

Assessment of confinement scaling models for W7-AS high- β data

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Introduction

System studies require testable confinement predictions for reactor relevant plasma β . Parameter scans for tokamak data are known to reveal different β scalings if compared to global regressions. Hence, first principle based physical transport models should be tested to identify significant signatures in confinement, e.g., due to electromagnetic effects.

The stellarator Wendelstein 7-AS was operated at different plasma- β up to reactor relevant values of $\beta=3.4\%$. The full set of confinement data, recently available from the International Stellarator Confinement Data Base (ISCDB) [1], allows one to perform model comparison studies on both low- and high- β data. Previous work already demonstrated that the collisional low- β Connor-Taylor model was found to be consistent with the experimental performance of W7-AS in low- β scenarios [2]. With the existence of high- β data it is possible to redo the analysis in order to check the Connor-Taylor behavior for the high- β data set as well.

Connor-Taylor Models

Global scaling laws are based on the assumption that the quantity to be scaled on – in this paper the plasma energy W – depends on a set of control variables, linear combinations of which are mapped on physics parameters. Here, the electron density n , effective minor radius a , toroidal magnetic field B and absorbed heating power P are employed. For the W7-AS subsets the rotational transform t is not considered as a scaling quantity but as a reference value to classify the data. From the Connor-Taylor (CT) invariance principle [3] constraints are imposed on the control variables compiling them to up to three terms with scaling exponents ξ_1 , ξ_2 and ξ_3 . The terms are assigned to the so-called collisionless/collisional low-/high- β model according to Tab. 1. Additionally, two fluid models are examined.

$$W^{theo} = na^4 B^2 \sum_{k=1}^E c_k \left(\frac{P}{na^4 B^3} \right)^{\xi_1} \left(\frac{a^3 B^4}{n} \right)^{\xi_2} \left(\frac{1}{na^2} \right)^{\xi_3} . \quad (1)$$

Note, that beyond most scaling approaches this is an expansion into a series of power law terms, however, still within the invariance approach of Connor and Taylor. Since an ever increasing expansion order E would finally end in a total description of the data (where as many

parameters as data points exist) over-fitting needs to be prevented. This principle, called Occam's razor, is automatically obeyed by Bayesian probability theory, a data analysis method employed throughout this work. The optimum expansion order E_{max} is part of the outcome inferred by model comparison techniques [2].

Choice of data sets

The data under consideration are drawn from the W7-AS high- β subset of ISCDDB. They are shown in Fig. 1 as a plot over those two Connor-Taylor variables which are the determining factors for collisionality and high- β character. As can be seen in Fig. 1 the data set is widely spread over the space spanned by the two vari-

ables. To investigate the conditioning of the data in the CT-variables is of special importance since any data analysis method can distinguish only between those models which show a sufficient variation in the respective variables. Moreover, the choice of a data set due to the subjective notion, that the data form a cloud in a graphical depiction (e.g. a scaling plot) could be misleading: the reason for forming a

'cloud' may be traced back to experimental conditions rather than to physical considerations. Therefore, in order to test the robustness of the model comparison and to check dependency on τ , a subgroup of the high- β data with

$0.45 < \tau < 0.49$ was generated (see Fig. 1). Contrarily to the low- β case where W varies between distinct rational numbers of τ by a factor of two [4], in the high- β case the dependence on τ is much smoother and passes through a broad maximum with variation of less than 10% [5].

Discussion of the results

The result of the model comparison is shown in table 1. The collisional high- β model has the highest probability for both data sets. This finding is different to the low- β case where the collisional low- β model is the most probable one [2]. The notion of a collisional model

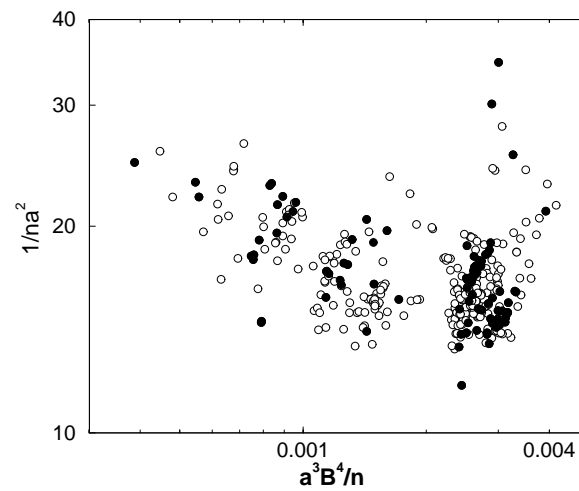


Figure 1: W7-AS high- β data in the space of the Connor-Taylor variables: $a^3 B^4 / n$ is assigned to a collisional model, $1/na^2$ to high- β . Open circles: full set of $N=380$ data. Filled circles: subset of $N=96$ data for $0.45 < \tau < 0.49$.

