From linear perturbation computation to fusion yield prediction: application to ICF double shell targets

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Fusion yield of inertial confinement fusion (ICF) targets may be impaired by the mixing which occurs at the pusher-fuel interface as a result of hydrodynamic instabilities. Numerical simulations of these intrinsically 3D instability and mix phenomena are extremely expensive in terms of computational burden.

As an alternative approach, we propose a suite of purely 1D methods—CALIMERO—consisting of: a linear perturbation code—SILEX [1]— for computing linear instability growth, the use of a hydrodynamic instability saturation model [7] for estimating the mixing zone time evolution, and a 1D mix model—MeDiC-2F [8, 9]—for performing fusion yield calculations.

**Linear perturbation code SILEX**

The retained physical modeling consists of a single-temperature fluid model for nonlinear heat-conducting, inviscid fluids, described by

\[
D_t \frac{1}{\rho} \frac{\partial}{\partial \rho} \vec{v}, \quad D_t \vec{v} = -\frac{1}{\rho} \vec{\nabla} p, \quad D_t e = -\frac{1}{\rho} \text{div} \left( p \vec{v} + \vec{\Psi} \right),
\]

where \( \vec{\Psi} = -\kappa \vec{\nabla} T \). Fluid conductivities are supposed to be of the form

\[
\kappa = K(\rho, T) T^\nu, \quad \nu \in \mathbb{R}^+, \quad (1)
\]

where \( K \) is an analytical function or tabulated data. Equations of state may be analytical functions (perfect or stiffened gas, . . .) or tabulated data interpolants provided they be continuously differentiable. The proposed linear perturbation computation method is based on a Lagrangian description of a planar (spherically) symmetric 1D mean flow and its 3D linear perturbations. Both the nonlinear equations for the mean flow and the linear equations for its perturbations are incompletely parabolic systems of 1D equations. Through operator splitting, these two systems are decomposed into reduced hyperbolic systems and parabolic scalar equations. Mean flow discontinuities (shock waves), inducing Dirac singularities for the linear perturbations, call for
appropriate numerical schemes when computing solutions of such hyperbolic systems, namely Godunov-type schemes [3, 4]. The nonlinear/linear parabolic equations are solved using classical iterative/direct methods [2]. This code has turned out to produce reliable and fairly accurate results of hydrodynamic instability evolutions [3].

**MeDiC-2F: a 1D diffusive mixing model**

Starting from the 1D two-fluid mixing model proposed by Saurel and Gavrilyuk [6] where each fluid occupies its own subvolume with its own density, velocity, temperature and pressure, the two-fluid flow is split between a mean flow and a relative flow. The resulting system of equations is closed assuming that mass and enthalpy exchanges are modeled by a diffusive process; fluids are in an isothermal and isobaric equilibrium; momentum exchange and its work are neglected; the mixing ion (i) and electron (e) species thermal fluxes are modeled by an effective thermal flux $\kappa_s^{\text{eff}} \partial_r T_s$ with $s = i, e$. The resulting system of equations describing flows in planar or spherical symmetry ($a = 0, 2$) reads

$$\rho D_r \frac{1}{r} \partial_r (r^a U) = 0, \quad \rho D_r c - \frac{1}{r^a} \partial_r (r^a \partial_r c) = 0, \quad \rho D_r U + \partial_r p = 0,$$

$$\rho D_r \varepsilon_s + p_s \frac{1}{r^a} \partial_r (r^a U) - \frac{1}{r^a} \partial_r (r^a \rho \partial_r \Delta h_s \partial_r c) = \frac{1}{r^a} \partial_r \left( r^a \kappa_s^{\text{eff}} \partial_r T_s \right) + S_s, \ s = i, e,$$

$$D_r E_{\text{rad}} + E_{\text{rad}} \frac{1}{r^a} \partial_r (r^a U) + p_{\text{rad}} \frac{1}{r^a} \partial_r (r^a U) = \frac{1}{r^a} \partial_r (r^a \kappa_{\text{rad}} \partial_r T_{\text{rad}}) + S_{\text{rad}},$$

where $\rho$ is the mix density, $c$ the fuel mass fraction, $U$ is the mass center radial velocity, $p$ is the pressure, $\varepsilon_s$ is the ion/electron internal energy, $E_{\text{rad}}$ is the radiative energy and $\Delta h_s$ is the difference between the shell and the fuel (ion/electron) enthalpies.

