Simulation of Raman Backscattering Instability in the Interaction of Two Laser Beams in Plasma

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In this investigation the suitable equations for describing the nonlinear dynamics of the interaction of two laser pulses in cold plasma are obtained. For deriving the equations of amplitudes of laser pulses and motion of electrons are assumed that wave's envelopes are slowly varying. A numerical code has been developed to simulate Raman backscattering instability when the group velocities of electromagnetic waves very less than light velocity.

I. Introduction

Stimulated Raman scattering (SRS) [1-4] is a parametric instability in which an incident electromagnetic wave decays into an electron plasma wave and a scattered light wave. Recent interest in the interaction of laser pulses with plasma has motivated a number of SRS experiments around the world [5]. A novel scheme of amplifying a short laser pulses in plasma was proposed by Shvets et al. in which a long pump laser transfers its energy to a short seed pulse via the Raman backscattering (RBS) instability [6]. Most of the analytical and numerical works on RBS have concentrated on the cases, in which the group velocity of waves roughly equal light velocity [6-8]. In this paper RBS with arbitrary group velocity is studied.

II. Derivation of the equations

In order to obtain the suitable equations for the interaction of laser pulses in the cold plasma we use the approach developed in plasma electronics and free electron lasers [9]:

$$\frac{\partial^2 A_\perp}{\partial t^2} - c^2 \frac{\partial^2 A_\perp}{\partial z^2} = 4\pi c j_\perp \qquad (1) \qquad \frac{\partial^2 \varphi}{\partial z^2} = -4\pi\rho \qquad (2) \qquad j_\perp = e n_0 \frac{\lambda}{N} \sum_j V_{\perp j} \delta(z - z_j) \qquad (3)$$

$$\rho = e n_0 \left(\frac{\lambda}{N} \sum_j \delta(z - z_j) - 1 \right) \qquad (4) \qquad V_{\perp j} = -\frac{e}{mc} \frac{A_\perp(z,t)}{\gamma_j} \bigg|_{z = z_j} \qquad (5) \qquad \frac{dz_j}{dt} = v_{\parallel j} \qquad (6)$$

$$\frac{dv_{\parallel j}}{dt} = -\frac{e}{m} \frac{1}{\gamma_j \gamma_{\parallel j}^2} \frac{\partial \varphi}{\partial z} - \frac{1}{2} \left(\frac{e}{mc\gamma_j} \right)^2 \left(\frac{\partial}{\partial z} + \frac{v_{\parallel j}}{c^2} \frac{\partial}{\partial t} \right) |A_\perp|^2 \qquad (7) \qquad \gamma_j = \gamma_{\parallel j} \left(1 + \frac{1}{c^2} (\frac{e}{mc})^2 A_\perp^2 \right)^{1/2} \bigg|_{z = z_j} \qquad (8)$$

$$\gamma_{\parallel j} = \left(1 - (v_{\parallel j}/c)^2 \right)^{-1/2} \qquad (9) \qquad A_\perp = A_x + iA_y ,\ V_{\perp j} = v_{xj} + iv_{yj} \qquad (9)'$$

here: A_x, A_y, v_{xj}, v_{yj} are the transverse components of vector potential and velocity of jth electron. λ is the characteristic scale length, N number of macro particles within λ, $z_j(t), v_{\parallel j}(t)$, are the longitudinal coordinate and velocity of jth electron. We assume two circularly polarized electromagnetic waves with following total vector potential:

$$A_{\perp} = A_0 \exp(-i\omega_0 t + ik_0 z) + A_s \exp(-i\omega_s t + ik_s z) \quad (10)$$

Where the 0,s indexes are for pump and seed lasers, respectively. The interaction of these electromagnetic waves in plasma causes the excitation of plasma wave with frequency $\omega_l = \omega_0 - \omega_s$ and wave number $k_l = k_0 - k_s$. The scalar potential φ can be written as:

$$\varphi = (\tilde{\varphi}/2)\exp(-i\omega_l t + ik_l z) + c.c. \quad (11)$$

We insert Eq. (10) into Eq. (1), multiply the resulting equation by $\exp(i\omega_0 t - ik_0 z)$ ($\exp(i\omega_s t - ik_s z)$), and integrate it respect to z in the interval $[0, \lambda = 2\pi / k_l)$. In this way, after omitting the second derivatives we can obtain:

$$\begin{cases} \frac{\partial a_0}{\partial t} + V_{g0}\frac{\partial a_0}{\partial z} + \frac{i\omega_p{}^2}{\omega_0}(<\gamma^{-1}> -1)a_0 = -\frac{i\omega_p{}^2}{4\omega_0}\hat{\rho}a_s \\ \frac{\partial a_s}{\partial t} + V_{gs}\frac{\partial a_s}{\partial z} + \frac{i\omega_p{}^2}{\omega_s}(<\gamma^{-1}> -1)a_s = -\frac{i\omega_p{}^2}{4\omega_s}\hat{\rho}^* a_0 \end{cases} \quad (12)$$