CT-model M_j	ξ_1	ξ_2	ξ_3	$N=380$		$N=96$	
				E_{max}	$p(M_j)$	E_{max}	$p(M_j)$
Collisionless low- β	x	0	0	3	10^{-52}	3	10^{-24}
Collisional low- β	x	y	0	3	10^{-43}	4	10^{-19}
Collisionless high- β	x	0	z	2	10^{-36}	2	10^{-23}
Collisional high- β	x	y	z	3	1	3	1
Ideal fluid	x	0	$1-x/2$	3	10^{-45}	2	10^{-20}
Resistive fluid	x	y	$1-x/2+y$	3	10^{-41}	3	10^{-15}

Table 1: Results of the assessment of W7-AS high- β data with respect to different Connor-Taylor models M_j . The ξ_i are the exponents in Eq. (1). E_{max} , i.e. the expansion order with highest probability and the model probabilities $p(M_j|\vec{W}^{exp}, \vec{\sigma}, I)$ are shown for the complete set of all W7-AS high- β data ($N=380$) and a subset ($N=96$).

coincides with the expectation that the collisionalities of the ions are of importance even for the low- β case since they are still in the plateau regime. The two fluid models appear to describe the data much less significantly. Additional columns in table 1 show the expansion order E_{max} with the largest contribution. For both data sets, $E_{max}=3$ evidently demonstrates the deviation of the present approach from a simple power law behavior. Note, that model probabilities are actually a sum over all relevant expansion orders. Eventually, such higher expansion orders facilitate the description of density or power saturation effects which are not described by a single term scaling law [6, 7].

For a global validation, Fig. 2 shows a scatter plot for the experimentally obtained plasma- β against predictions from the collisional and collisionless high- β model. Indicating agreement, the values are distributed along the diagonal. The error-bars are shown not only for the experimental data but also for the prediction. They are a measure for the importance of a single value within the model and the data-set under consideration: large differences of the experimental and predicted error indicate either unreliable errors in variables or failure of the respective physical model.

Deviations from the diagonal exceeding the error bars indicate the presence of additional physical phenomena in the plasma not covered by the scaling law approach within the CT-models. Among those are: (i) the wall condition and recycling behavior, (ii) the impurity content, (iii) the speed of the density ramp (gas puff), (iv) the plasma ion species (H,D), (v) the NBI deposition and heating efficiency. Altogether they can lead to a larger uncertainty as indicated.

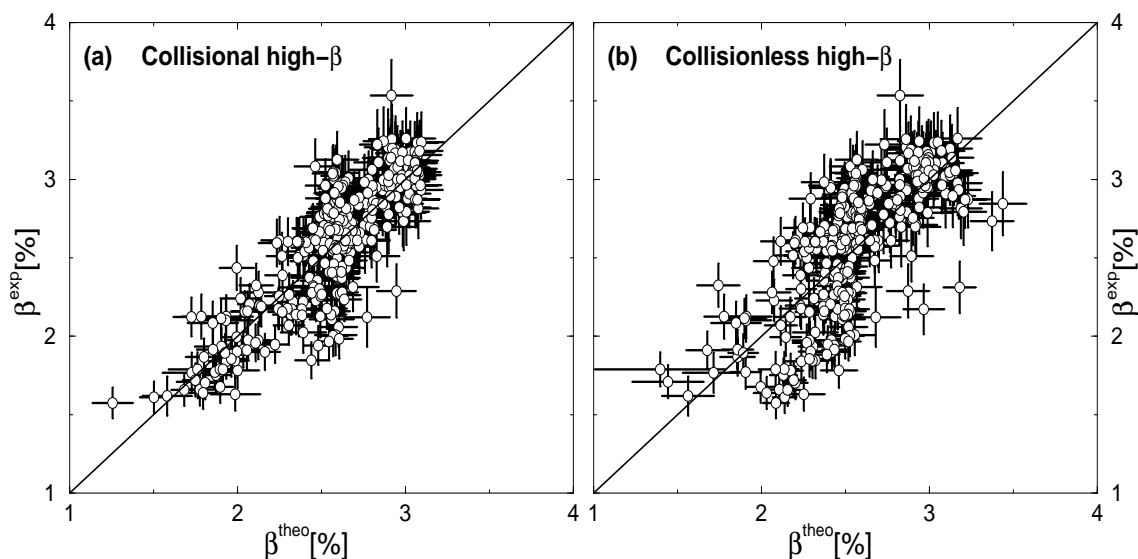


Figure 2: Experimental versus predicted plasma- β for (a) the most probable model (collisional high- β) and (b) the second most probable model (collisionless high- β).

Moreover, in order to obtain high plasma- β the discharges are close to operational limits (of density and of equilibrium β). In order to be still representative, the data were selected to be stationary, i.e. variations of less than 10% in all quantities over $5 \tau_E$.

Finally, the comparison in Fig. 2 between the best (collisional high- β) and the next-best model (collisionless high- β) shows that, as expected, the latter model has a broader, less clear distribution around the diagonal with larger standard deviations of the theoretically calculated beta values.

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