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Initial reference-spectrum resistive wall mode feedback control simulation and multivariable design

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Introduction

In line with experimental results from the EXTRAP-T2R reversed field pinch (RFP) device, e.g. [1], a general control-oriented formulation of resistive wall mode (RWM) dynamics has been compiled. A state-space form is considered, where the control quantities directly correspond to cylindrical Fourier modes. A feedback controller strategy capable of steering Fourier modes along essentially arbitrarily preset reference spectrum trajectoria is proposed. Initial simulations are performed; hinting interesting possibilities. General mode control is a natural development of the PID-based local field annihilation and Fourier mode suppression techniques experimentally tested so far [2], where stabilization-only is the main objective. Specifically, the actual EXTRAP-T2R active- and sensor coil geometry and $m = 1$-wiring is investigated; in practice meaning 64 control signals and 64 sensor signals; capable of affecting and sensing polodially odd ($m = 1, 3, 5, \ldots$) RWMs, for a toroidal (n) resolution of $N = 32$; m, n both alias-infested due to coil discreteness. Model parameters have been checked and calibrated against experiments. It is demonstrated, assuming proper applicability of the single-pole ideal cylindrical RFP Fourier mode RWM model, that successful control of $(m = 1, n = -16 \ldots +15)$ is, in this context, straighforward and principially possible [3].

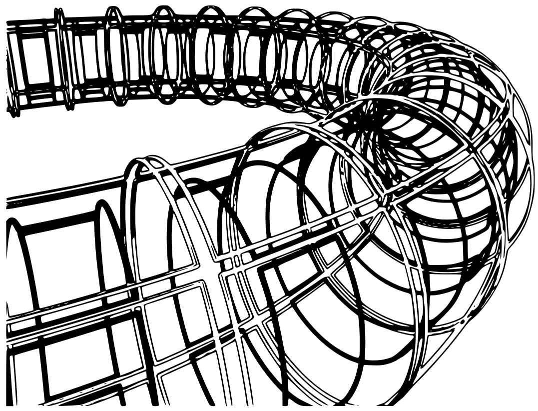

Figure 1: Cartoon of EXTRAP-T2R coil array geometry. Full surface coverage by active saddle coils. Half-width sensor coils inside vessel. Vessel not depicted.

Simulation results encourage development of the experimental controller system.

Modeling and model-based control design

Linear resistive-wall mode (RWM) reversed-field pinch (RFP) stability theory and analysis of experimental data agree at $\tau_{mn}\dot{B}_{mn} = \tau_{mn}\gamma_{mn}B_{mn} + B^{ext}_{mn}$ for a first-order radial field dynamics description in the cylinder approximation. Modal growth-rates γ_{mn} are calculated at a particular RFP equilibrium situation. Using planar Fourier series, Biot-Savart's and Faraday's laws this is repackaged to a general state-space format of the s.c. RWM-Plant

$$\begin{cases} \dot{x} &= Ax + Bu + Nv_1 \\ z &= Mx \\ y &= Cx + v_2 \end{cases} \tag{1}$$

suited for multivariable control design. Vector state x contains the truncated spectrum, arbitrarily filled with any selection of cylindrical Fourier modes $\{m,n\}$. Main point here is to include high poloidal mode numbers $m \geq 1$ to capture transient mode behaviour in the controller. With u being the signal bundle of 64 active coil currents and y 64 ideally integrated sensor coil voltages, the model represents the coil-interfaced EXTRAP-T2R RFP device. Matrix elements

$$\begin{array}{lll} A_{mn,m'n'} & \sim & \gamma_{mn}\delta_{mn,m'n'} \\ B_{mn,ij} & \sim & \tau_{mn}^{-1}\int_\Omega \left(\hat{\mathbf{r}}(\theta,\phi) \cdot \oint_{l_{ij}} \frac{\mathbf{dl}_{ij}\times(\mathbf{r}(\theta,\phi)-\mathbf{r}_{ij})}{|\mathbf{r}(\theta,\phi)-\mathbf{r}_{ij}|^3} \right) e^{-\imath(m\theta+n\phi)} d\theta d\phi \\ C_{pq,mn} & \sim & \int_\Omega f_{pq}(\theta,\phi) A_{pq}(\theta,\phi) e^{+\imath(m\theta+n\phi)} d\theta d\phi \end{array} \tag{2}$$

where δ is the Kronecker symbol. Matrix M has at most one 1 on each row selecting a specific mode for the controlled RWM spectrum subspace, with zeroed elements elsewhere. Typically $\dim v_1 = \dim x$ and diagonal $N_{mn,m'n'} = \tau_{mn}^{-1}\delta_{mn,m'n'}$, but there is plenty of room for improvements in modeling errors; e.g. coupled excitations. This scheme can furthermore easily handle dispersive field diffusion through the shell, introducing off-diagonal elements in matrix A, or any other linear coupling. Note that mn, ij and pq enumerates Fourier mode, active coil and sensor coil respectively. There are several simplifications to be made when the coefficients are calculated in a perfect cylinder geometry, and particularly when all coils are equally shaped. Only one line integral field is then needed; all other fields are yielded by translation (phase-shifts in spatial spectrum). The same goes for the Fourier coefficients of the sensor coil aperture and area functions f_{pq}, A_{pq}. The integration set Ω is here a full period of the toroidal surface $(\theta,\phi) \in [-\pi,\pi[\times[-\pi,\pi[$.

