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Angle dependent acoustic mode in weakly ionized plasmas with magnetized electrons

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We investigate the behavior of an ion acoustic mode driven by an equilibrium electron flow along the magnetic field lines, in the presence of ion-neutral, electron-neutral, and electron-ion collisions. Neutrals can also be perturbed and this introduces an additional (neutral) gas acoustic mode. Moreover, these two modes may interact and such effects are studied in the present work in both the electrostatic and the electromagnetic regimes. In such a weakly ionized plasma the ions may be un-magnetized and electrons magnetized. An ion acoustic mode can propagate almost at any angle with respect to the magnetic field lines as long as the electrons are capable of participating in the perturbations by moving only along the field lines.

Using the neutral momentum and continuity equations, one finds that the perturbed velocity of neutrals is coupled to the perturbed ion velocity as $v_{n1} = v_{i1}[i\omega\nu_{ni}/(\omega\omega_n - k^2 v_{Tn}^2)]$. We have assumed small longitudinal perturbations of the form $\sim \exp(-i\omega t + i\vec{k}\cdot\vec{r})$, propagating in an arbitrary direction $\vec{r}$ which makes an angle ψ with the magnetic field lines $\vec{B}_0 = B_0\vec{e}_z$. Here, $\omega_n \equiv \omega + i\nu_{ni}$, $v_{Tn}^2 = \kappa T_n/m_n$, and v_{i1} is the perturbed ion velocity in the same $\vec{r}$ direction given by

$$m_i n_{i0}\frac{\partial v_{i1}}{\partial t} = -en_{i0}\frac{\partial \phi_1}{\partial r} - \kappa T_i\frac{\partial n_{i1}}{\partial r} - m_i n_{i0}\nu_{in}(v_{i1} - v_{n1}) + \mu_L\frac{\partial^2 v_{i1}}{\partial r^2}. \quad (1)$$

The ions are not magnetized, $\nu_{in} \gg \Omega_i$. Here, $\mu_L\nabla^2\vec{v}_i$ is an effective viscosity term that accounts for the Landau damping [1]. The parameter μ_L is chosen to quantitatively describe the well-known properties of the Landau effect. Taking note of the fact that the ratio between the attenuation length δ and the wavelength λ is independent both of the wavelength and the plasma density n, and dependent on the electron/ion temperature ratio $\tau = T_e/T_i$ in a prescribed way one writes [1] $\mu_L = m_i n_{i0} v_s\lambda/(2\pi^2\delta/\lambda)$. Here, $v_s = (c_s^2 + v_{Ti}^2)^{1/2}$ is the ion sound speed, $c_s^2 = \kappa T_e/m_i$, while the dependence of the ratio δ/λ on τ is such that the attenuation is strong at $\tau \approx 1$ and weak for higher values of τ. The 'fluid' attenuation length can be expressed by the following approximate fitting formula to give the same decrement as the corresponding kinetic expression $d \equiv \delta/\lambda \approx 0.2751 + 0.0421\,\tau + 0.089\,\tau^2 - 0.011785\,\tau^3 + 0.0012186\,\tau^4$. More details on that issue are available in Refs. [1], [2]. Eq. (1) and the ion continuity equation yield

$$v_{i1} = \frac{ek}{m_i\omega_2}\left(1+\frac{k^2v_{Ti}^2}{\omega\omega_2-k^2v_{Ti}^2}\right)\phi_1, \quad \omega_2=\omega+i(\nu_{in}+\mu_0k^2)+\frac{\nu_{in}\nu_{ni}\omega}{\omega\omega_n-k^2v_{Tn}^2}, \quad \mu_0=\frac{\mu_L}{m_in_{i0}}, \tag{2}$$

$$\frac{n_{i1}}{n_{i0}} = \frac{ek^2}{m_i(\omega\omega_2-k^2v_{Ti}^2)}\phi_1. \tag{3}$$