Where $V_{gi} = k_i c^2 / \omega_i$, $(i = 0, s)$ is the group velocity, $\omega_p = \sqrt{4\pi n e^2 / m}$ plasma frequency and:

$$<\gamma^{-1}> = \frac{1}{N}\sum_{j(z)}\frac{1}{\gamma_j}, \qquad \hat{\rho} = \frac{2}{N}\sum_{j(z)}\frac{\exp(i\omega_l - k_l z_j)}{\gamma_j}, \qquad a_i = \frac{|e|}{mc^2}A_i \quad (i = 0, s) \quad (13)$$

The summation in the above expressions is carried out over those particles (electrons) whose coordinates lie within the region $z_j \in [z - \lambda/2, z + \lambda/2)$. Now with substituting Eq. (11) into Eq. (2), and doing the same averaging procedure we can fined $\tilde{\varphi}$. Substituting $\tilde{\varphi}$ in the equation of motion of the electrons leads to:

$$\frac{dv_{\parallel j}}{dt} = -\frac{i\omega_p{}^2}{2k_l\gamma_j\gamma_{\parallel j}{}^2}\left(\rho\exp(-i\omega_l t + ik_l z_j) - c.c.\right) - \frac{iq_j}{2\gamma_j{}^2}\left(a_0 a_s{}^* \exp(-i\omega_l t + ik_l z_j) - c.c.\right) \quad (14)$$

Where: $$q_j \equiv k_l c^2 - \omega_l v_{\parallel j}, \qquad \rho = \frac{2}{N}\sum_{j(z)}\exp(i\omega_l t - ik_l z_j) \quad (15)$$

The Eqs. (12) and (14) together with Eqs. (8),(9),(13) and (15) establish the perfect set of equations that describes the nonlinear evolution of laser pulses in SRS.

III. Numerical results

We introduce the following dimensionless quantities and variables:

$$\overline{\omega}_i = \frac{\omega_i}{\omega_p}, \quad \overline{k}_i = \frac{k_i c}{\omega_p}, \quad \overline{V}_{gi} = \frac{V_{gi}}{c} = \frac{\overline{k}_i}{\overline{\omega}_i} \quad (i = 0, s), \quad \tau = \omega_p t, \quad \xi = \frac{\omega_p}{c}z, \quad \eta = \frac{v_{\parallel j}}{c} \quad (16)$$

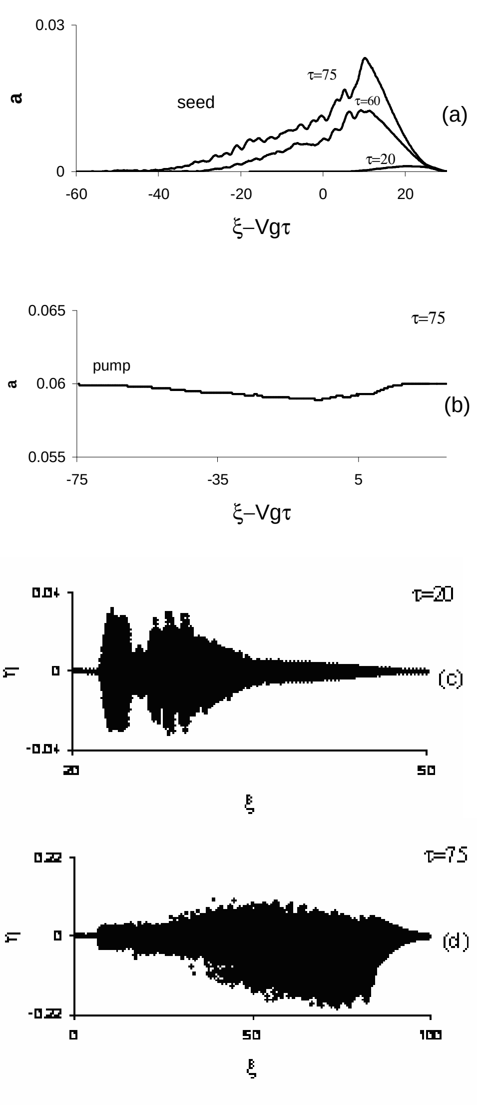

Fig. 1. Dynamics of the Raman amplification of a Gaussian seed pulse (a), pump depletion (b) and phase planes (c), (d) when $a_s = 0.001, a_0 = 0.06, \overline{\omega}_0 = 10$

