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The investigation of parametric instability in a collisional dusty plasma

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The presence of charged grains plays an important role in the astrophysical and space environment. For example, the spoke formation in the Saturns ring, the Jovian ring formation and formation of the protoplanetary disks are but a few examples where dust dynamics plays an important role. The planets are understood to have formed from a disk of gas and dust orbiting around the protostars.

The parametric instability of finite amplitude, circularly polarized Alfvén wave has been studied in the space and astrophysical plasmas for last four decades (Hollweg, 1974; Derby, 1978; Goldstein, 1978; Nariyuki & Hada, 2006). Such an investigation in a dusty medium is relatively recent phenomena (Hertzberg et al, 2003). It should be expected that the role of the plasma-dust collision may as well be important for the parametric instability. It is known that the collision between the plasma particles and the grain are responsible for some of the novel features in dusty plasma. As an example, the new collective behaviour is known to exist in such a plasma due to the charge fluctuations - an offshoot of collision (Vladimirov, 1994; Bhatt & Pandey, 1994).

The present work investigates the parametric instability of collisonal dusty plasma. By dusty plasmas a three component plasma consisting of electrons, ions and charged grains will be implied. The simplest description of dusty plasma is given in terms of continuity and momentum equations for respective species with a suitable closure model, viz., an equation of state. We shall define mass density of the bulk fluid as $\rho = \rho_e + \rho_i + \rho_d \approx \rho_d$. Then the bulk velocity $\mathbf{v} = (\rho_i \mathbf{v}_i + \rho_e \mathbf{v}_e + \rho_d \mathbf{v}_d)/\rho \approx \mathbf{v}_d$. The continuity and the momentum equations for the bulk fluid becomes

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{1}$$

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla P + \frac{\mathbf{J} \times \mathbf{B}}{c}. \tag{2}$$

Here $P = P_e + P_i + P_d$ is the total plasma pressure.

Making use of plasma quasi-neutrality condition, $n_e = n_i + Z n_d$, in the expression for the current density $\mathbf{J} = e\,(n_i \mathbf{v}_i - n_e \mathbf{v}_e)$, and assuming $\mathbf{v}_e \simeq \mathbf{v}_i$, and taking curl of the electron momentum equation and making use of the Maxwell's equation, in the $\omega_{ce} \gg \nu_{ed}$ limit, the induction equation can be written as

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times \left[(\mathbf{v}_d \times \mathbf{B}) - \left(\frac{\mathbf{J}\times\mathbf{B}}{Zen_d} \right) \right] . \tag{3}$$

One can investigate the dusty plasma dynamics with the help of Eqs. (1), (2) and (3) along with an isothermal equation of state. In what follows, an uniform background magnetic field $\mathbf{B} = (0,0,B)$ is assumed along the z direction. It will be assumed that all physical quantities depend on z only. Then an exact solution of the resulting equations is finite amplitude circularly polarized Alfvén wave

$$\begin{aligned} B_x = A_0 \cos\phi \, , \; B_y = A_0 \sin\phi \, , \\ V_x = V_0 \cos\phi \, , \; V_y = V_0 \sin\phi \, . \end{aligned} \tag{4}$$

Here $\phi = k_0 z - \omega_0 t$. The wavenumber k_0 and frequency ω_0 of the pump wave are related by the following dispersion relation

$$\omega_0^2 = k_0^2 V_A^2 \left(1 \pm \frac{\omega_0}{\omega_{cd}} \right) , \tag{5}$$

where $V_A^2 = B^2/\mu_0 \rho$ is the Alfvén speed and $\omega_{cd} = ZeB/m_d$ is the dust-cyclotron frequency. The amplitudes A_0 and V_0 of the pump waves are related $V_0/A_0 = -V_A^2 k_0/B\omega_0$. After linearizing Eqs. (1), (2) and (3) with $\delta B_z = 0$, and defining

$$\begin{aligned} \delta B_+ = \delta B_x \cos\phi + \delta B_y \sin\phi \, , \; \delta B_- = \delta B_x \sin\phi - \delta B_y \cos\phi \, , \\ \delta v_+ = \delta v_x \cos\phi + \delta v_y \sin\phi \, , \; \delta v_- = \delta v_x \sin\phi - \delta v_y \cos\phi \, , \end{aligned} \tag{6}$$

the linearized equations can be reduced to following set of equations with constant coefficient.