Theory for control of linear models (1) is well-developed [4] and a good starting point is the general multivariable LQ-synthesis. This mature technique systematically designs state-estimation filter K and feedback gain L matrices to minimize a quadratic cost functional of expected weighted state and control vector magnitudes subject to a Gaussian noise error assumption. The model-based LQ-optimal linear controller dynamics is

$$\begin{array}{lll} \dot{\hat{x}} &=& A\hat{x} + Bu + K(y - C\hat{x}) \\ u &=& -L\hat{x} + L_r r \end{array} \tag{3}$$

where matrix K usually is known as the Kalman gain, $\hat{x}$ is the RWM state-estimate, and $L_r = (M(BL-A)^{-1}B)^{-1}$ achieves unity gain for each closed-loop reference transfer channel $r_i \rightarrow z_i$ at static angular frequency $\omega = 0$.

Initial simulation results of reference-spectrum shots

The model (1)-(2) has been implemented in the `MATLAB/SIMULINK` computational environment, figure (2). It has been confirmed that aliasing emerges in both toroidal and poloidal bands for the discrete coil arrangements. This is typically seen by diagonalization of B and C by Singular Value Decomposition (SVD) methods when model mode-inclusion is appropiately large. The model has been specifically compiled and calibrated for the EXTRAP-T2R device. This

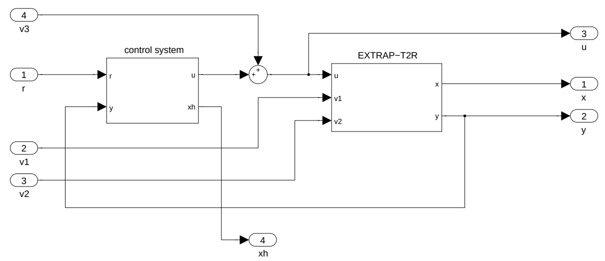

Figure 2: High-level view of `SIMULINK` setup. Model (1)-(2) feedback-controlled with system (3). Controller objective is to keep a subset of EXTRAP-T2R RWMs at the preset shot reference spectrum $r(t)$.

has been done by initial vacuum shot data, and then open-loop plasma shots. Model output can be reproduced very close to measured output by tayloring a field-error signal. Knowing typical amplitude and characteristic frequency content of the field-error is important for both control design and error-modeling, effectively outlining an internal plasma excitation process. An illustrative simulation computed for EXTRAP-T2R spectrum $m = \{1,3,5\}$, $n = \{-64,+63\}$ is shown in figure (3). The controller is based on a smaller model $m = \{1,3\}$, $n = \{-24,+23\}$. Process and measurement noises are of realistic intensities, w.r.t. experimental data. Here all RWMs are desired at zero amplitude except for $(m,n) = (1,-10)$, $(1,+2)$. These two modes are ramped up, phase-rotated and simultaneously scaled in amplitude, and then ramped down. It is seen that the control system approximately achieves its objective to keep the modes at their preset reference during the shot, and this is done with coil-currents confirmed safely below the EXTRAP-T2R power limit. This simulation however assumes a perfect amplifier and coil response and therefore only exhibits a proof of principle (a clean control problem). A key issue here is the excited aliased modes. These modes are stable and die out, but only in a static situation. Precision control of RWMs should take these modes into account. The Kalman filter (3)

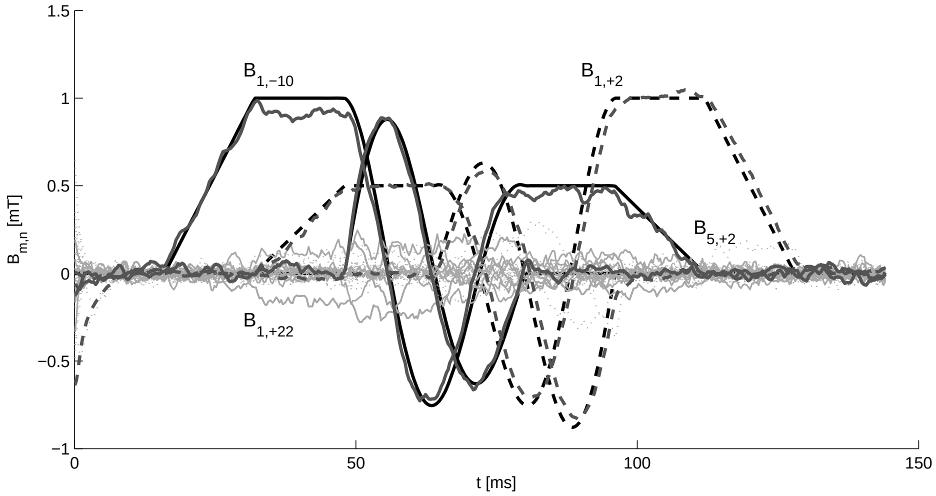

Figure 3: Reference spectrum tracked by control system. Two RWMs are simultaneously kept at non-zero amplitudes. Real- and imaginary Fourier coefficients are plotted, corresponding to horizontal and vertical components of the radial field B_r. Aliased modes are significantly excited during transients.

systematically does exactly this.

Discussion and outlook

High-performance monitoring and control of RWMs in EXTRAP-T2R may require model-based filtering and feedback. By tracking higher toroidal and poloidal modes (effectively subtracting them from the sensor signals) it is possible to accurately follow Fourier modes during active coil transients. EXTRAP-T2Rs spectra amplifies a broad repertoire of modes (w.r.t. vacuum shots) most still highly stable. Knowledge of modal growth-rates and field diffusion characteristic times might provide the information needed not to misinterpret sensor signals and wrongly apply coil currents.

General control of phase and amplitude for both stable and unstable RWMs might prove a useful instrument in future experimental work. Consideration of robustness issues, specifically while including non-ideal amplifier and coil dynamics is then high-priority.

References

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