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The parametric instability in weakly ionized molecular clouds

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Molecular clouds are thought to be the nursery of star formation (Lada, 1985). These clouds generally consists of very small $(2-3\%)$ fraction of ionized matter and yet, depending upon the level of plasma-neutral coupling, the magnetic field can play an important role in the dynamics of such a cloud. The parametric instability of a finite amplitude circularly polarized Alfvén wave have been studied for last several decades (Hollweg 1974; Derby 1978; Goldstein 1978;). Recently, this problem has been studied in the single fluid MHD framework for a self-gravitating molecular cloud (Fukuda & Hanawa 1999). The investigation of parametric instability in a dusty medium is relatively recent phenomena (Hertzberg et al. 2003). However, the dynamics of the neutrals has been left out, making it inapplicable to the interstellar medium. It is known that collisonal processes in weakly ionized plasma can strongly influence the nonlinear ambipolar diffusion and the ensuing current sheet formation (Cramer & Vladimirov 1994). The role of the plasma-dust collision may as well be important for the parametric instability. For example, collisions between the plasma particles and grains are responsible for some of the novel features in dusty plasma (Bhatt & Pandey 1994). Therefore, it is desirable to investigate the propagation of the large amplitude Alfvén wave in weakly ionized collisonal plasma. We note that in dark molecular clouds, the momentum of the bulk fluid is carried by the neutrals and the current is carried by the ionized component.

One of the difficulties with the investigation of the parametric instability of the circularly polarized Alfvén wave is associated with the appearance of the periodic coefficients in the linearized MHD equations. However, a recently proposed linear transformation (Ruderman & Simpson, 2003) reduces the periodic coefficients in the linearized MHD equations to the constant coefficient. In the present work such a transformation is applied to the dispersive Alfvén modes.

We shall assume a weakly ionized molecular cloud consisting of plasma particles, i.e. electrons and ions, charged grains and neutral particles. We shall define mass density of the bulk fluid as $\rho \approx \rho_n$. Then the bulk velocity $\mathbf{v} \approx \mathbf{v}_n$. The continuity and momentum equations for the bulk fluid are

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\, \mathbf{v}) = 0\,. \tag{1}$$

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla P + \frac{\mathbf{J}\times\mathbf{B}}{c}\,. \tag{2}$$

Making use of plasma quasi-neutrality condition, $n_e = n_i + Z\,n_d$, in the expression for the current density $\mathbf{J} = e\,(n_i\,\mathbf{v}_i - n_e\,\mathbf{v}_e)$, and assuming $\mathbf{v}_e \simeq \mathbf{v}_i$,, taking curl of the electron momentum equation and making use of the Maxwell's equation, in the $\omega_{ce} \gg \nu_{ed}$ limit, the induction equation can be written as

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times\left[(\mathbf{v}_d\times\mathbf{B}) - \left(\frac{\mathbf{J}\times\mathbf{B}}{Zen_d}\right)\right]. \tag{3}$$

We investigate the parametric instability with the help of Eqs. (1), (2) and (3) and $P = c_s^2\,\rho$. In what follows, an uniform background magnetic field $\mathbf{B} = (0,0,B)$ is assumed along with z dependence for all physical quantities. Then an exact solution of the resulting equations is finite amplitude circularly polarized Alfvén wave $(B_x,V_x) = (A_0,V_0)\cos\phi$,$(B_y,V_y) = (A_0,V_0)\sin\phi$, with $\phi = k_0\,z - \omega_0\,t$. The wavenumber k_0 and frequency ω_0 of the pump wave are related by the dispersion relation $\omega_0^2 = k_0^2 V_A^2\left(1 \pm \frac{\omega_0}{\omega_{cd}}\right)$, where $V_A^2 = B^2/\mu_0\,\rho$ is the Alfvén speed and $\omega_{cd} = Z\,eB/m_d$ is the dust-cyclotron frequency. The amplitudes A_0 and V_0 of the pump waves are related $V_0/A_0 = -V_A^2\,k_0/B\,\omega_0$. We assume now that the steady-state background consists of the unperturbed as well as the circularly polarized pump waves, i.e. $\mathbf{B} = (B_x(z), B_y(z), B)$ and $\mathbf{v} = (V_x(z), V_y(z), 0)$ with constant density. After linearizing Eqs. (1), (2) and (3) with $\delta B_z = 0$, and defining $\delta B_+ = \delta B_x\cos\phi + \delta B_y\sin\phi$, $\delta B_- = \delta B_x\sin\phi - \delta B_y\cos\phi$, $\delta v_+ = \delta v_x\cos\phi + \delta v_y\sin\phi$, $\delta v_- = \delta v_x\sin\phi - \delta v_y\cos\phi$, the linearized equations can be reduced to following set of equations with constant coefficient.

