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# Using mutual information to quantify spatial correlation between simultaneous spacecraft measurements of solar wind plasma turbulence

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The analysis of measurements of solar wind plasma turbulence raises widely applicable questions. Given one or more datasets, in practical terms how does one distinguish: a nonlinear structure of external origin (here, solar), from an internally evolved turbulent structure; a wave packet representing a superposition of harmonic waves (here, Alfvénic), from a nonlinear structure; or turbulence evolving *in situ*, from turbulence that is convected past? In the last ten years multiple point measurements of the highly supersonic and super-Alfvénic solar wind flow have become available. These enable direct analysis of the spatial characteristics of turbulent structures in the solar wind plasma. By calculating the correlation between simultaneous measurements made by different spacecraft at known separations, the scale and physical nature of these structures can be inferred.

Until now, solar wind correlation has been calculated using linear measures [1]. Here we present the first multiple spacecraft mutual information measurements of the solar wind: this technique [2] enables the detection of nonlinear correlations and clustering that are not easily detected by linear or Fourier-based approaches [3].

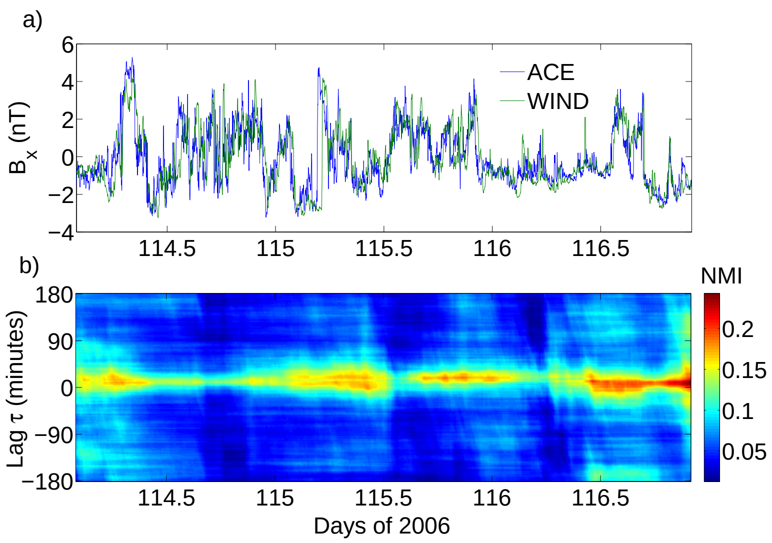


Figure 1: *a) Magnetic field data in the x direction (GSE coordinates) taken from the ACE (blue) and WIND (green) spacecraft in 2006. b) Timeseries of lagged mutual information measurements for this data.*

In [2], mutual information was calculated between WIND and terrestrial magnetometer data, whereas here both datasets are from within the solar wind. We use data from the ACE and WIND spacecraft to calculate the mutual information over large scales. The data is from 1998, when ACE is arriving at, and WIND is leaving, the Earth-Sun libration point; and 2005 and 2006, when both ACE and WIND are around this point. These times are among the closest to solar minimum available when these spacecraft are operational, which minimises the effect of coherent structures of solar origin on the measurements of the turbulent scale size. We also choose “quiet” periods, where fluctuations are mostly below five standard deviations in size. A running 24 hour average is subtracted from the data to remove variations on the largest and slowest scales.

Mutual information $I(A;B)$ [4] is a simple relation between the entropies $H$ of two signals $A$, $B$ whose values are subdivided into alphabets $\{a_i\}$, $\{b_i\}$ with measured probabilities of occurrence $P(a_i)$, $P(b_i)$ and joint probability $P(a_i;b_i)$:

$$H(A) = -\sum_{a_i \in A} P(a_i)\log_2(P(a_i)); \quad H(A;B) = -\sum_{a_i \in A}\sum_{b_i \in B} P(a_i;b_i)\log_2(P(a_i;b_i)) \quad (1)$$

$$I(A;B) = H(A) + H(B) - H(A;B); \quad I'(A;B) = \frac{H(A)+H(B)}{H(A;B)} - 1 \quad (2)$$