Electron momentum equation is of the form

$$m_en_e\left[\frac{\partial\vec{v}_e}{\partial t}+(\vec{v}_e\cdot\nabla)\vec{v}_e\right] = en_e\nabla\phi - en_e\vec{v}_e\times\vec{B}-\kappa T_e\nabla n_e - m_en_e\nu_{en}(\vec{v}_e-\vec{v}_n)-m_en_e\nu_{ei}(\vec{v}_e-\vec{v}_i). \tag{4}$$

The electrons are assumed to be magnetized, $\Omega_e > \nu_{ei}+\nu_{en}$, their perpendicular dynamics is negligible and from Eq. (4) we take the parallel part only. The electron continuity yields

$$\frac{n_{e1}}{n_{e0}} = \frac{\nu_e\omega_0}{\nu_e\omega_0+ik_z^2v_{Te}^2}\left(\frac{iek_z^2}{m_e\nu_e\omega_0}\phi_1+\frac{\nu_{en}}{\nu_e}\frac{k_z^2}{k\omega_0}v_{n1}+\frac{\nu_{ei}}{\nu_e}\frac{k_z^2}{k\omega_0}v_{i1}\right). \tag{5}$$

Here, $\omega_0 = \omega - k_zv_0$, $\nu_e = \nu_{ei}+\nu_{en}$. We have expressed the ion and neutral velocity components in the z-direction by $v_{j1z} = v_{j1}\cos\psi = v_{j1}k_z/k$. This yields the dispersion equation:

$$\left[\omega^2-k^2c_s^2\left(1+\frac{1}{\tau}\right)\right](\omega\omega_n-k^2v_{Tn}^2) = -\nu_{ni}\omega^2\left(\nu_{in}+\nu_{en}\frac{m_e}{m_i}\right)$$

$$-i(\omega\omega_n-k^2v_{Tn}^2)\left\{\omega(\nu_{in}+\mu_0k^2)+\frac{m_e}{m_i}\left[\omega\left(\nu_{ei}\left(\frac{k^2}{k_z^2}-1\right)+\nu_{en}\frac{k^2}{k_z^2}\right)-k_zv_0(\nu_{ei}+\nu_{en})\frac{k^2}{k_z^2}\right]\right\}. \tag{6}$$

We discus first the case of perturbations in the plasma species only, by assuming static neutrals. From Eq. (6) we find a modified ion sound mode that is unstable provided that

$$V > \frac{\kappa}{1+\nu}\left(1+\frac{1}{\tau}\right)^{1/2}\left[\nu\left(\frac{1}{\kappa^2}-1\right)+\frac{1}{\kappa^2}+\frac{\mu}{\widehat{\nu}_{en}}\left(\frac{\widehat{\nu}_{en}b}{(\mu\tau)^{1/2}}+\frac{(1+1/\tau)^{1/2}}{\pi d}\right)\right]. \tag{7}$$

Here, $\widehat{\nu}_{en}$ is normalized to $\omega_r \simeq kc_s$ and should be chosen in accordance with the assumptions introduced earlier. From $\nu_{en} = \sigma_{en}n_{n0}v_{Te}$ and $\nu_{in} = \sigma_{in}n_{n0}v_{Ti}$, we obtain $\widehat{\nu}_{in} = \widehat{\nu}_{en}b/(\mu\tau)^{1/2}$, where $\mu = m_i/m_e$, $b = \sigma_{in}/\sigma_{en}$ is the ratio of the corresponding collision cross sections, $\kappa = k_z/k$, $\nu = \widehat{\nu}_{ei}/\widehat{\nu}_{en}$, $\widehat{\nu}_{ei}$ is also given in units of kc_s, and $d(\tau)$ is given above. For larger values of ν, the second and third terms in Eq. (7) are reduced, the latter implying that the minimum in the threshold velocity profile reduces. This behavior is presented in Fig. 1 (left) for a hydrogen plasma in a neutral hydrogen gas. Here, $\tau = 1$, and $\sigma_{en} = 2.5\cdot10^{-19}$ m^2, $\sigma_{in} = 9.24\cdot10^{-19}$ m^2 at the temperature of 1 eV, and we have chosen $\widehat{\nu}_{en} = 30$. For these parameters $d \simeq 0.4$, and $\widehat{\nu}_{in} = 2.6$. The electron-ion collisions drastically reduce the velocity threshold at small angle of propagation (i.e., for k_z/k close to 1).

