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Experimental estimation of nonlinear energy transfer in two-dimensional plasma turbulence

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Two- and three-dimensional fluid turbulence is distinguished by the direction of energy transfer. In the second case the energy is transferred via a direct cascade from small to large scales, in the first case via an inverse cascade in the opposite direction, while the enstrophy is transferred by a direct cascade. Turbulence develops from the nonlinearity of the system by wave-wave interactions. For quadratic nonlinearities, this process is known as three-wave coupling. In a Fourier decomposition in terms of wave numbers $\mathbf{k}$, waves (or modes) can interact, which satisfy the constraint $\mathbf{k} = \mathbf{k_1} + \mathbf{k_2}$. A possibility to investigate the wave-wave interaction is bispectral analysis, which measures the amount of phase correlation between three spectral components.

The Kim method [1] models turbulence by the assumption, that the measured fluctuations satisfy the nonlinear wave coupling equation

$$\frac{\partial \phi(\mathbf{k},t)}{\partial t} = \Lambda^L_{\mathbf{k}} \phi(\mathbf{k},t) + \frac{1}{2} \sum_{\mathbf{k}=\mathbf{k_1}+\mathbf{k_2}} \Lambda^Q_{\mathbf{k}}(\mathbf{k_1},\mathbf{k_2}) \phi(\mathbf{k_1},t) \phi(\mathbf{k_2},t), \tag{1}$$

where $\phi(\mathbf{k},t)$ is the spatial Fourier spectrum of a measured quantity. From this the linear transfer function $\Lambda^L_{\mathbf{k}} = \gamma_{\mathbf{k}} + i\omega_{\mathbf{k}}$, composed out of the growth rate $\gamma_{\mathbf{k}}$ and the dispersion relation $\omega_{\mathbf{k}}$ can be calculated. The strength of coupling between the modes $\mathbf{k_1}$, $\mathbf{k_2}$ and $\mathbf{k}$ is given by the coupling coefficient or quadratic transfer function $\Lambda^Q_{\mathbf{k}}(\mathbf{k_1},\mathbf{k_2})$. Thus the method is appropriate to describe turbulence, dominated by three-wave coupling in a heuristic model, which does not consider multiple interacting fields. Quasi-linear effects (interactions between a mode and the background) are also neglected. By expanding the discrete form of Eq. (1) into a series of moment equations up to the fourth order with subsequent ensemble averaging $\langle \cdot \rangle$ the linear and quadratic transfer function are estimated. From the spatial Fourier spectrum the auto-, cross-bispectrum and the fourth order moment are calculated, from which the linear and quadratic transfer functions are estimated by a 2D version of Eqs. (15) and (16) in Ref. [1].

In both cases the nonlinear spectral power transfer

$$T_{\mathbf{k}}(\mathbf{k_1},\mathbf{k_2}) := \mathrm{Re}(\Lambda^Q_{\mathbf{k}}(\mathbf{k_1},\mathbf{k_2}) \langle \phi(\mathbf{k_1},t)\phi(\mathbf{k_2},t)\phi(\mathbf{k},t)^* \rangle) \tag{2}$$

results from power conservation [2]. The energy transfer function is calculated from this power transfer function by $(1+k_\perp^2)T_\mathbf{k}(\mathbf{k_1},\mathbf{k_2})$.

Tests on simulated Hasegawa-Wakatani turbulence [3] have shown, that the scheme is appropriate for the envisaged study. The analysis has been carried out at realistic values for the parallel resistivity, which was high enough to decouple density from potential, so that density is convected as a passive scalar with the flow. In the analysis of 2D density and potential fluctuation data with only a few modes taken into account the analytically known growth rate and dispersion relation have been recovered and clear evidence for the expected dual cascade was found [4].

