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Numerical modeling of dust spinning in a magnetized plasma

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Introduction. Spinning of dust particles, which is observed in many laboratory experiments (e.g. see Ref. 1 and the references therein), may be important ingredient in physics of processes associated with dust and ranging from laboratory to astrophysical phenomena (e.g. see Ref 1, 2 and the references therein). A few mechanisms resulting in the spinning of dust particle were suggested so far. They are associated with: i) shear of plasma flow [3], ii) plasma flow and asymmetry of dust shape [4], iii) synergy of the effects of electric field, $\vec{E}$, plasma flow with velocity $\vec{V}_p$, and electric dipole, $\vec{D}$, caused by plasma flow [5].

Dust spinning in uniform magnetic field. Analytic estimates. In the presence of magnetic field a new mechanisms of dust spinning becomes available. In Ref. 6 it was shown that the interaction of electric dipole, induced in dust grain by $\vec{E}\times\vec{B}$ plasma flow, and electric field itself results in the torque acting on the dust grain. This torque is proportional to E^2 so that both laminar and fluctuating turbulent electric field can spin up the grain. In rather hot edge plasmas of fusion devises, where dust also was observed (e.g. see Ref. 7 and the references therein), the ablation of dust, which imposes quite large force on the grain [8], can also result in the spinning of dust. In Ref. [9] two other mechanisms of dust particle spin up in uniform magnetized plasma were suggested. They are associated with: i) gyro-motion of ions striking the grain and ii) in case of conducting material of the grain, the $\vec{j}\times\vec{B}$ force caused by cross-field current flowing through the grain and which closes the different paths of magnetized plasma electrons and ions reaching the grain.

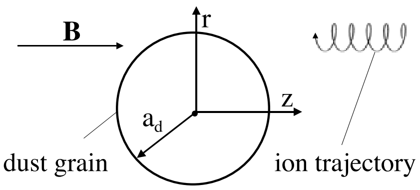

Fig. 1

a) Effect of gyro-motion of impinging ions. To illustrate the physics of the first one we consider a spherical grain of radius a_d in a uniform magnetic field $\vec{B}$ as it is shown in Fig. 1 (r and z are the cylindrical coordinates). In order to make qualitative estimates of the magnitude of the torque, τ, imposed on the grain by magnetized plasma we will assume that: i) dust particle is made of an insulating material and ii) electron temperature, T_e, is much lower than ion one, T_i, and iii) $a_d > \lambda_D, \rho_{T_e}$, where ρ_{T_e} is the electron gyro-radius and λ_D is the Debye length. As a result, we can neglect both impact of grain charging on ion dynamics and the contribution of electrons to the transport of total angular momentum to the grain.

Let consider the motion of individual ions and transfer of their momentum to the grain. For the magnitudes of perpendicular, $V_\perp$, and parallel, $V_\parallel$, ion velocity components such that $V_\parallel \sim V_\perp$ and ion gyro-radius, $\rho_\perp \equiv V_\perp/\Omega_i << a_d$ ($\Omega_i = eB/Mc$, B is the strength of the magnetic field, e is the elementary charge, M is the ion mass, and c is the light speed), practically all ions striking the surface of the grain have the same direction of azimuthal components of ion velocities for practically all striking radii a (see Fig. 2). The reason for such orientation of ions velocity is the interplay between the topology of the grain surface and parallel and perpendicular motion of the ions. It predetermines the azimuthal direction of ions striking the surface even though there is no averaged azimuthal direction in incoming ion flow. For $\rho_{T_i} \equiv V_{T_i}/\Omega_i << a_d$ (where $V_{T_i} = \sqrt{T_i/M}$ is the ion thermal speed) the motion of majority of ions has these features. Therefore, taking into account Fig. 1, 2 and the fact that ion flux to the grain is of the order of $nV_{T_i}a_d^2$, (where n is the plasma density) one can easily find the following estimate of the torque imposed to the grain by plasma