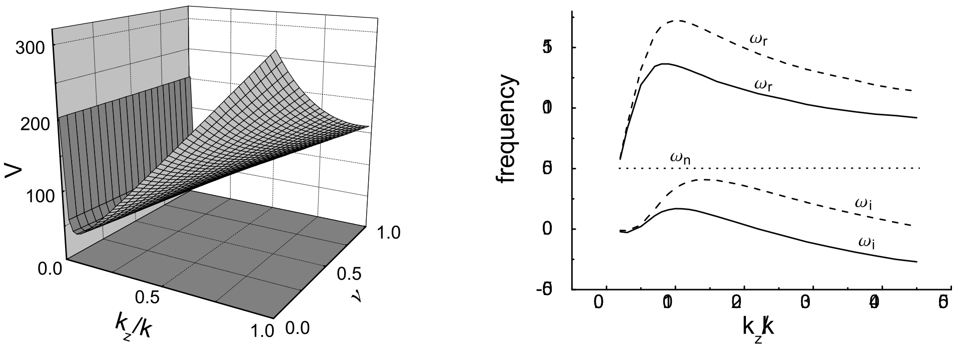

Figure 1: Left: The normalized threshold velocity $V \equiv v_0/c_s$. Right: Normalized real ω_r and imaginary ω_i parts of the angle dependent IA frequency for $\nu = 0$ (full lines) and $\nu = 0.916$ (dashed lines), in terms of k_z/k. The dotted line describes the neutral acoustic mode.

When the neutral gas is perturbed or when the perturbations in the ionized component induce (due to the friction) perturbations of the neutral background, the full dispersion equation (6) needs to be solved. In dimensionless form it becomes

$$\widehat{\omega}^2 = 1 + \frac{1}{\tau} - \frac{\widehat{\omega}^2 \widehat{\nu}_{ni} \widehat{\nu}_{en} \left[b(\mu\tau)^{-1/2} + \mu^{-1} \right]}{\widehat{\omega}(\widehat{\omega} + i\widehat{\nu}_{ni}) - \frac{\mu_n}{\tau_n}}$$

$$-i \left\{ \widehat{\omega} \left[\widehat{\nu}_{in} + \frac{(1+\tau^{-1})^{1/2}}{\pi d} \right] + \frac{1}{\mu} \left[\widehat{\omega}\widehat{\nu}_{en} \left(\nu \left(\frac{1}{\kappa^2} - 1 \right) + \frac{1}{\kappa^2} \right) - \frac{V \widehat{\nu}_{en}(1+\nu)}{\kappa} \right] \right\}. \quad (8)$$

All frequencies are normalized to kc_s, and we have introduced new parameters $\tau_n = T_e/T_n$, $\mu_n = m_i/m_n$. It is solved for the parameters $\tau = 4$, $\tau_n = 4$, $\mu = 1838$, $\mu_n = 1$, $\widehat{\nu}_{en} = 30$, and $V = 30$. We have taken $n_{e0} = n_{i0} = 6 \cdot 10^{16}$ m^{-3} and $n_{n0} = 10^{19}$ m^{-3}. The results are presented in Fig. 1 (right), with the remarkable angle dependent behavior of the IA mode. The neutral acoustic mode has a nearly constant frequency $\omega_n \simeq 0.5$ and a very small decrement $\simeq -0.005$. The real and imaginary parts of the ion acoustic mode frequency change in the presence of electron-ion collisions ν although the ionization is relatively small.

