*34th EPS Conference on Plasma Phys. Warsaw, 2 - 6 July 2007 ECA Vol.**31F**, P-2.086 (2007)*

Wave propagation in molecular clouds

B.P. Pandey[1], and, J. Vranjes[2]

[1] *Department of Physics, Macquarie University, Sydney, NSW 2109, Australia*

[2] *Centre for Plasma Astrophysics, Celestijnenlaan 200B, 3001 Leuven, Belgium*

The molecular clouds are the site of star formation and generally consist of very small ($\sim 10^{-4}-10^{-8}$) fraction of ionized matter and yet, depending upon the level of plasma-neutral coupling, the magnetic field can play an important role in the dynamics of such a cloud. The magnetic field can provide either static or dynamic (by magnetohydrodynamic waves) support to such clouds (Dewar, 1970). The Alfvén waves are not easily dissipated (Zweibel & Josafatsson, 1983) making them an attractive candidate for the dynamic support of the cloud. The origin of the non-thermal line broadening in molecular cloud is attributed to the Alfvenic turbulence owing to the non-steepening of the large amplitude Alfvén waves (Mouschovias et al. 2006). The supernova shocks, gravitational collapse or the large scale motion of the cloud could act as the source of such waves in the medium.

In dense molecular cloud cores charged grains are more numerous in number than the electrons and ions (Wardle and Ng, 1999). The ionization fraction of the plasma is strongly affected by the abundance and size distribution of the grains through the recombination process on the grain surface. At the densities relevant to cloud cores and at higher densities occurring during the formation of protostar and protostellar discs (densities $\geq 10^7 - 10^{11}$ cm^{-3}), the role of the grains can not be neglected in the dynamics (Wardle & Ng, 1999). The presence of the charged grains in the planetary and interstellar medium causes the excitation of very low frequency Alfvén waves (Pillip et al. 1987). These waves provide important physical mechanism for the transport of angular momentum and energy in differentially rotating dusty discs (Wardle, 1999; Pandey and Wardle 2006).

The dynamical processes in the star forming clouds are strongly influenced by the coupling of the largely neutral medium to the magnetic field. The non-ideal magnetohydrodynamic effect not only determines the behaviour of such a medium but also becomes crucial to the various models of cloud support and star formation. The investigation of the various non-ideal MHD effect in the molecular clouds has a rich history. For example, the role of the ambipolar diffusion in the star formation has been extensively investigated (Mouschovias et al. 2006). The self-initiated collapse of the cloud core is assisted by the ambipolar diffusion. At the densities relevant to cloud cores and at higher densities occurring during the formation of protostar and protostellar discs (densities $\geq 10^7 - 10^{11}$ cm^{-3}), the collisonal effects of the charged grain may significantly modify the ambipolar time-scale of the collapse. The charged grain plays an important role in generating the Hall electric field. The Hall field is a generic feature of any multi-component plasma (Pandey & Wardle, 2006; Pandey & Vranjes, 2006). The present work investigates the propagation of Alfvén wave in a dark molecular cloud. For this purpose, we assume that the electrons and ions are well coupled to the magnetic field and move together whereas grains are weakly magnetized. We demonstrate that the Alfvén waves in such a medium is dispersive in nature and its wavelength compares favourably with the size of the cloud cores.

The dynamics of a partially ionized mixture of plasma particles, i.e. electrons and ions, charged grains and neutral particles is described by the following multi-fluid system.

$$\frac{\partial \rho_j}{\partial t} + \nabla \cdot \left(\rho_j \mathbf{v}_j\right) = 0\,. \tag{1}$$

Here, ρ_j is the mass density and $\mathbf{v}_j$ is the velocity of the various plasma components and neutrals. We note that the plasma velocities $\mathbf{v}_j$ are written in the neutral frame. The momentum equations for electrons, ions, grains and neutrals are

$$0 = -q_j n_j \left(\mathbf{E}' + \frac{\mathbf{v}_j \times \mathbf{B}}{c}\right) - \rho_j \nu_{jn} \mathbf{v}_j\,, \tag{2}$$

$$\rho_n \frac{d}{dt}\mathbf{v}_n = -\nabla P + \sum_{e,i,d} \rho_j \nu_{jn} \mathbf{v}_j\,. \tag{3}$$

Here $q_j n_j \left(\mathbf{E}' + \mathbf{v}_j \times \mathbf{B}/c\right)$ is the Lorentz force and $\mathbf{E}' = \mathbf{E} + \mathbf{v}_n \times \mathbf{B}/c$ is the electric field in the neutral frame with $\mathbf{E}$ and $\mathbf{B}$ as the electric and magnetic fields respectively, n_j is the number density, and, j stands for electrons ($q_e = -e$), ions ($q_i = e$) and dust ($q_d = -e$) and c is the speed of light.