The diffusive coefficient $\varnothing$ is defined as a function of the mixing half lengths $L_{\text{mix}}^-$ ($-$ for $r < R_{\text{int}}$ and $+$ for $r > R_{\text{int}}$), the spatial location $r$ and the interface mean location $R_{\text{int}}$ [8]:

$$\varnothing^\pm = \max \left( \frac{1}{2\xi^2} L_{\text{mix}}^- d_t L_{\text{mix}}^+ \left( 1 + \frac{r - R_{\text{int}}}{L_{\text{mix}}^-} \right) \left( 1 - \frac{r - R_{\text{int}}}{L_{\text{mix}}^+} \right) \frac{\bar{\rho}^\pm}{\rho}, 0 \right),$$

with $\xi$ a model constant and $\bar{\rho}^\pm$ the mean density on the corresponding mixing zone. The ion/electron effective conduction coefficients are defined by the relation $\kappa^{\text{eff}}_s = f \kappa_s$ where $f \geq 1$ account for the interfacial transverse thermal flux and 1D averaging effects [8]:

$$f = 1 + k_{\text{eff}}^2 \left( L_{\text{mix}}^+ + L_{\text{mix}}^- \right)^2 \frac{(2R_{\text{int}} + L_{\text{mix}}^- - L_{\text{mix}}^+)}{2\pi R_{\text{int}}^3} (4c (1 - c)),$$

with $k_{\text{eff}}$ the effective mode modeling of the interface wrinkling. When the interface wrinkling exceeds a certain threshold, the transverse thermal gradient may be neglected and the functions $f$ is set to 1. The mixing half lengths $L_{\text{mix}}^\pm$ and the corrugation $k_{\text{eff}}$ are provided by the previous stage of the CALIMERO suite as described in figure 1.
Application to indirect-drive high-Z pusher target implosions

A first application has been to compare instability growths and their effect on the thermonuclear yield for the same double-shell target (see figure 2) irradiated by either a high-power (I1) or a low-power (I2) laser pulse. We focus on the pusher-DT interface from the start of the implosion up to ignition time, so that only the DT core and part of the pusher layer are treated. Time-dependent heat flux and pressure recorded at an interior point of the pusher in a reference multi-group radiation diffusion calculation were taken as mean-flow boundary conditions at the pusher external surface in SILEX computations.

![Figure 2: target design](image)

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Figure 3: CALIMERO results

Thanks to the reduced cost of linear perturbation computations, complete instability growth factors (see figure 3.a) —— for Legendre mode numbers up to several hundreds with initial grids such that the number of points per wavelength exceeds 100 —— have been computed and convolved with a "real" internal pusher interface defect spectrum, here the NIF cryogenic CH-ablator specification with a total roughness of 10 nm for modes greater than 10 [5].

The obtained saturated growth (see figure 3.b) is larger with the low-power pulse because the

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![Image of CALIMERO results](image)
Rayleigh-Taylor deceleration phase lasts longer, inducing a lower fusion yield (see figure 3.c). We can also note that this kind of target is very sensitive to the initial pusher roughness due to the mass combustion of the DT fuel.

**Conclusion**

The CALIMERO suite of 1D methods is dedicated to perform calculation of ICF target fusion yield including mixing at fluid interfaces. CALIMERO can give reliable and rapid results in the framework of sensitivity studies to laser pulse, as in the present work, or target geometry characteristics. It is an interesting alternative to time consuming multi-dimensional calculations during the process of ICF target design.

Since in the fusion capsules the mixing zone induced by hydrodynamic instabilities has not enough time to become fully turbulent, the linear and non-linear stages of the interface perturbation amplification require particular attention. This explains the choice of a linear perturbation code, SILEX, to describe the crucial first stage of amplification. In MeDiC-2F calculations, the last component of CALIMERO, the effect of the interpenetration of the two fluids at large scales is mimicked by diffusion processes. This treatment —only available for material fronts, with the exception of ablation fronts— has been already successfullly compared to 2D simulations of ICF capsules with perturbed interfaces [8].

The planned improvements in the CALIMERO suite are the introduction of a multi-temperature fluid description in SILEX and the modeling of the momentum exchange in MeDiC-2F.

**References**