$$
\begin{aligned}
\frac{\partial\delta\rho}{\partial t} + \rho\,\frac{\partial\delta v_z}{\partial z} &= 0\,,\\
\frac{\partial\delta v_+}{\partial t} - \omega_0\,\delta v_- &= \frac{B}{\mu_0\,\rho}\left(\frac{\partial\delta B_+}{\partial z} + k_0\,\delta B_-\right),\\
\frac{\partial\delta v_-}{\partial t} + \omega_0\,\delta v_+ - k_0\,V_0\,\delta v_z &= \frac{B}{\mu_0\,\rho}\left(\frac{\partial\delta B_-}{\partial z} - k_0\,\delta B_+\right) + k_0\,\frac{BA_0}{\mu_0\,\rho^2}\,\delta\rho\,,\\
\frac{\partial\delta v_z}{\partial t} &= -\frac{C_s^2}{\rho}\,\frac{\partial\delta\rho}{\partial z} - \frac{A_0}{\mu_0\,\rho}\,\frac{\partial\delta B_+}{\partial z}\,,\\
\frac{\partial\delta B_+}{\partial t} - \omega_0\,\delta B_- &= B\left(\frac{\partial\delta v_+}{\partial z} + k_0\,\delta v_-\right) - \alpha\left(\frac{\partial^2\delta B_-}{\partial z^2} - 2\,k_0\,\frac{\partial\delta B_+}{\partial z} + k_0^2\,\delta B_-\right)\\
&\quad - A_0\,\frac{\partial\delta v_z}{\partial z} - \frac{\alpha\,k_0\,A_0}{\rho}\,\frac{\partial\delta\rho}{\partial z}\,,\\
\frac{\partial\delta B_-}{\partial t} + \omega_0\,\delta B_+ &= B\left(\frac{\partial\delta v_-}{\partial z} - k_0\,\delta v_+\right) + \alpha\left(\frac{\partial^2\delta B_+}{\partial z^2} + 2\,k_0\,\frac{\partial\delta B_-}{\partial z} - k_0^2\,\delta B_+\right)\\
&\quad + k_0\,A_0\,\delta v_z\,,
\end{aligned}
\tag{4}
$$

Here $\alpha = B/(\mu_0\,Z\,e\,n_d)$. Fourier analysing the spatial and temporal dependence of the fluctuations as $\sim\exp(i\,\omega\,t - i\,k\,z)$ and denoting $\omega = \omega/\omega_0$, $k = k/k_0$, $F_\pm = 1/(1\pm\omega/\omega_{cd})$, $\beta = C_s^2/V_A^2$

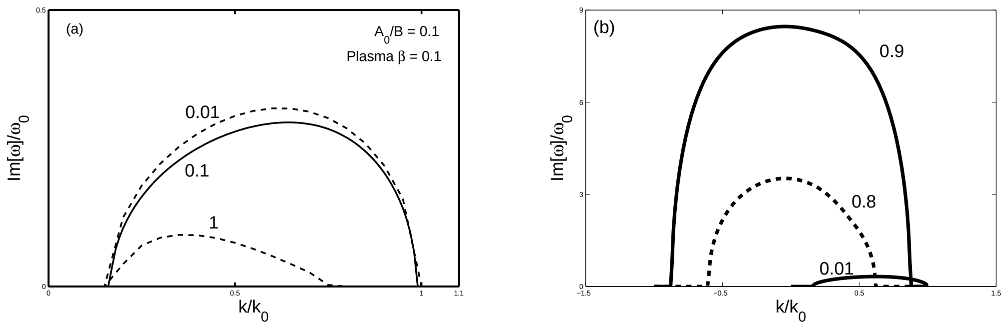

Figure 1: The growth rate of left-circularly polarized (Fig. 1(a)) and right-circularly polarized (Fig. 1(b)) is shown in the figure.

and $C_s^2 = k^2 \beta F$ following 8^{th} order dispersion relation is derived from Eq. (4).