The collision frequency is

$$\nu_{jn} \equiv \gamma_{jn} \rho_n = \frac{< \sigma v >_j}{m_n + m_j} \rho_n\,. \tag{4}$$

Here $< \sigma v >_j$ is the rate coefficient for the momentum transfer by the collision of the j^{th} particle with the neutrals. We shall define the mass density of the bulk fluid as $\rho \approx \rho_n$. Then the bulk velocity is $\mathbf{u} \approx \mathbf{v}_n$. The continuity equation (summing up equation (1)) for the bulk fluid becomes

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\, \mathbf{u}) = 0\,. \tag{5}$$

The momentum equation can be derived by adding Eqs. (2) and (3)

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla P + \frac{\mathbf{J} \times \mathbf{B}}{c}\,. \tag{6}$$

As noted above, we shall assume that the electrons and ions are well coupled to the magnetic field. Defining $\beta_j \equiv \omega_{cj}/\nu_{jn}$ as the ratio of cyclotron $\omega_{cj} = q_j B/m_j c$ to the collision ν_{jn} frequencies, we shall thus assume that $\beta_e \gg \beta_i \gg 1$, i.e. plasma particles are tied to the magnetic field and move together $\mathbf{v}_e \simeq \mathbf{v}_i$. The dust grains will be assumed unmagnetized, i.e. $\beta_d \ll 1$. Then, taking curl of the electron momentum equation (2) and making use of Maxwell's equation, in the $\beta_e \gg 1$ limit the induction equation can be written as

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[(\mathbf{u} \times \mathbf{B}) - \frac{\mathbf{J} \times \mathbf{B}}{e n_d} \right] . \tag{7}$$

We shall note that in the absence of dust, $\mathbf{J} \times \mathbf{B}/Z e n_d \to 0$, and, Hall term will disappear in the induction Eq. (7). In molecular clouds generally $n_d \leq n_e$ and thus, Hall drift is always present.

To investigate the wave characteristics of the medium, we utilize Eqs. (5) $-$ (7) along with an isothermal equation of state $P = c_s^2 \rho$ where $c_s = \sqrt{k_B T m_n}$ is the sound speed of the medium. We assume an uniform background magnetic field with only z component, i.e $\mathbf{B} = (0, 0, B)$. For physical quantities depending upon z only, Eqs. (5) $-$ (7) become

$$\begin{aligned} \frac{\partial \rho}{\partial t} &= -\rho \frac{\partial \mathbf{u}_z}{\partial z} , \\ \frac{\partial V_1}{\partial t} &= -\mathbf{u}_z \frac{\partial V_1}{\partial z} + \frac{B}{4\pi\rho} \left(\frac{\partial}{\partial z} B_1 \right) , \\ \frac{\partial \mathbf{u}_z}{\partial t} &= -\frac{\partial}{\partial z} \frac{\mathbf{u}_z^2}{2} - \frac{1}{\rho} \frac{\partial}{\partial z} \left(P + \frac{B_1^2}{8\pi} \right) , \\ \frac{\partial B}{\partial t}_1 &= -\frac{\partial}{\partial z} \left(\mathbf{u}_z B_1 - V_1 B + \frac{icB}{4\pi\rho e n_d} \frac{\partial B}{\partial z}_1 \right) . \end{aligned} \tag{8}$$

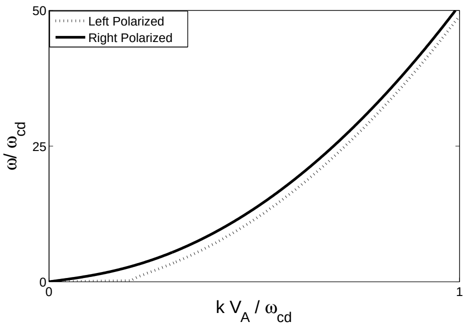
Here $V_1 = v_x + i v_y$ and $B_1 = B_x + i B_y$. Finite amplitude, circularly polarized Alfvén wave is an exact solution of Eq. (8)