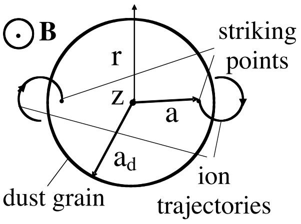

Fig. 2

$$\tau_{gyro}(\rho_{T_i} << a_d) \sim \tau_0 \equiv nT_i a_d^3. \qquad (1)$$

However, for the case $\rho_{T_i} >> a_d$ the ions striking the grain may have different directions of azimuthal velocities and in the limit of no magnetic field the torque should disappear. To estimate the magnitude of the torque for the case $\rho_{T_i} >> a_d$ we need to make more accurate assessment of the impact of different groups of ions. Analyzing the motion of ions with $\rho_\perp >> a_d$ but different values of $V_\parallel$ we find that the dynamics of collisions of ions with the grain crucially depends on the magnitude $\rho_{||} \equiv |V_{||}|/\Omega_i$. For $\rho_{||} >> a_d$ the motion of ions along the magnetic field is so fast that no interlay between the topology of the grain surface and parallel and perpendicular motion of the ions occurs and azimuthal direction of the ions striking the grain is just a blueprint of the azimuthal motion of incoming ions, which we neglect here. As a result, such ions practically do not contribute to torque. After a very cumbersome algebra, which we omit here, one finds that the contribution of ions with $\rho_{||} >> a_d$ gives the magnitude of torque not larger than $o(a_d/\rho_{T_i}) \times \tau_0$. Contrary to the dynamics of ions with $\rho_{||} >> a_d$, the approach of ions with $\rho_{||} << a_d << \rho_\perp$ to the grain is very similar to that of ions with $\rho_\perp \sim \rho_{||} << a_d$. However, in the case of interest, $\rho_\perp >> a_d$, the gyro-centers (GC) of ions striking the grain at local radius a are located at radii $r_{(-)} = \rho_\perp - a$ and $r_{(+)} = \rho_\perp + a$. Each ion having GC at $r_{(-)}$ and $r_{(+)}$ delivers the torque on the grain of the same magnitude but opposite direction. Since simply due to geometrical reasons there are more ions with GC at $r_{(+)}$ than at $r_{(-)}$, we find the following estimate for the torque imposed by ions with $\rho_{||} << a_d$:

$$\tau_{gyro}(\rho_{T_i} >> a_d) \sim \int_{\rho_{||} << a_d} d\vec{V} f_i(\vec{V}) M |V_{||}| V_\perp a_d^3 \sim \tau_0 \left(a_d/\rho_{T_i}\right) << \tau_0, \tag{2}$$

where $f_i(\vec{V})$ is the distribution function of incoming ions far away from the grain. In (2) we neglect the contribution, $\sim o(a_d/\rho_{T_i}) \times \tau_0$, of ions with $\rho_{||} >> a_d$. Since only ions with very small parallel speed $|V_{||}|/V_{T_i} \lesssim a_d/\rho_{T_i} << 1$ contribute to torque, collisions have an impact on dynamics of such ions and, hence, the torque magnitude can be affected by collisions while the bulk of ions is still magnetized.

b) Effect of electric current. Now we consider the torque associated with $\vec{j} \times \vec{B}$ force acting on a spherical grain of radius a_d in a uniform magnetic field $\vec{B}$ (see Fig. 1) and caused by cross-field current flowing through the grain and which closes the different paths of magnetized plasma electrons and ions reaching the grain. To simplify the physics we assume that: i) dust particle is made of a perfectly conducting material and ii) $T_e << T_i$ and $\rho_{T_i} > a_d > \rho_{T_e}, \lambda_D$. As a result, we can neglect both electron gyro-motion and impact of grain charging on ion dynamics. Then ions will come to the grain homogeneously from all direction, while more electrons will arrive to the grain surfaces, normals to which are closer to parallel to the magnetic field. Thus, radial electric current, J_r, of the magnitude equivalent to plasma particle flux $\sim enV_{T_i} a_d^2$ will flow through the grain to close the loops of electron and ion fluxes. As a result, due to azimuthal component of $\vec{j} \times \vec{B}$ force the following torque is applied to the grain

$$\tau_{\vec{j}\times\vec{B}} \sim \tau_0 \left(a_d/\rho_{T_i}\right) << \tau_0. \tag{3}$$

As one sees from Eq. (2, 3) the magnitudes of $\tau_{\vec{j}\times\vec{B}}$ and τ_{gyro} for the case $\rho_{T_i} >> a_d$ are of the same order and it is also easy to see the directions of these torques are the same also. Let us now estimate what steady-state angular rotation frequency, Ω_d, will be reached by the grain under the influence of the torques τ_{gyro} and $\tau_{\vec{j}\times\vec{B}}$, providing that the dust is affected by plasma only. For the case $a_d > \rho_{T_i}$ all ions contribute to the torque τ_{gyro} and the only way to balance the effect of ions impinging the grain is to have rotational velocity of the grain surface, $\sim \Omega_d a_d$, to be of the order of ion thermal speed. As a result we find the estimate for steady-state angular rotation frequency for $\tau_{gyro}(a_d > \rho_{T_i})$

$$\Omega_d^{(gyro)}(a_d > \rho_{T_i}) \sim \omega_{Bi} \left(\rho_{T_i}/a_d\right). \tag{4}$$