$$\begin{aligned}
\frac{\partial \delta\rho}{\partial t} + \rho \frac{\partial \delta v_z}{\partial z} &= 0 \, , \\
\frac{\partial \delta v_+}{\partial t} - \omega_0 \delta v_- &= \frac{B}{\mu_0 \rho} \left(\frac{\partial \delta B_+}{\partial z} + k_0 \delta B_- \right) , \\
\frac{\partial \delta v_-}{\partial t} + \omega_0 \delta v_+ - k_0 V_0 \delta v_z &= \frac{B}{\mu_0 \rho} \left(\frac{\partial \delta B_-}{\partial z} - k_0 \delta B_+ \right) + k_0 \frac{BA_0}{\mu_0 \rho^2} \delta\rho \, , \\
\frac{\partial \delta v_z}{\partial t} &= -\frac{C_s^2}{\rho} \frac{\partial \delta\rho}{\partial z} - \frac{A_0}{\mu_0 \rho} \frac{\partial \delta B_+}{\partial z} \, , \\
\frac{\partial \delta B_+}{\partial t} - \omega_0 \delta B_- = B \left(\frac{\partial \delta v_+}{\partial z} + k_0 \delta v_- \right) &- \alpha \left(\frac{\partial^2 \delta B_-}{\partial z^2} - 2k_0 \frac{\partial \delta B_+}{\partial z} + k_0^2 \delta B_- \right) \\
& - A_0 \frac{\partial \delta v_z}{\partial z} - \frac{\alpha k_0 A_0}{\rho} \frac{\partial \delta\rho}{\partial z} \, , \\
\frac{\partial \delta B_-}{\partial t} + \omega_0 \delta B_+ = B \left(\frac{\partial \delta v_-}{\partial z} - k_0 \delta v_+ \right) &+ \alpha \left(\frac{\partial^2 \delta B_+}{\partial z^2} + 2k_0 \frac{\partial \delta B_-}{\partial z} - k_0^2 \delta B_+ \right) \\
& + k_0 A_0 \delta v_z \, ,
\end{aligned} \tag{7}$$

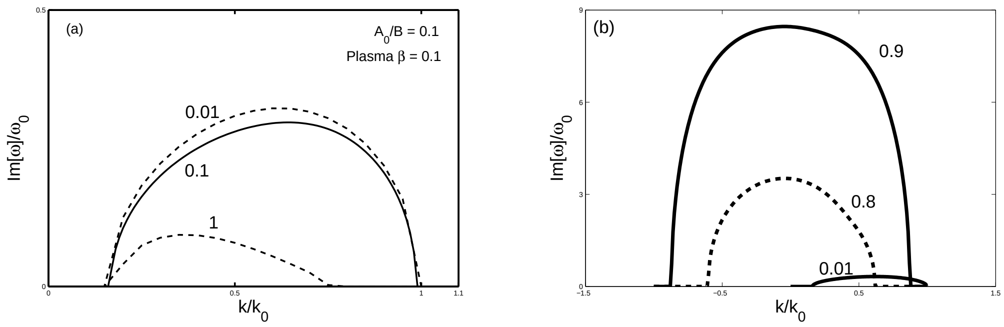

Figure 1: The growth rate of left-circularly polarized (Fig. 1(a)) and right-circularly polarized (Fig. 1(b)) is shown in the figure.

Here $\alpha = B/(\mu_0 Z e n_d)$. Fourier analysing the spatial and temporal dependence of the fluctuations as $\sim \exp(i\omega t - ikz)$ and denoting $\omega = \omega/\omega_0$, $k = k/k_0$, $F_\pm = 1/(1 \pm \omega/\omega_{cd})$, $\beta = C_s^2/V_A^2$ and $C_s^2 = k^2 \beta F$ following 8^{th} order dispersion relation is derived from Eq. (7).

