omega' - \omega)^2}{2|k' - k|^2 v_{T_i}^2} \right) \]  

(3)
in presence of dust the factor \( F \) is described by a is power-law

\[ F = \frac{|k' - k|^2 v_{T_i}^2 (\nu_{abs} + \nu_{scat})}{[\nu_{abs}^2 + 2\nu_{abs}\nu_{scat} - (\omega_{k'} - \omega_k)^2]^2 + 4(\omega_{k'} - \omega_k)^2(\nu_{abs} + \nu_{scat})^2} \]  

(4).

If the \( \nu_{scat} \) is much larger than \( \nu_{abs} \), than for \( \sqrt{\nu_{abs}\nu_{scat}} \ll |\omega_{k'} - \omega_k| \ll \nu_{scat} \) the intensity of scattered wave is \( \propto 1/(\omega' - \omega)^2 \) with a coefficient inverse proportional to \( \nu_{scat} \) and for \( |\omega' - \omega| \ll \nu_{scat} \) the asymptotic tail of scattering is \( \propto 1/(\omega' - \omega)^4 \) with a coefficient proportional to \( \nu_{scat} \). These dependencies are induced by dust. This analysis shows that the electromagnetic wave scattering by ions in dusty plasmas can be rather sensitive to the dust-plasma particle collisions. The condition of validity of relation (4) can be written through the angle of scattering \( \theta \). For forward scattering \( \theta \ll 1 \) this relation is often fulfilled since \( v_{T_i}/c \ll 1 \). Contrary for back scattering the criteria of validity of (4) is \( |\omega - \omega'|/\omega \gg v_{T_i}/c \) can be often not fulfilled.

3) The screening factors in the denominators of the amplitude of screening in (1) in presence of dust can be close to zero describing the resonance scattering. This occurs if \( (\omega' - \omega)^2 \ll |k - k'|^2 v_{T_i}^2 \approx \nu_{abs}\nu_{scat} \) and is important for backscattering for central part of the ion scattered signal determined by dust. This resonance is pronounced if the temperature of electrons is much larger than the temperature of ions. Thus if the plasma electrons are heated the scattering is decreased since the ion scattering in absence of resonance is proportional to \( (T_i/T_e)^2 \). But during the subsequent electron cooling eventually the scattering will pass the resonance, the intensity of scattering will overshoot the intensity of scattering before electron heating and after that will relax to the initial scattering intensity. The further analysis is required to understand wether this effects can explain the observed overshooting of dust scattering in the last radar experiments with electron heating of lower ionosphere [12].

References