For the case of dust spin up due to $\tau_{gyro}(a_d < \rho_{T_i})$ and $\tau_{\vec{j}\times\vec{B}}$, majority of ions do not produce any torque on not rotating grain. In the case where dust is spinning, majority of ions will produce a slowing down torque

$$\tau_{sd} \sim \tau_0 \left(\Omega_d a_d/V_{T_i}\right). \tag{5}$$

Then, from the balance of the torques $\tau_{gyro}(a_d < \rho_{T_i})$ and $\tau_{\vec{j}\times\vec{B}}$ from (2) and (3) by τ_{sd} from (5) we find corresponding steady-state angular rotation frequencies:

$$\Omega_d^{(gyro)}(a_d < \rho_{T_i}) \sim \Omega_d^{(\vec{j}\times\vec{B})} \sim \omega_{Bi}. \tag{6}$$

However, to reach such steady-state values of Ω_d the grain needs time $t_{ss} \sim (a_d/V_{T_i})(\rho_d/Mn)$, where ρ_d is the mass density of grain material. For a tokamak edge plasma we have $t_{ss} \sim 1$ s. But in practice, the life-time of the grain, t_{lt}, in standard regimes of edge plasma is smaller than t_{ss} due to evaporation and collisions with the walls. Therefore, dust grain angular rotation frequency will be smaller than those given by Eq. (4, 6) by the ratio t_{lt}/t_{ss}, which (for rather typical value $t_{lt} \sim 0.01$ s) is about 0.01 [8, 10]. Therefore, in a tokamak environment for $B = 3\,T$ we find the following estimate for a dust grain angular rotation frequency: $\Omega_d \sim 10^6\ s^{-1}$.

Numerical simulation. To verify the torque value imposed on the grain by gyro-motion of ions in uniform magnetic field a two dimensional (2D3v) particle-in-cell (PIC) Dust in Magnetic field (DiMag) code was developed. The code is based on PIC engine of previously developed 2D3v code simulating single dust particle in non-magnetized plasma wall transition layers [11] and inherits its main features, which include axially symmetrical 2D simulation domain with uniform rectangular mesh, a dust particle placed on the axis of symmetry, and two component plasma with any given velocity distribution function. In the DiMag code the direction of the magnetic field is naturally selected along the axial direction. The simulated system is limited in both the axial, z, and the radial, r, directions with boundaries transparent for plasma particles. It is assumed that bulk plasma outside the simulated domain has fixed velocity distribution function that provides input plasma fluxes through the boundaries along z-direction. The plasma particles are also allowed to freely leave the simulated system along this direction. The particles leaving the domain trough the outer radial boundary are returned to the system assuming analytical continuation of the particles gyro-motion outside the domain. In the case when the continued trajectory goes out of the z-boundaries, such particles are re-injected into the system with a velocity randomized according to the velocity distribution function in the bulk plasma. Therefore, the plasma particle flux trough the outer radial boundary is forced to be zero and is not restricted for the z-boundaries. This implies negligible radial electric field at the outer r-boundary that requires sufficiently large radius of the simulated system as compared to the dust particle radius. Equations of motion of the plasma particles in the magnetic field are solved employing Buneman-Boris integrator. Trajectories of plasma particles are traced until they either go out of the system or cross the dust surface.

The simulated dust particle can be placed at any z position at the axis of symmetry of the system and may have any cylindrically symmetrical shape (spherical for present simulations). The plasma particles, which trajectories cross the dust surface, are assumed to be absorbed by the dust transferring to it their charge, momentum and energy. The two limiting cases of dust conductivity can be simulated, when dust surface is a perfect conductor or an ideal insulator. In the latter case, it is assumed that absorbed charges are stick at the point of absorption at the dust surface. The momentum transferred to the dust particle both in the axial and the azimuthal directions is accumulated over a given time period to calculate force and torque acting on the dust. The electric field in the simulated system is calculated solving the Poisson's equation using the immersed boundary method [12], which describes the dust particle as a region with a given dielectric permittivity on the simulated spatial mesh. The conductive dust particles are simulated as having very large ($\sim 10^6$) dielectric permittivity. Thus, the spatial mesh should be fine enough to reproduce shape of the dust particle well. However, for the purpose of plasma absorption by the dust we introduce a dust surface mesh, which represents actual geometrical shape of the grain. The absorbed by dust plasma charges are accumulated on the surface mesh and then distributed to the spatial simulation mesh employing CIC weighting procedure. Using the immersed boundary method for solving the Poisson's equation no boundary conditions at the dust surface are required. The boundary conditions used assume zero potential at the z-boundaries and zero radial electric field at the outer radial boundary of the simulated system and at the axis of symmetry. To satisfy these boundary conditions physically it is required that size of the simulated system is sufficiently large in comparison with size of the dust particle. The boundary conditions also allow analytical continuation of plasma particle trajectories going out of the radial boundary of the system assuming simple gyro-motion without drifts in combined electric and magnetic fields.