$$\omega^8 + a_7\omega^7 + \ldots + a_1\omega + a_0 = 0. \tag{8}$$

The transition from stability to instability will proceed through $\omega = 0$. In the $\omega \to 0$ limit,

$$\omega = -\frac{a_0}{a_1} \tag{9}$$

In the long wavelength limit retaining only $\sim O(k)$, $O(k^2)$ terms in the coefficients a_1 and a_0 one may write

$$\frac{a_1}{kF_\pm} = -2\frac{A_0^2}{B^2}\left[1 - F_\pm\left(1 + \frac{\omega_0}{\omega_{cd}}\right)\right] + k\frac{A_0^2}{B^2}(1 + F_\pm), \tag{10}$$

$$\frac{a_0}{kF_\pm} = \left[\beta\left(1 + F_\pm\frac{\omega_0}{\omega_{cd}}\right) - F_\pm\frac{A_0^2}{B^2}\right]\left[1 - F_\pm\left(1 + \frac{\omega_0}{\omega_{cd}}\right)\right] \tag{11}$$

Recall that for the left-circularly polarized (LCP) pump waves, $F_+ = 1/(1 + \omega_0/\omega_{cd})$ and hence, from Eq. (11), $a_0 = 0$. Thus in view of Eq. (9), one should anticipate that the instability will disappear in the vicinity of $k = 0$.

The dependence of the growth rate on the ratio of the left circularly polarized pump wave to the dust-cyclotron frequency is shown in Fig. 1(a) for the corresponding physical parameters in the box. In the vicinity of $k = 0$, wave does not grow. This is caused by the dissipation of the long wavelength fluctuations by the plasma-dust collisions. The increase in ω_0/ω_{cd} implies the increasing importance of the Hall term ($\mathbf{J} \times \mathbf{B}$), which appears due to the relative drift between the plasma and the dust. This drift is caused entirely by the collisonal momentum exchange. Therefore, the increase in ω_0/ω_{cd} implies the increased dissipation of the free energy. Hence with the increasing ω_0/ω_{cd}, one would expect a decrease in the growth rate. With decreasing ω_0/ω_{cd}, the growth rate is due to non-dispersive Alfvén pump and will correspond to a non-dissipative, ideal regime. Hence one sees the saturation of the growth rate with decreasing ω_0/ω_{cd}.

For the right-circularly polarized (RCP) pump waves, when $F_- = 1/(1-\omega_0/\omega_{cd})$, the growth rate can be written as

$$Im[\omega] = \frac{F_-}{2}\frac{A_0^2}{B^2}\left(\beta - \frac{A_0^2}{B^2}\right). \tag{12}$$

As is clear from above Eq. (12), the growth rate is inversely proportional to the factor $(1-\omega_0/\omega_{cd})$. This implies that near $\omega_0 \simeq \omega_{cd}$, when pump is operating near the dust-cyclotron frequency, the instability can grow resonantly.

The dependence of the growth rate on the amplitude of the right handed circularly polarized pump wave is shown in the Fig. 1(b). When $\omega_0/\omega_{cd} = 0.9$, the growth rate becomes very large. The relative drift between the plasma and the dust causes a Hall field over the dust-cyclotron time. If the Alfvén wave propagation time ω_0^{-1} becomes comparable to the dust-cyclotron time, the energy is freely fed to the fluctuation by the pump to the gyrating dust particles. The resonant driving is indirectly related to the dust-plasma collisions. Therefore, the growth of the parametric instability is quite different for the left and right-circularly polarized pump. Whereas for the left-circularly polarized waves, the instability does not exist in the neighbourhood of $k = 0$, for the right-circularly polarized pump, the instability may become large near $k = 0$. Clearly, for the right circularly polarized pump, unbounded growth of the instability is not possible. The linear approximation in which the dispersion relation (8) has been derived breaks down for any dependent physical variable f once $\delta f \leq f$.

To summarize, the collision of the dust with other plasma species results in the dispersive nature of the pump waves. By using a mathematical transformation, the linearized equations with periodic coefficients can reduced to equations with constant coefficient permitting the normal mode analysis at all wavelengths. The parametric instability is sensitive to the change in the ratio of the pump to the cyclotron frequencies. However, the growth rate is not sensitive to the plasma β. Furthermore, the instability can become an order of magnitude larger for the right hand circularly polarized pump particularly near the resonance, i.e. when $\omega_0 \simeq \omega_{cd}$.

References

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