$$B_1 = -B_0 \, exp(i\phi) , \quad V_1 = V_0 \, exp(i\phi) , \tag{9}$$

Where $V_0 = -(kB_0/\omega B)(1 \pm (\rho_n/\rho_g)(\omega/\omega_{cd})$ and $\phi = \omega t - kx$ is the phase of the wave. The following dispersion relation is satisfied by the phase of the circularly polarized Alfvén wave

$$\omega = \frac{1}{2} \left(\frac{\rho_n}{\rho_g} \right) \frac{\omega_A^2}{\omega_{cd}} \left[1 \pm \left(1 + 4 \left(\frac{\rho_g}{\rho_n} \right)^2 \left(\frac{\omega_{cd}}{\omega_A} \right)^2 \right)^{1/2} \right] . \tag{10}$$

Above dispersion relation, Eq. (10) describes the waves known in the literature as helicoidal or whistler waves. This dispersion relation is same as one derived previously by Wardle and Ng (1999; their Eq. 25). However, their derivation is more general in scope and non-ideal effects have been expressed in terms of conductivity tensor. The positive and negative sign inside the square bracket in (10) corresponds to the left and circularly polarized waves.



In the figure we plot the dispersion relation (10) for both left and right circularly polarized modes. The frequency and thus the phase and group velocity of the wave is dependent upon the handedness introduced by the Hall term in the dynamics.

The circularly polarized Alfvén waves undergoes parametric instability (Pandey & Vladimirov) and thus it is of interest to investigate the typical wavelength over which this mode can be excited. This has implications for the flux redistribution and hence for the star formation in the cloud cores. The molecular clouds span a large range of mean radii ($R \sim 1-100\,\text{pc}$), masses ($M \sim 10^2 10^6\,\text{M}_\odot$) and mean number density ($n_n \sim 10^2 - 10^3\,\text{cm}^{-3}$). The correlation of these quantities with the velocity dispersion σ is given by the line width-size-density relation (Solomon et al., 1987)

$$\sigma = 7.2 \times 10^4 \left(\frac{R}{\text{pc}}\right) \frac{\text{cm}}{\text{s}}, n_n = 2.3 \times 10^3 \left(\frac{R}{\text{pc}}\right)^{-1} \text{cm}^{-3}, \tag{11}$$

For typical cloud temperature $T \sim 10\,\text{K}$ the velocity dispersion is typically supersonic since sound speed $c_s \approx 0.2\,\text{km/s}$. The observed persistence of turbulent motion in molecular clouds could be due to the transverse Alfvén waves (Arons & Max, 1975). The whistler wave may affect the width of the molecular lines. To see that we note that group velocity of the whistler is $\sim (\rho_n/\rho_g)kV_A^2/\omega_{cd}$. Assuming $\rho_n/\rho_g \sim 0.01$ and $V_A \sim 10^4\,\text{cm/s}$ and $\omega_{cd} \sim 10^{-10}\,\text{s}^{-1}$ for $B \sim 10\,\mu\text{G}$ and $n_n \sim 10^5\,\text{cm}^{-3}$ and equating group velocity with the velocity Dispersion $0.3\,\text{cm/s}$, we get wavelength $\sim 0.1\,\text{pc}$ and wave period $\sim$ Myr. This time scale is shorter than the corresponding ambipolar time scale. Therefore, the flux transfer by the whistler in the molecular cloud may play vital role in the star formation.

References

[1] P. M. Solomon, A. R. Rivolo, J. Barrett and A. Yahil, ApJ, **319**, 730 (1987). [2] J. Arons & C. E. Max, ApJ, **196**, L77 (1975). [3] M. Wardle, C. Ng, MNRAS, **303**, 239 (1999). [4] B. P. Pandey & M. Wardle, preprint, **astroph/0608008** (2006). [5] B.P.Pandey & J. Vranjes, Phys. Plasmas, **14**,122106 (2006). [6] B.P.Pandey and S.V. Vladimirov, ApJ, (in press) (2007). [7] T.Ch. Mouschovias, K. Tassis & W.M. Kunz, ApJ, **646**, 1043 (2006). [8] E.G. Zweibel & K.Josafatsson, ApJ, **270**, 511 (1989). [9] R. Dewar, Phys. Fluids, **13**, 2710 (1970). [11] W. Pilipp, T. W. Hartquist, O. Havnes & G. E. Morfill, ApJ, **314**, 3411 (1987).