We used the DiMag code to simulate torque imposed on dust by magnetized plasma. The results shown in Fig. 3 are obtained for the spherical conductive dust particle placed in the center of the simulated system. Induction of the simulated magnetic field varied to obtain ratio of the ion Larmor radius to the dust radius ρ_{T_i} / a_d from 1/150 to 1.6. The size of the simulated system was 5 times larger than the dust radius in the radial direction and 10 times larger in the z-direction. The bulk plasma was a deuterium plasma with Maxwellian electrons and ions having ratio of the electron and ion temperatures T_e/T_i=0.1 and the plasma density corresponding to the electron Debye length of 1/4 of the radius of the dust particle. The electron Larmor radius in these regimes was always much smaller than the radius of the dust particle. The simulations were performed in two modes with the self-consistent electric field calculated and without the electric field, in order to estimate the electric field effect on the ion torque. A few cases without the electric field and the ratio ρ_{T_i} / a_d varying from 2 to 50 were simulated for 10 times larger system size, but fixed plasma parameters. The simulations continued until a steady state was reached and then sufficient statistics was accumulated to obtain the average ion torque. The simulation time was limited by computational resources available and typically was hundreds of ion plasma periods.

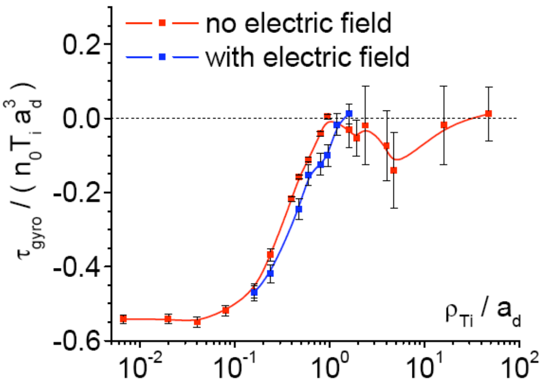

Fig. 3. Simulated plasma torque acting on the grain

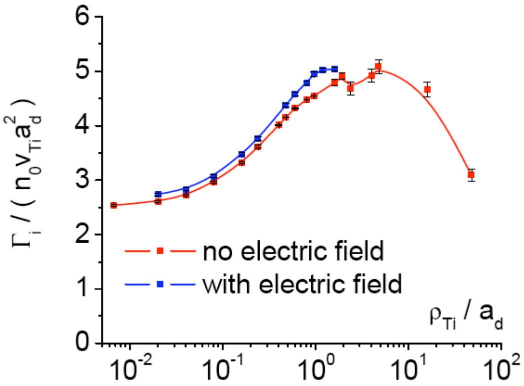

Fig. 4. Simulated plasma flow to the grain

As we see from Fig. 3, the value of torque found from modeling agrees with analytic estimates (1) for small ion gyro-radius, for the case $\rho_{T_i} >> a_d$, torque, in accordance with analytic estimates, decreases with increasing ion gyro-radius, but more accurate simulation is needed to have quantitative comparison. We also notice that numerical simulation reveals an interesting resonance-like phenomena occurring at $\rho_{T_i} \sim a_d$ and causing a non-monotonic dependence of the torque τ_{gyro} on the ratio ρ_{T_i} / a_d. The non-monotonic torque behavior is accompanied by non-monotonic dependence of ion flow to the grain on the ratio ρ_{T_i} / a_d, as shown in Fig. 4.Comparing Fig.3 and 4 one can see that the torque magnitude decreases for $\rho_{T_i} < a_d$ in spite of increase of the ion flow to the grain. This demonstrates an impact of the interlay between topology of the grain's surface and ion gyro-motion on the torque magnitude.

Conclusions. We find that i) both gyro-motion of magnetized plasma ions impinging the grain and $\vec{j} \times \vec{B}$ force caused by cross-field current flowing through the conducting grain and closing the different paths of magnetized plasma electrons and ions reaching the grain, can be effective mechanisms of dust grain spin up; ii) the $\vec{j} \times \vec{B}$ force impose the torque $\tau_{\vec{j}\times\vec{B}} \sim \tau_0 (a_d / \rho_{T_i}) \propto B$; iii) for a large magnetic field stress B, where $\rho_{T_i} < a_d$, the magnitude of torque cased by ion gyro-motion is $\sim \tau_0 \equiv n T_i a_d^3$; and iv) at small B, where $\rho_{T_i} >> a_d$ gyro-motion of ions imposes the torque in the same direction and similar magnitude, $\sim \tau_0 (a_d / \rho_{T_i}) \propto B$, as $\vec{j} \times \vec{B}$ force does; however, only ions with small parallel speed such that $| V_{\|} | / V_{T_i} \tilde{<} a_d / \rho_{T_i}$ contribute to the $\tau_{gyro}(\rho_{T_i} << a_d)$ so that the collisions of slow ions can alter the torque estimate. Preliminary results of numerical simulation of the impact of gyro-motion of magnetized plasma ions on the torque, obtained with the DiMag code, in a ballpark support our analytic estimates.

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