Assume now that the magnetic lines are perturbed. Typically this is due to the difference in the parallel motion of electrons and ions, which implies a perturbed parallel current and a perturbed perpendicular magnetic component according to the Ampère law. In the case of magnetized ions that are tied to the magnetic field lines, such perturbations propagate along the field lines at the Alfvén speed. However, this is not the situation in the present model. For the electromagnetic (EM) perturbations, the dynamics of neutrals is unchanged. For not so small plasma β, assuming only perpendicular bending of the magnetic field lines, we express the perturbations of the

EM field in terms of potentials $\vec{E}_1 = -\nabla\phi_1 - \partial\vec{A}_{z1}/\partial t$ and $\vec{B}_1 = \nabla\times\vec{A}_{z1} = -\vec{e}_z\times\nabla_\perp A_{z1}$. The ion momentum equation (1) now includes the new term $-en_{i0}\,\partial_t A_{z1}\,\vec{e}_z$. Consequently, the ion dynamics in $\vec{k}$-direction comprises the new term $A_k = k_z A_{z1}/k$, so that

$$v_{i1} = \frac{ek}{m_i\omega_2}\left(1+\frac{k^2v_{Ti}^2}{\omega\omega_2 - k^2v_{Ti}^2}\right)\left(\phi_1 - \frac{\omega}{k}\frac{k_z}{k}A_{z1}\right). \tag{9}$$

This results in the modified Eq. (3):

$$\frac{n_{i1}}{n_{i0}} = \frac{ek^2}{m_i(\omega\omega_2 - k^2v_{Ti}^2)}\left(\phi_1 - \frac{\omega}{k}\frac{k_z}{k}A_{z1}\right). \tag{10}$$

The electron parallel dynamics now includes the vector potential, and as a result there appears a new term, $-ie\omega k_z A_{z1}/(m_e\nu_e\omega_0)$ within the brackets in Eq. (5).

From the Ampère law $\nabla\times\vec{B} = \mu_0\vec{j}$, and equating Eq. (10) and the electromagnetically modified Eq. (5), with the help of Eq. (9), in the collision-less case without the electron flow the dispersion equation reads $(\omega^2 - k^2v_{Tn}^2)\left[k^4k_z^2v_s^2\lambda_i^2 - \left(k^2(1+k_z^2\lambda_i^2) - k_z^2\right)\omega^2\right] = 0$. Here, we have the GA mode uncoupled with the IA mode which is modified due to the electromagnetic effects yielding $\omega^2 = k^2c_s^2k_z^2\lambda_i^2/(1+k_z^2\lambda_i^2 - k_z^2/k^2)$. Hence, in the absence of Alfvén waves (unmagnetized ions), the parallel propagation ($k_z = k$) yields an ordinary ion sound mode. For any other angle of propagation (except for $k_z \to 0$) the ion sound mode is electromagnetically modified and becomes dispersive.

The collisions couple the two modes and the full dispersion equation is given by

$$k^4\lambda_i^2v_{Tn}^2\left(k_z^2v_s^2 - i\nu_e\omega_0\frac{m_e}{m_i}\right) + \omega_n\omega^3\left(1-\frac{k_z^2}{k^2}\right) + k_z^2\omega^2\left[\lambda_i^2\frac{m_e}{m_i}(\nu_{en}\nu_{ni} - i\nu_{ei}\omega_n) + v_{Tn}^2 + \omega_n\omega_2\lambda_i^2\right]$$

$$-k^2\omega\left\{\omega v_{Tn}^2 - i\lambda_i^2\nu_e\omega_0\omega_n\frac{m_e}{m_i} + k_z^2\lambda_i^2\left[v_{Tn}^2\left(\omega_2 - i\nu_{ei}\frac{m_e}{m_i}\right) + \omega_n v_s^2\right]\right\} = 0. \tag{11}$$

$$\omega_0 = \omega - k_z v_0, \quad \omega_n = \omega + i\nu_{ni}, \quad \omega_2 = \omega + i(\nu_{in} + \mu_0 k^2) + \frac{\nu_{in}\nu_{ni}\omega}{\omega\omega_n - k^2v_{Tn}^2}.$$

In order to compare the ES and EM cases, we use the same set of parameters as in the previous cases. The most visible difference is in the graph of the IA mode increment, which is now much higher for the velocity v_0 above certain critical value. Physically, here we have the bending of the magnetic lines representing an additional obstacle for the electron motion in the z-direction, and as a result the mode is more unstable.

References

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