$$\omega^8 + a_7 \omega^7 + \ldots + a_1 \omega + a_0 = 0\,. \tag{5}$$

The transition from stability to instability will proceed through $\omega = 0$. In the $\omega \to 0$ limit,

$$\omega = -\frac{a_0}{a_1} \tag{6}$$

In the long wavelength limit retaining only $\sim O(k), O(k^2)$ terms in the coefficients a_1 and a_0 one may write

$$\frac{a_1}{kF_\pm} = -2\frac{A_0^2}{B^2}\left[1 - F_\pm\left(1 + \frac{\omega_0}{\omega_{cd}}\right)\right] + k\frac{A_0^2}{B^2}\left(1 + F_\pm\right), \tag{7}$$

$$\frac{a_0}{kF_\pm} = \left[\beta\left(1 + F_\pm\frac{\omega_0}{\omega_{cd}}\right) - F_\pm\frac{A_0^2}{B^2}\right]\left[1 - F_\pm\left(1 + \frac{\omega_0}{\omega_{cd}}\right)\right]. \tag{8}$$

Recall that for the left-circularly polarized pump waves, $F_+ = 1/(1 + \omega_0/\omega_{cd})$ and hence, from Eq. (8), $a_0 = 0$. Thus in view of Eq. (6), one should anticipate that the instability will disappear in the vicinity of $k = 0$.

The dependence of the growth rate on the ratio of the left circularly polarized pump wave to the dust-cyclotron frequency (Fig. 1(a)) shows that in the vicinity of $k = 0$, wave does not grow. This is caused by the dissipation of the long wavelength fluctuations by the plasma-dust collisions. The increase in ω_0/ω_{cd} implies the increasing importance of the Hall term ($\mathbf{J} \times \mathbf{B}$), which appears due to the relative drift between the plasma and the dust. This drift is caused entirely by the collisonal momentum exchange. Therefore, the increase in ω_0/ω_{cd} implies the increased dissipation of the free energy. Hence with the increasing ω_0/ω_{cd}, one would expect a decrease in the growth rate. With decreasing ω_0/ω_{cd}, the growth rate is due to nondispersive Alfvén pump and will correspond to a non-dissipative, ideal regime. Hence one sees the saturation of the growth rate with decreasing ω_0/ω_{cd}.

For the right-circularly polarized pump waves, when $F_{-} = 1/(1-\omega_0/\omega_{cd})$, the growth rate can be written as

$$Im[\omega] = \frac{F_{-}}{2}\frac{A_0^2}{B^2}\left(\beta - \frac{A_0^2}{B^2}\right). \tag{9}$$

As is clear from above Eq. (9), the growth rate is inversely proportional to the factor $(1-\omega_0/\omega_{cd})$. This implies that near $\omega_0 \simeq \omega_{cd}$, when pump is operating near the dust-cyclotron frequency, the instability can grow resonantly.

The dependence of the growth rate on the amplitude of the right handed circularly polarized pump wave is shown in the Fig. 1(b). When $\omega_0/\omega_{cd} = 0.9$, the growth rate becomes very large. The relative drift between the plasma and the dust causes a Hall field over the dust-cyclotron time. The resonant driving is indirectly related to the dust-plasma collisions. Therefore, the growth of the parametric instability is quite different for the left and right-circularly polarized pump. Whereas for the left-circularly polarized waves, the instability does not exist in the neighborhood of $k = 0$, for the right-circularly polarized pump, the instability may become large near $k = 0$.

The observations of the molecular cloud suggest a magnetically threaded supersonically turbulent environment. Thus taking the upper value of the pump wave frequency (Folini et al. 2004), $\omega_0 \simeq 10^{-4}\,\mathrm{yr}^{-1}$, the growth rate of the left-circularly polarized wave $0.3\,\omega_0$ suggests that the parametric instability of the Alfvén wave could be relevant to the onset of turbulence. The Alfvén wavelength lies in the range of $0.07-0.35$pc (Folini et al. 2004) and since the maximum growth rate occurs at $k/k_0 = 0.5$ (Fig. 1(a)), the parametric instability of the left-circularly polarized mode will operate between $0.1-0.7$ pc.

To check whether the resonance condition can prevail in the molecular clouds, assuming a typical magnetic field $B \sim 10\mu$G, and taking interstellar grain mass 10^{-15}g, one gets $\omega_{cd} \sim 10^{-4}\,\mathrm{yr}^{-1}$ which is comparable to the pump frequency. Therefore, the right circularly polarized wave can resonantly excite this instability. Furthermore, it can operate on very long wavelengths.

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