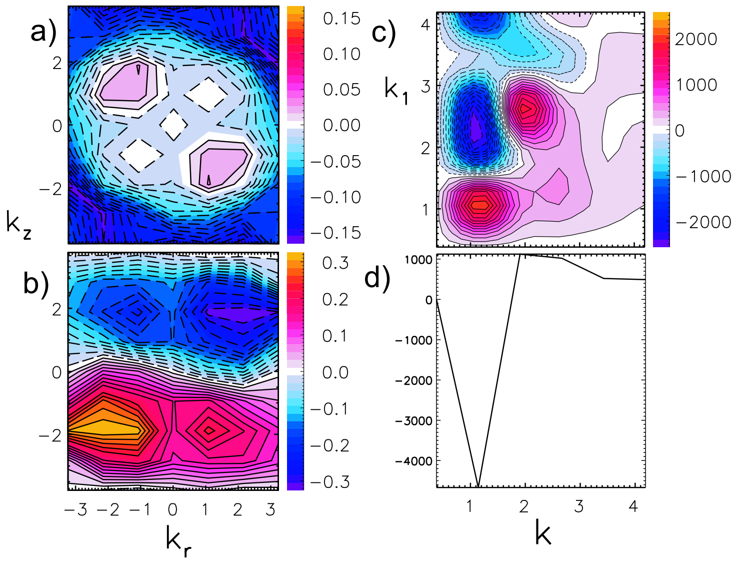

Figure 1: Growth rate (a), dispersion relation (b) and spectral power transfer (c,d) of density fluctuations in a helium discharge in TJ-K.

The experimental data analysed in this work are from the torsatron TJ-K [5], with a major radius of $R_0 = 0.6\,\mathrm{m}$ and the effective plasma radius of $a = 0.1\,\mathrm{m}$. The toroidally confined low-temperature plasma in TJ-K is dimensionally similar to the edge of fusion plasmas [6]. The excellent accessibility for probe arrays directly provides the possibility to measure spatial wave-vector spectra in 2D over long time series, which are necessary for bispectral analysis methods. The ion temperature is less than 1 eV and the electron temperature is about $T_e = 8$ eV. The working gas was helium at a neutral gas pressure of $p = 4\cdot10^{-5}$ mbar. Plasma is generated by microwaves at 2.45 GHz and 1.8 kW. The line-averaged density is about $\bar{n} = 10^{17}$ m^{-3} and the magnetic field strength is $B = 72$ mT.

Fluctuations in potential or density are measured with an 8×8 array of Langmuir probes with a spatial resolution of 1 cm in vertical and horizontal direction [7]. The density fluctuations $\tilde{n}$ are deduced from the ion-saturation current and the floating potential fluctuations $\tilde{\phi}$ are interpreted as plasma potential fluctuations, which has been shown to be valid for plasmas in TJ-K [8]. Data are recorded for 1 s at 1 MHz and 16 bit.

The fluctuation data are normalized as $r \rightarrow r/\rho_s$ and $t \rightarrow (c_s/L_n)t$, where $\rho_s = \sqrt{(m_iT_e)}/(eB)$ is the drift-scale parameter, $c_s = \sqrt{T_e/m_i}$ the sound speed and L_n is the density gradient length.

According to this the normalized fluctuation parameters are

$$\tilde{\phi} \rightarrow \frac{L_n}{\rho_s}\frac{e\tilde{\phi}}{T_e}, \qquad \tilde{n} \rightarrow \frac{L_n}{\rho_s}\frac{\tilde{n}}{n_0}. \tag{3}$$

The equilibrium density n_0 and temperature profiles T_e are recorded with a radially movable probe. From these measurements, $\rho_s = 0.012$ m and $L_n = 0.08$ m are obtained. To investigate stationarity, the data sets are separated into four ensembles of 250.000 realisations each. Considering an auto correlation time of about 50 μs, this corresponds to 5000 independent realisations. For every time step, the fluctuations are spatially Fourier transformed.