The mutual information is normalised (NMI, $I'$) to a value between zero and one by dividing by the joint entropy $H(A;B)$; this quantifies the information shared between signals $A$ and $B$ as a fraction of their joint entropy. Solar wind variables, such as magnetic field, are continuous. We therefore use a discretisation method [5] to construct an alphabet of values, $a_i$, for the measurements, and calculate the probabilities $P(a_i)$. The standard deviation $\sigma$ of the whole data set over all time periods (1998, 2005 and 2006) for a given variable is the basic unit of this discretisation: values within $\pm 5\sigma$ are accepted and put into 20 evenly spaced bins. From this a time series of mutual information between ACE and WIND measurements at different time lags $\tau$ is calculated as shown in Eq. (3) and Fig. 1, where 480 data points at 2 minute resolution are used for each measurement of mutual information:

$$I'\left(B_x^A(t); B_x^W(t+\tau)\right) = \frac{H\left(B_x^A(t)\right) + H\left(B_x^W(t+\tau)\right)}{H\left(B_x^A(t); B_x^W(t+\tau)\right)} - 1 \quad (3)$$

Figure 1 shows that mutual information peaks for correlated structures at a time lag $\tau \approx 660s$. This equates to the spacecraft separation $274000km$ in the $x$-direction (GSE coordinates), divided by the solar wind speed $V_{sw} \approx 400km/s$, implying that structures seen in the mutual information are convecting with the solar wind speed. By calculating MI over periods where the spacecraft are at different separations we measure its decline with distance. Mutual information is calculated at $\tau = 0$, giving a measure of two point correlation, and plotted against distance. Figure 2 shows that this yields a clear difference between the ACE-WIND correlation in the magnetic field magnitude and in the ion density.

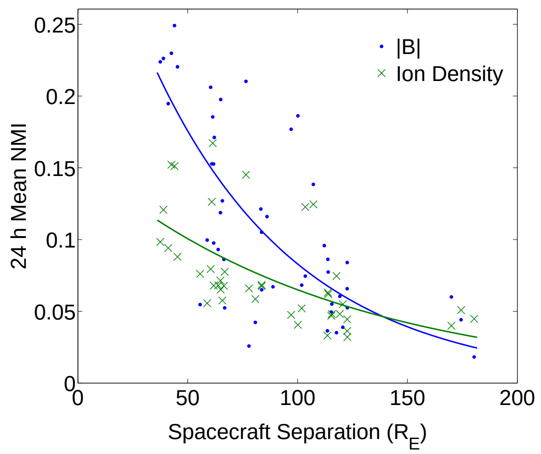


Figure 2: *24 hour mean NMI between ACE and WIND for selected periods in 1998, 2005 and 2006, magnetic field strength* $|B|$ *(blue dots) and ion density* $\rho$ *(green crosses). The lines plotted are exponential fits to the data.*

This implies that the data cannot purely represent incompressible wave turbulence, which would create no correlation in these variables. Conversely if the data simply reflected convected nonlinear structures of solar origin, $|B|$ should show similar behaviour to ion density. To quantify the difference evident in Fig. 2, following [1] we attempt an exponential fit $y = a\exp(x/\lambda)$ for comparison with linear correlation measurements of the turbulent correlation length. The values of $\lambda$ measured here are $\lambda_{|B|} = 70R_E$ and $\lambda_\rho = 115R_E$, which are smaller than obtained earlier [1]. The magnetic field components behave in a similar fashion to the magnitude, except $B_x$, which is not as correlated at smaller distances. The velocity components also display lower correlation, but the calculated Elsässer variables resemble the magnetic field measurements.

