Interactions of moving charges with 2D strongly coupled dusty plasmas

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Introduction: We use both analytical theories and numerical simulations to study interactions of a charged particle with a 2D strongly coupled dusty plasma (SCDP), with an emphasis on forces acting on the projectile due to polarization of the system, namely, the stopping force and its fluctuation and the image force, which have been defined in Fig. 1.

Such a problem is of fundamental interest and also of importance in understanding related experiments, where interactions between a monolayer of plasma crystal with an incomplete layer of particles levitating right beneath crystal layer are concerned \citep{1, 2}. Our model system is a simplification of these real experiments. We consider here a monolayer of highly charged dust particles, with an external dust particle moving parallel to it, immersed in a large volume of ion-electron plasma. The displacements $\mathbf{r} = \{x, y\}$ of dust particles are confined to the plane $z = 0$ in a Cartesian coordinate system with $\mathbf{R} = \{x, y, z\}$, and electrons and ions provide only Debye screening, responsible for the Yukawa-like inter-particle potential. Such a system is fully characterized by the following two parameters: the coupling parameter $\Gamma = (Ze)^2/(ak_B T) \geq 1$, where $k_B T$ is the system temperature and $a$ is the average inter-particle distance, and the screening parameter $\kappa = a/\lambda$, where $\lambda$ is the (Debye) screening length.

Simulation: Our simulation is based on the so-called Brownian Dynamics (BD) method \citep{3}, in which we track the Brownian motions of a number of charged dust particles in a rectangle with periodic boundary conditions. Our simulation consists of two steps. Firstly, no external perturbation is applied on the system, consisting of $N = 1600$ charged particles, which are initially placed at random positions in the computational domain. They then undergo Brownian motions while interacting with each other, till the system reaches the equilibrium. This calcula-
tion reveals some dynamical and static properties of the system, such as the radial distribution function (RDF), which will be used as an input of our theoretical analysis in the next section.

In the second step, a charged particle or simply a test particle (TP), which carries $Z_t$ elementary charges, is projected horizontally into the system at a constant height $h$ over or beneath the dust layer with velocity $v_t$. The details of the interactions between the TP and all particles in the dust layer are recorded. In particular, the force $F_t$ exerted on the TP by the dust particles is measured directly. Fig. 2 shows some examples of direct measurements of the force along the moving direction of the TP, i.e., $-\hat{v}_t \cdot F_t$, when the TP is moving from left-hand side to the right-hand side of the dust plane. We then do the time average to these forces, and the resulting cumulative distributions are given in Fig. 3(a), where one may clearly see that the averages all go to steady values that depend on the projectile velocity. The same tendency may also be found in the average of the standard deviation of the forces, as is shown in Fig. 3(b), as well as in the average of image force.

Note that the steady values: the means of forces, give actually the so-called stopping force, while those of the standard deviation of the forces give the so-called stopping force straggling or energy loss straggling. So-measured stopping force and image force will be compared with those obtained from analytical theories nextly.

**Theoretical analysis:** Our theoretical analysis is based on linear dielectric response theory, in which the dust layer is treated as a dielectric medium and its response to external electromagnetic perturbation is fully determined by its dielectric response function. In our case, the perturbation comes from a charged particle out of the plane. It will
then disturb the 2D dusty plasma and thereby generate a density perturbation (space-charge distribution or polarization). We are here concerned only with a linear response, so that the potential field $\Phi_{\text{ind}}(r, z, t)$ associated with such a space-charge distribution (or the induced potential) may be written in Fourier space as

$$
\Phi_{\text{ind}}(k, k_z, \omega) = \frac{1}{\varepsilon_L(k, \omega)} \Phi_{\text{ext}}(k, k_z, \omega) - 1,
$$

where $\Phi_{\text{ext}}$ is the Yukawa-like external potential[3], and $\varepsilon_L(k, \omega)$ is the longitudinal dielectric response function of the 2D dust layer.

$\varepsilon_L(k, \omega)$ may contain the microscopical details of particle interactions and structures in the system, depending on the theoretical model to be used. Here to take into account the short range correlation effect in the dust layer, we base the analysis mainly on the so-called quasilocalized charge approximation (QLCA) [4], in which $\varepsilon_L(k, \omega)$ may be expressed as a functional of the equilibrium RDF of the dust layer.

Given the expression of $\varepsilon_L(k, \omega)$ in QLCA [4], one may apply reverse Fourier transform to Eq. (1), and obtain $\Phi_{\text{ind}}(r, z, t)$, while the stopping and image forces are defined respectively by

$$
S(v_t) \equiv eZ_t \frac{\partial \Phi_{\text{ind}}}{\partial x} \bigg|_{z=h, r=v_t}, \quad \text{and} \quad I(v_t) \equiv eZ_t \frac{\partial \Phi_{\text{ind}}}{\partial z} \bigg|_{z=h, r=v_t}.
$$

In Fig. 4 we show the (normalized) stopping force $S$ and its straggling versus the speed $v_t$, obtained from the BD simulation, and the QLCA model, for different heights of the TP. Characteristic hill shapes with peaks at the TP speeds around the dust acoustic speed $v_s$, indicate the onset of resonance effects [3].

One may also find that while the agreement between the QLCA and MD data is very good at all speeds for distances $h \geq a$, this agreement deteriorates spectacularly when the projectile gets closer to the dust layer than the average inter-particle spacing $a$. This disagreement may be well explained as a result of the nonlinear effects during the interaction[3].

The same tendency may be found in the behavior of the image force, as is shown in Fig. 5, the normalized image force versus the speed $v_t$, obtained from the BD simulation, and the
QLCA and standard Vlasov random phase approximation (RPA). Putting aside the first figure in Fig. 5, we see that one of the most pronounced features in Fig. 5 is that the QLCA results agree reasonably well with those from the MD simulations at all speeds, whereas the low-speed losses are heavily suppressed in the RPA. This deficiency of the RPA results is seen to increase with $\Gamma$, on the account of the correlation effects [3].

The nonlinear effects may be assessed by adopting the parameter $\Theta(h,r,v_t)$ [3], which indicates the coupling strength between the TP and dust particles. Weak (linear) perturbation is realized for $\Theta \ll 1$, whereas $\Theta > 1$ indicates the onset of the nonlinear effects during the interaction. This qualitatively explains the good as well as bad agreement between the QLCA and BD data observed in these figures.

**Conclusion:** We use both analytical theories and numerical simulations to study interactions of a charged particle with a 2D SC/PD, when the particle is moving parallel to the system. The emphasis is put on forces acting on the projectile due to polarization of the system. In particular, the stopping force and its straggling and image force are studied in detail. Strong correlation effects in the dust layer and nonlinear effects during the interactions are clearly demonstrated by comparing the results obtained from the BD simulation, the QLCA and RPA models.

**Acknowledgement:** This work was supported by NSERC and PREA (Z.L.M.).

**References**