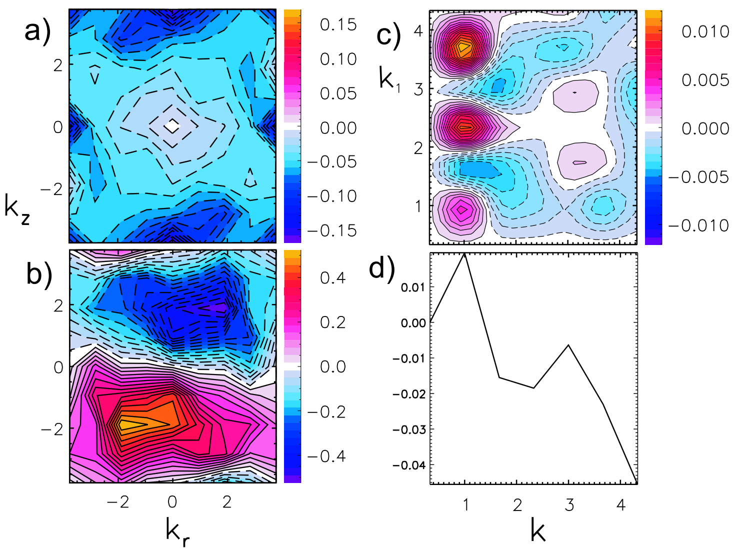

Figure 2: Same as in Fig. 1 for potential fluctuations.

The growth rate of the density fluctuations (Fig. 1a) increases slightly to local maxima at $(k_r, k_z) \approx (-2.0, 1.0)$ and $(2.0, -1.0)$ and then decreases steeply for small scales. This is typical for the growth rate and has previously been studied in experiments [2, 9] and simulations [10]. The transfer functions are the same for the four ensembles. Thus the turbulence is saturated and the number of realisations is sufficient. The dispersion relation in Fig. 1b has two maxima at $(-2.0, -2.0)$ and $(1.0, -2.0)$, and it is antisymmetrical around zero.

Comparing this to the potential fluctuations (top of Fig. 2), the growth rate still decreases for large k, but is more flat. The dispersion relations look quite similar. The global extremal values are the same, whereas the local ones do not appear. The absolute values are of the same magnitude. According to the dispersion relation of the Hasegawa-Wakatani equations, which is given by [10]

$$\omega = \frac{(1/L_n)k_y - i\mu k^4 + (1/C)(-\mu\omega k^4 + i\omega^2 k^2)}{1 + k^2} \tag{4}$$

one notices a great structural similarity, because the Hasegawa-Wakatani turbulence and the TJ-K plasma have drift waves as the driving instability [6]. Here μ is the viscosity and C is the adiabatic parameter.

In order to obtain an appropriate visualisation of the four-dimensional $(k_x, k_y, k_{1x}, k_{1y})$ spectral power and energy transfer functions, sums of all contributions at given $|k|$ and $|k_1|$ have been taken and divided by the number of contributions. In Fig. 1c,d and Fig. 2c,d the result for the energy transfer functions as a function of the absolute values of the wave vectors $\mathbf{k}$ and $\mathbf{k_1}$ are shown. An evidence of the direction of the enstrophy transfer is the direction of the spectral power transfer of the density fluctuations (free energy). In incompressible fluid turbulence, which corresponds to the plasma dynamics perpendicular to the magnetic field, the density perturbations are advected by the flow as a passive scalar [11].

The free energy transfer of the density fluctuations, as shown in Fig. 1d, is transferred from $k \approx$1.1 to $k \approx$1.9 by a direct cascade. As shown in detail in Fig. 1c, the power is locally transported from $(k, k_1) \approx$(1.1,2.8) to (1.9,2.8). Considering the $E \times B$ energy transfer calculated from potential fluctuations in Fig. 2, the absolute value of the $E \times B$ energy transfer is much smaller than that of the free energy, which is consistent with the result from computations in the strongly non-adiabatic regime [12]. Fig. 2d shows, that most of the $E \times B$ energy is transferred <u>from</u> $(k, k_1) \approx (4.3, 3.0)$ and $(3.7, 1.0)$, and most $E \times B$ energy is transferred <u>to</u> $(k, k_1) \approx (1.0, 3.7)$ and $(1.0, 2.3)$. Hence the $E \times B$ energy transfer shows an inverse cascade.

In conclusion, two-dimensional density and potential fluctuation data from helium discharges have been investigated. For the first time, the two-dimensional energy transfer has been calculated from measured fluctuation data. For the $E \times B$ energy transfer of the potential fluctuations an inverse cascade and for the spectral power transfer of the density fluctuations a direct cascade is observed. This can be seen as an experimental evidence of the dual cascade in magnetized plasma turbulence.

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