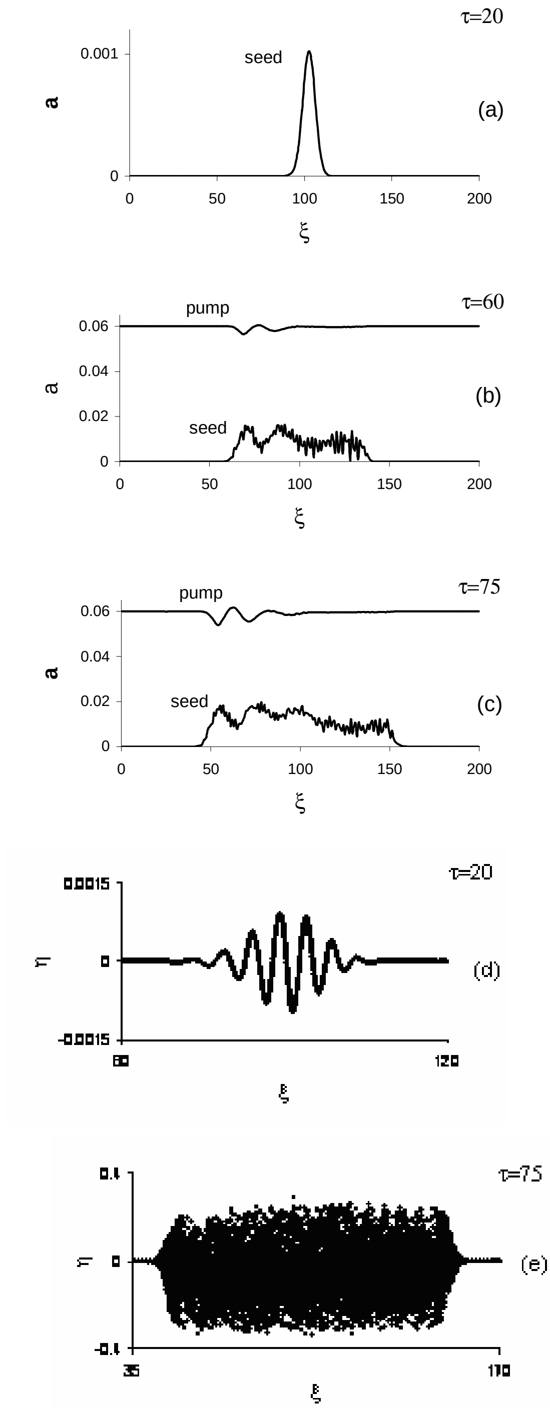

Fig. 2. Dynamics of the Raman amplification of a Gaussian seed pulse (a), (b), (c) and phase planes (d), (e) when $a_s = 0.001, a_0 = 0.06, \overline{\omega}_0 = 2.01$

Some results of numerical simulation of the pulse dynamics during the Raman instability are illustrated in Figures 1 and 2. The seed pulse is right-going ($V_{gs} > 0$) and the pump wave is left-going ($V_{g0} < 0$). For initial seed pulse is considered a Gaussian form. We assume the resonance condition for three waves interaction: $\omega_0 - \omega_s = \omega_p$. Fig. 1 shows the results of simulation with high group velocities ($V_{gi} \approx c$). The front point of the pulse moves with the velocity $\bar{V}_{gs} \approx 1$. The pump laser begins to be depleted as the seed pulse grows and nonlinear interaction between the three waves starts. Energy transfer is almost completely in one direction: from the pump to the seed (Figs.1-a and 1-b). In Figs. 1-c and 1-d display the phase planes of the electrons at two different dimensionless times. Fig. 2 shows the amplification of seed laser and the depletion of the pump when the group velocities less than the light velocity: $\bar{V}_{g0} = -0.87$ and $\bar{V}_{gs} = 0.14$. In the initial stage ($\tau < 20$) the velocity of electrons less than V_{gs} (see Fig. 2-d) and the front point of the pulse moves with the group velocity V_{gs}. But in the latter stages the velocity of electrons grows up V_{gs} and this effect leads to the fast propagation of laser pulse (the velocity of front point of pulse is nearly c). Where the seed amplitude is significant the back of the seed gives energy to the pump (see Figs. 2-b and 2-c). In Figs. 2-d and 2-e are shown the phase planes. It is obvious that the velocity of electrons in the past time is greater than the similar time in the previous case.

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