The anisotropy of solar wind turbulence during low solar activity is also investigated using this data. The spacecraft separation vector is broken into field parallel ($r_\parallel$) and field perpendicular ($r_\perp$) components, and the NMI calculated for given values of ($r_\parallel$) and ($r_\perp$) is plotted in Fig. 3. There is slight anisotropy, especially in the y and z components. It is also useful to calculate the mutual information between different measurements from the same spacecraft. Figure 4 shows the mutual information calculated between $B_x$ and $v_x$ data from ACE in 2006. Strong correlation measured by mutual information identifies episodes where the signals reflect underlying harmonic Alfvén wave properties.

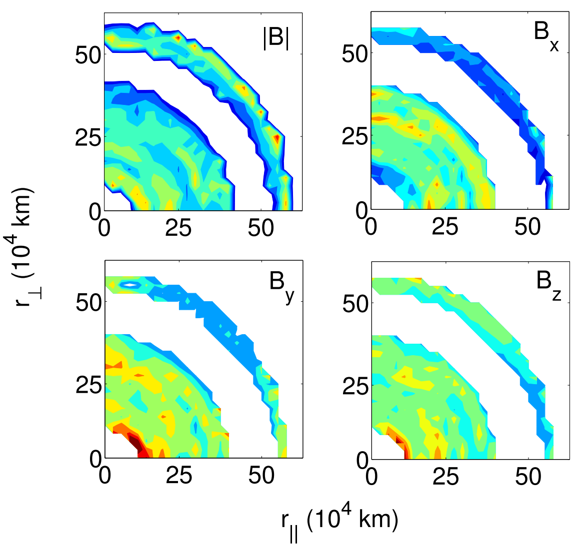


Figure 3: *Anisotropy measurements using NMI for* $|B|$ *and the components of the magnetic field for selected periods in 1998, 2005 and 2006.*

We expect that further analysis of this data will provide insight into why the nonlinear spatial correlation properties of the density and the magnetic field do not decline with distance at the same rate. This bears on the fundamental question of the extent to which structures in the solar wind plasma are of solar origin, or are really turbulent.

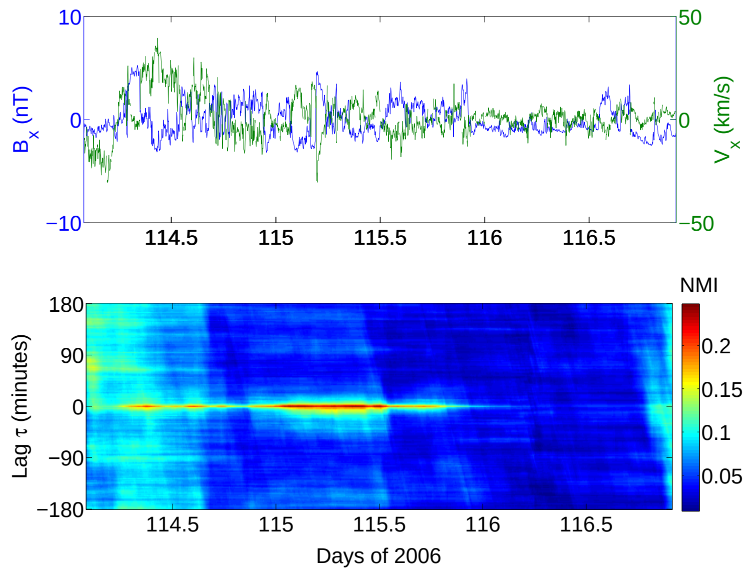


Figure 4: *a) Magnetic field (blue) and velocity data (green) in the x direction (GSE coordinates) taken from the ACE spacecraft in 2006. b) Timeseries of lagged mutual information measurements for this data.*

Insofar as the structures reflect convected turbulence, we need to understand whether the turbulence is in steady state, or evolving towards it; and over what range of lengthscales and timescales. The phase of the solar cycle is also relevant: if structures of solar origin increase the correlation length, this should vary with the solar cycle. This is important for future studies, as it is difficult to find long time periods in the ecliptic which do not contain such structures.

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